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G.S.M: a Grating Scale Monitor for atmospheric turbulence measurements

II. First measurements of the wavefront outer scale at the O.C.A*.

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Abstract. The spatial and temporal properties of atmospherically degraded wavefronts are relevant for high angular resolution techniques like adaptive optics or long baseline interferometry. The wavefront outer scale has been estimated on two sites by a new instrument described in the previous paper. The normalized covariance of angle of arrival fluctuations at two wavefront points separated by a variable distance from 0.14 m to 10 m is measured and compared to theoretical set of curves computed in the framework of Von Karman or Greenwood–Tarazano turbulence models. At the Calern site of Observatoire de la Côte d'Azur (O.C.A.) G.S.M. was installed near the long–baseline interferometer GI2T. The results from two nights show that in stable seeing conditions, and taking in account the models, the normalized angle of arrival covariance corresponds to turbulence outer scale $\mathcal{L}_0 = 300m$. Systematic deviation from the models was observed on another night, corresponding to $\mathcal{L}_0$ from 10 m to 300 m. Similar results were obtained on two nights at the Mont Gros site of O.C.A., giving $\mathcal{L}_0 = 50m$. Extensions of the technique and plans are briefly discussed.

Key words: Atmospheric effects – Methods: observational – Site testing

1. Introduction

In a first paper (Martin et al. 1994) the G.S.M. experiment intended to measure atmospheric parameters relevant for high angular resolution interferometry (Roddier 1983; Roddier & Léna 1984a) has been presented. As it has been recalled, one of these parameters is the outer scale $\mathcal{L}_0$ of the perturbed wavefront incident on the ground. It can be physically interpreted as a global outer scale of turbulence in the whole atmosphere (Borgnino 1990) taking into account altitude profile $L_0(h)$ of turbulence outer scales in different layers and a profile of the structure coefficient $C_N^2(h)$ of the air refractive index fluctuations which characterizes the strength of the turbulence at an altitude h.

Determination of $\mathcal{L}_0$ is essential for long baseline interferometry because its values will influence the signal to noise ratio that can be expected from a passive interferometric experiment. The standard deviation of the phase fluctuations is :

$$\sigma = 2.62\,(d/r_0)^{5/6}\,, \tag{1}$$

for short baselines d where Kolmogorov's law is assumed to be valid, i.e. in the inertial range (Roddier 1981). When the baseline is increased beyond the wavefront outer scale $\mathcal{L}_0$, the phase fluctuations saturate at the level (Roddier 1983)

$$\sigma_{max} \propto 2.62\,(\mathcal{L}_0/r_0)^{5/6}\,, \tag{2}$$

The distance where the scale of turbulence ceases to obey to Kolmogorov's law can be smaller than the effective baseline d of modern long baseline interferometers. The fact that the usable optical bandwidth $\Delta\lambda$ in interferometers is limited by the spectral coherence and varies as σ^{-1} leads to an expression :

$$\Delta\lambda \propto {\sigma_{max}}^{-1} \propto {\mathcal{L}_0}^{-5/6} \tag{3}$$

in the case when $d > \mathcal{L}_0$. This expression shows clearly that the lower is the value of $\mathcal{L}_0$, the higher will be the allowable bandwidth, and consequently the signal to noise

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Periods	References	Values of $\mathcal{L}_0$ (m)	Techniques	Instruments
1984	Mariotti et al.	~ 8	Interferometer	I2T (CERGA)
1987	Colavita et al.	>2000	Interferometer	Mark III (Mt–Wilson)
1988	Beland et al .	0.1–1	*In situ* soundings	Radiosonds and meteorology
1988	Coulman et al.	~ 5	*In situ* soundings	SIDAR and meteorological radiosond data
1989	Tallon	5–8	Wavefront analysis	Hartmann–Shack
1990	Borgnino	~ 8	*In situ* soundings	Average $C_N^2(h)$ profile (Hufnagel)
1991	Rigaut et al.	~ 50	Wavefront analysis	COME ON
1991	Buscher et al.	30	Interferometer	Mark III (Mt–Wilson)
1993	Ziad et al.	$5 \sim 100$	Wavefront analysis	Hartmann–Shack (OHP)

Table 1. Summary of previous results of turbulence outer scale measurements

ratio in a long baseline interferometric experiment without fringe tracking. This is the fundamental reason why many authors have been interested in measuring the value of this parameter.

Let us see now what are the values of $\mathcal{L}_0$ obtained by different authors using several methods (Table 1). A brief review of these methods has been recently given by Ziad et al. (1994). They can be classified into two groups: (i) indirect methods where the external scale of turbulence is estimated from *in situ* measurements of $C_N^2(h)$ in the atmosphere (Beland & Brown 1988; Borgnino 1990; Coulman et al. 1988), and (ii) direct measurements by interferometric wavefront sensing (Mariotti & Di Benedetto 1984; Colavita et al. 1987; Buscher et al. 1991) or angle of arrival analysis (Tallon 1989; Rigaut et al.1991; Rousset et al. 1991; Conan et al. 1992; Ziad et al. 1994).

The large dispersion of these values explains why we felt necessary to build G.S.M, a monitor devoted to $\mathcal{L}_0$ measurements. The estimates of this parameter deduced from recent experiments at the Calern and Mont Gros sites of O.C.A. are presented below after a brief theoretical treatment of longitudinal and transverse covariances of atmospheric angle of arrival fluctuations.

2. Theory

In a G.S.M. experiment the angle of arrival fluctuations are measured in two beams and their correlation coefficient is calculated. Measurements at several baselines are made to estimate the normalized covariance function of angle of arrival fluctuations. Let us examine its relation to the phase structure function. If $\varphi_0(x, y)$ describes the phase of the perturbed wavefront at the ground, the fluctuations of angle of arrival of the light rays in x and y directions are defined by

$$\alpha(x, y) = -\frac{\lambda}{2\pi}\frac{\partial}{\partial x}\varphi_0(x, y), \tag{4}$$

$$\beta(x, y) = -\frac{\lambda}{2\pi}\frac{\partial}{\partial y}\varphi_0(x, y). \tag{5}$$

Their covariance is defined as

$$C_\alpha(x_0, y_0) = \langle \alpha(x, y)\, \alpha(x + x_0, y + y_0)\rangle. \tag{6}$$

where $\langle\rangle$ stands for an ensemble average.

When the baseline is parallel to the direction of angle of arrival measurements, i.e. $y_0 = 0$, it is said that the *longitudinal* covariance is measured. In our experiment the baseline is perpendicular to the measurement direction and the *transverse* covariance is measured. It can easily be shown that the longitudinal covariance is proportional to the second derivative of the phase structure function (Roddier 1981). So, if the phase structure function approaches saturation at some baseline the corresponding covariance is expected to be negative. This purely qualitative test would indicate the existence of a finite outer scale of turbulence. If the phase structure function obeys a power law d^t ($t = 1.667$ for Kolmogorov turbulence) then the longitudinal and transverse covariances have also a power law dependence with exponent $t - 2$.

In the real world the angle of arrival is measured by an aperture of finite size D. Aperture filtering has to be taken into account by the theory, and the above statements remain approximately true only for large baselines $d >> D$. The scale $\mathcal{L}_0$ can be introduced explicitly into the theory as an additional parameter. The exact form of the phase structure function in the transition region, from Kolmogorov power–law growth to saturation, depends of course on the formula used to model it. Several such formulae have been proposed in the literature, and two of them, namely the models of Von Karman (Strohbehn 1971; Bouricius & Clifford 1970; Lutomirski & Yura 1971; Ishimaru 1978) and Greenwood–Tarazano (Greenwood & Tarazano 1974; Gardner 1976), will be used here for comparison with observations. The behaviour of angle of arrival covariance function was studied by Borgnino et al. (1992). It has the form

$$C_\alpha(x_0, 0) = 0.0388 G(x_0, \mathcal{L}_0)\, \lambda^2 r_0^{-5/3} x_0^{-1/3}, \tag{7}$$

where the weighting factor G depends on the altitude distribution of turbulence $C_N^2(h)$, altitude distribution of outer scale $L_0(h)$, and aperture diameter D. It is also different for longitudinal and transverse covariances and is model–dependent.

As can be seen from (Eq.7), the angle of arrival covariance depends on the Fried parameter r_0 which is known

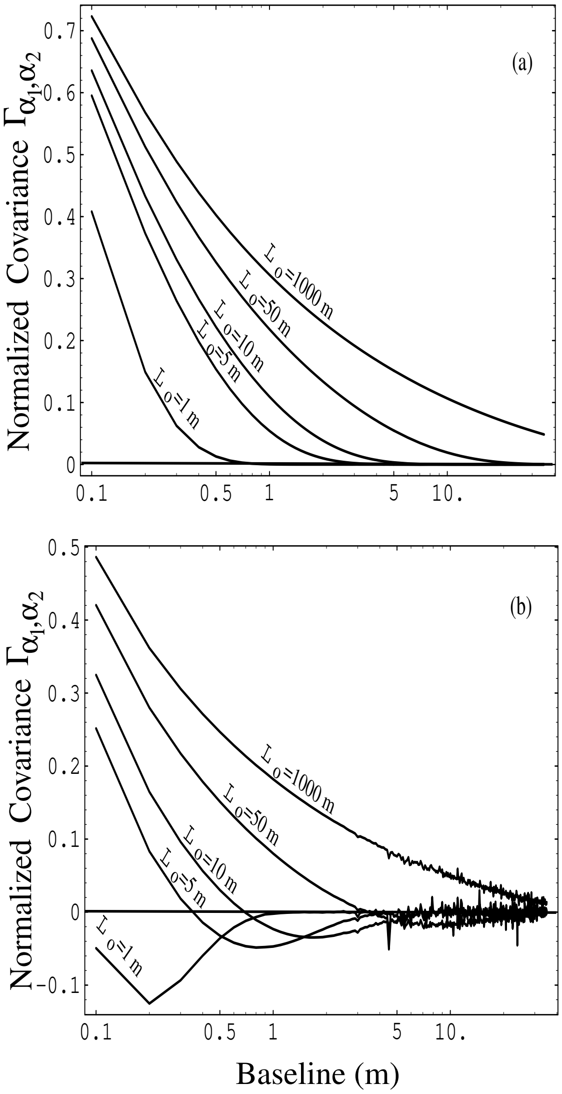

Fig. 1. Normalized covariance of angle of arrival fluctuations: (a) in the direction perpendicular to the baseline (transverse); (b) in the direction of the baseline (longitudinal). The noise appearing on the curves of (b) is only due to the numerical computation.

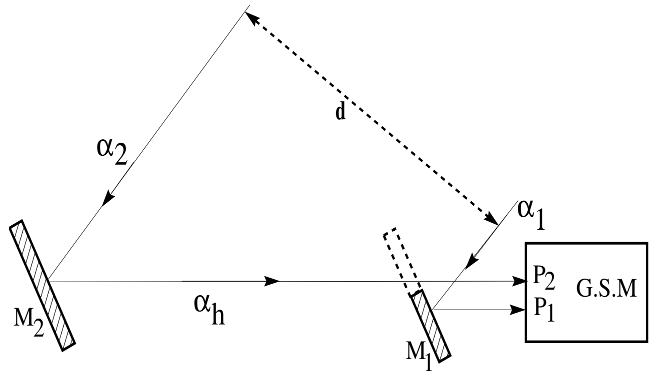

Fig. 2. With G.S.M, angle of arrival α_1 and α_2 are measured at the two subapertures P_1 and P_2 of a small telescope. Influence of the additional horizontal path from M_2 to M_1 is eliminated by computing the angle of arrival covariance. When a large mirror is used instead of M_1, as shown by the dashed line, the baseline of 14 cm is obtained.

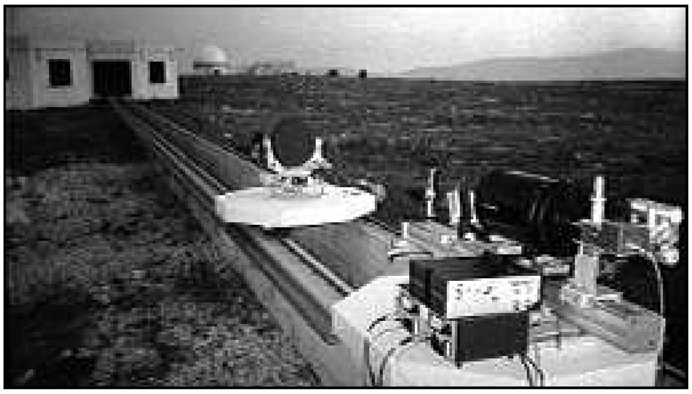

Fig. 3. View of the G.S.M. experiment at the Calern site of the O.C.A. One can see in the foreground the platform with telescope and detection module placed on the rails.The movable platform with a big mirror M_2 is seen further on.

to vary significantly with time. This is why we choose to measure the *normalized* covariance (called also correlation coefficient)

$$\Gamma_\alpha(d) = C_\alpha(d)/\sigma_\alpha^2, \tag{8}$$

where σ_α^2 is the variance of angle of arrival fluctuations. The r_0 dependence is now cancelled by normalization and results are expected to be more stable if the spatial structure of turbulence is itself stationary.

The dependence of longitudinal and transverse normalized covariance Γ_α on the baseline and wavefront outer scale can be seen in Fig.1 where we present the results of our calculations for Von Karman model. Pupil diameter was taken to be 6 cm and a unique turbulence outer scale $\mathcal{L}_0$ throughout all atmosphere was assumed. The behaviour of longitudinal and transverse covariance is very different for baselines of the order of the outer scale: while the transverse covariance remains always positive, the longitudinal covariance displays the expected negative regions. Another important point concerns the sensitivity of the normalized covariance to the turbulence outer scale even at baselines that are an order of magnitude smaller than the scale itself. With a baseline of 1 m, outer scales of 50 m and 1000 m can be distinguished.

3. Estimation of $\mathcal{L}_0$ at two O.C.A. sites

The G.S.M. instrument was described in our first paper. Here we remind only that the star α UMi is used as a light source without tracking. The two light beams are deflected to the measuring unit by plane mirrors as shown on Fig.2. The mirror separation can be changed and the angle of arrival correlation coefficient can thus be measured at different baselines d which are referred to the wavefront plane as shown on the figure. When both beams are reflected from one larger mirror, the smallest baseline of 14 cm is realized. Input pupils are a 6 by 6 cm square.

At Calern site the existing north–south rails of I2T interferometer were used to install the G.S.M. (Fig.3). It is located in the vicinity of the GI2T long–baseline interferometer with 1.5 m apertures (Mourard et al. 1994) and our results can be of some relevance to the operation of this instrument. The fixed part of G.S.M. comprising the mirror M_1, telescope and detection module was attached to the reinforced concrete platform placed on the rails. A 25 cm mirror M_2 whose alt–azimuth pointing is adjustable by a remote radio control unit was placed on a second platform that can move on wheels along the rails. This configuration permits easy and fast change of the baseline up to a horizontal separation of 40 m between the mirrors.

At Mont Gros site there are no suitable rails and different baselines are obtained by placing the mirror M_2 on one of the 7 pillars built for this purpose. The maximum horizontal separation between the mirrors is 12 m.

The time resolution of G.S.M. was set to 20 ms and data acquisition time was chosen to be 120 s. Six such samples were recorded for each baseline. Angle of arrival correlation coefficients were computed for each sample and then averaged, the error being estimated from the rms scatter of individual values. The duration of each run consisting in measurements on all baselines (from 0.14 to 9.6 m at Mont Gros and from 0.14 to 16 m at Calern) was between 3 and 4 hours.

The effects of the turbulence in the horizontal path between the mirrors M_1 and M_2 need some discussion. Referring to the Fig. 2, α_1 and α_2 are the angle of arrival fluctuations produced by the atmosphere respectively above the mirror M_1 and M_2. The angle of arrival fluctuations measured in the channel 2 are in fact a sum of α_2 and α_h produced by the turbulence in the horizontal path, which may be non–negligible, especially for longer baselines. The contribution of horizontal path is indeed seen because the Fried parameter estimated from channel 2 data is systematically smaller than that from channel 1, this difference increasing with the baseline. However we can safely assume that α_h and α_1 are uncorrelated and, consequently, the measured angle of arrival covariance is equal to $\langle\alpha_1\alpha_2\rangle$. The corresponding correlation coefficient deduced from (Eq.8) is :

$$\Gamma_{\alpha_1,\alpha_2}(d) = \langle\alpha_1\alpha_2\rangle/\sigma^2_{\alpha_1}, \tag{9}$$

where the variance used in the denominator is estimated only from channel 1 data.

The angle of arrival correlation coefficients measured at the Mont Gros site on the two nights, May 7–8 and May 11–12 1994, are presented on Fig. 4. The corresponding theoretical curves calculated for Von Karman and Greenwood–Tarazano models with different outer scales, from 5 m to infinity, are also plotted for comparison. The angle of arrival correlation coefficients measured at the Calern site on the nights of June 14–15 and June 15–16 1994 are plotted on Fig. 5 together with the same set of theoretical curves. Corresponding values of average Fried parameter r_0 ($\lambda = 0.5$ nm) and their dispersions are given in Table 2.

4. Discussion of results

It can be immediately seen from Fig. 4 and Fig. 5 that for all baselines the angle of arrival correlation coefficients are *smaller* than those predicted by the infinite outer scale Kolmogorov–Tatarski model, making apparent the effects of outer scale.

On some nights a fairly good correspondence between the experimental points and the theoretical curves can be noted, as for example in Figs. 4b and 5b where the curves for $\mathcal{L}_0 = 50$ m and $\mathcal{L}_0 = 300$ m respectively seem to offer a good approximation to the data. However the systematic departures from the models are also apparent in these examples and are more evident in Fig. 4a and Fig. 5a, in a sense that the correlation coefficient decreases with baseline more slowly that any model would predict. This behaviour can be understood as a natural consequence of the varying outer scale in different atmospheric layers and also with time. The phase structure function is in fact a sum of structure functions of each layer, and the difference of outer scales would result in a more gradual transition from inertial to saturation regimes. The night–to–night variations apparent on Fig. 4 and Fig. 5 can thus be interpreted as being due to the variations in the distribution of turbulent layers. On nights when the layer(s) with a given outer scale dominate(s) the rest of turbulence, a more close correspondence between the measured and theoretical curves is to be expected. This assumption is strengthened by the data on fluctuations of Fried's parameter r_o during these two nights (Table 2), greater variation of r_0 corresponding to greater departure from the models.

Another way to look at the data is just to try to see which structure function would correspond to the observed behaviour of the correlation coefficient. For example the experimental points can be fitted with a power law, and the best exponent turns out to be -0.5. It means that the phase structure function is likely to grow as $d^{1.5}$, i.e. more slowly than $d^{1.67}$ predicted by the Kolmogorov–Tatarski law. It might be interesting to note that a similar conclusion has been reached by Bester et al. (1992) with quite a different technique and at a different site. These authors stressed the importance of their result: a small departure from the Kolmogorov–Tatarski law exponent, however insignificant it might seem, has dramatic consequences on the chromatic behaviour of seeing and means that at infrared wavelengths the image size may be significantly

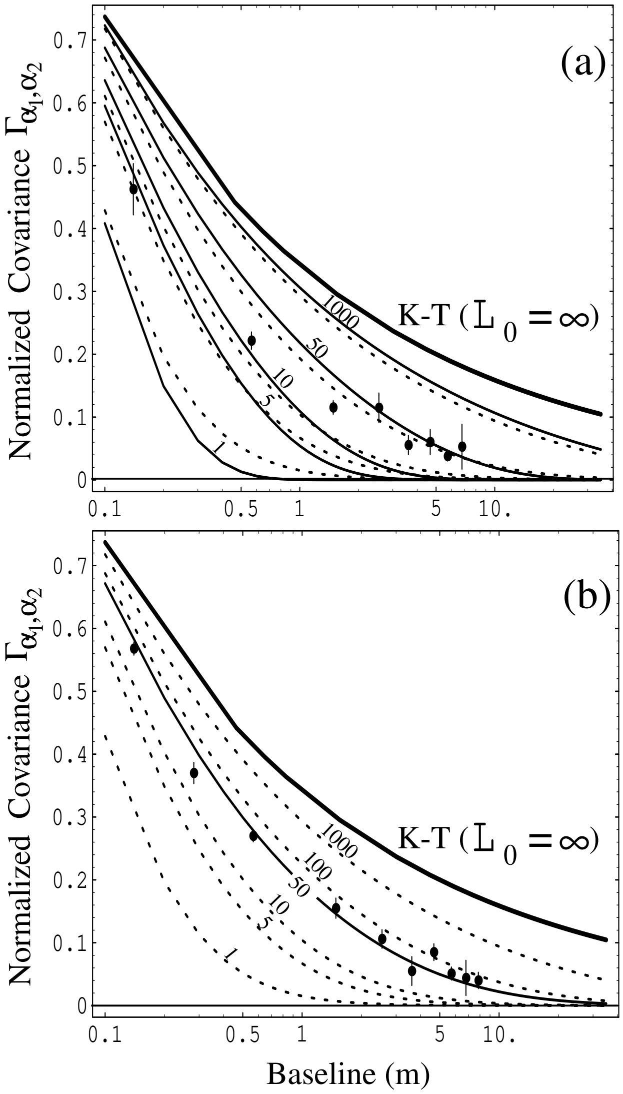

Fig. 4. Correlation coefficients of angle of arrival fluctuations measured at Mont Gros site during two nights: May 7–8 from 23 p.m. to 4 a.m. (a) and May 11–12 from 24 p.m. to 3 a.m. (b) is plotted versus baseline (points with error bars). The theoretical curves for different $\mathcal{L}_0$ corresponding to Von Karman (full line on (a)), Greenwood–Tarazano (dashed line) and Kolmogorov–Tatarski (thick line, $\mathcal{L}_0 = \infty$) turbulence models are given for comparison.

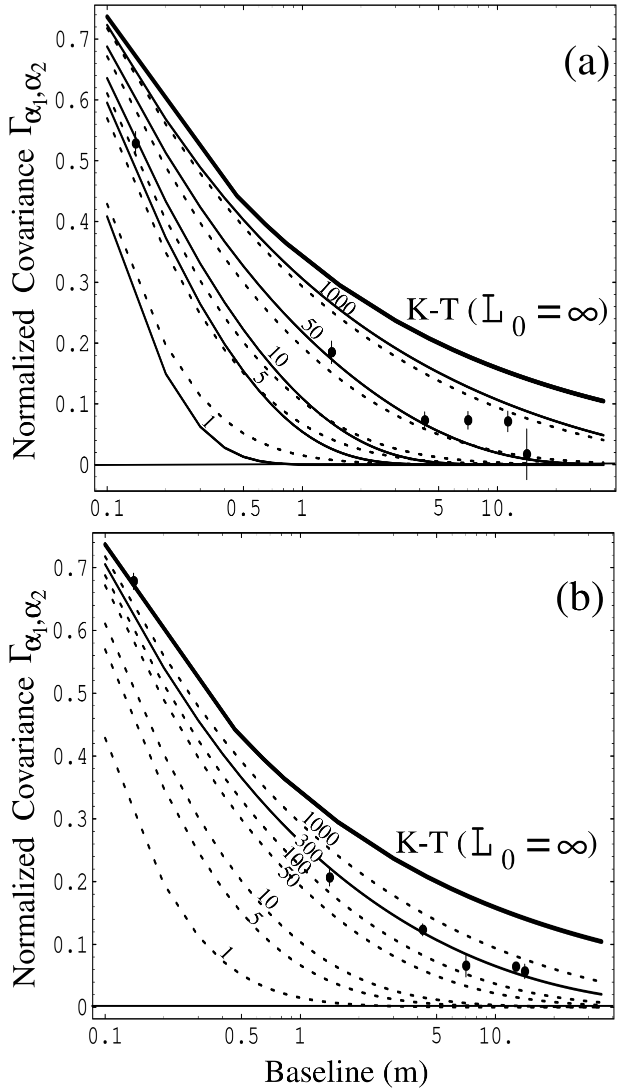

Fig. 5. Correlation coefficient of angle of arrival fluctuations measured at Calern site during two nights: June 14–15 from 23 p.m. to 4 a.m. (a) and June 15–16 from 24 p.m. to 4 a.m. (b) (See Fig. 4 for theoretical curves).

smaller than that predicted by image formation theory with infinite outer scale.

5. Conclusion

The measurements made at two sites, in the Southern of France, with the G.S.M. instrument indicate that at spatial scales from 1 to 20 m the atmospheric distortions of the wavefront definitively show signs of a finite outer scale. Models with a single value of outer scale are hardly applicable, the real behaviour of the phase structure function

Sites	**Figures**	$\mathcal{L}_0$ **(m)**	r_o **(cm)**	σ_{r_o} **(cm)**
Mont Gros	Fig.4a	$5 \sim 50$	10.5	2.7
Mont Gros	Fig.4b	~ 50	7.8	1.12
Calern	Fig.5a	$10 \sim 300$	8.9	1.35
Calern	Fig.5b	~ 300	7.5	0.34

Table 2. Turbulence outer scale $\mathcal{L}_0$, Fried parameter r_0 and its dispersion for the 4 nights

being more complex. An estimate of the wavefront outer scale can however be given when a model can be fitted to the data.: $\mathcal{L}_0$=50 m for Mont Gros and $\mathcal{L}_0$ =300 m for Calern.

Further accumulation of data is needed to provide statistically significant estimations of this parameter and of its variability at the Calern site. This is why systematic measurements of $\mathcal{L}_0$ are planned during the next 12 months. Values of Fried parameter, wind velocity and speckle life time will be also deduced to fully characterize this site for high angular resolution astronomy.

A deeper insight into the spatial structure of turbulence will be obtained from the combined spatio–temporal analysis of G.S.M. data. Another approach consists in simultaneous measurements at several baselines (for example, 6 baselines are possible with 4 units). It would enable to make "snapshots" of the phase structure function and thus to study its temporal behaviour. This work is already underway. Beam–deflecting mirrors will soon be replaced by independent units with their own small telescopes. Horizontal path will thus be completely eliminated and the measurements of longitudinal angle of arrival correlation will become possible.

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