Observatoire
Midi-Pyrénnées

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Light deflection in Weyl gravity

GREX

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Based on articles:

- *Light deflection in Weyl gravity: critical distances for photon paths*.
  S. Pireaux, Classical and Quantum Gravity 21(2004) 1897-1913.
  gr-qc/0403071

- *Light deflection in Weyl gravity: constraints on the linear parameter*.
  S. Pireaux, Classical and Quantum Gravity 21 (2004) 4317-4333.
  gr-qc/0408024

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1. Introduction

1.1 General relativity ≠ final theory…

Theoretical point of view:

- why Einstein-Hilbert action?
- no quantum field theory perturbation
- no conformal invariance
- validity of Newtonian potential on very short/long distances?
- …

Experimental point of view:

- flat velocity distribution at galactic distances?
- …

1.2 Interesting features of Weyl theory

- conformal invariance
- deviations Newtonian potential on long distances
- could explain flat rotation curves without dark matter?
- …

1.3 Light deflection = good probe for Weyl theory

2. The Weyl theory

2.1 Weyl action

$$I_{W\ \text{gravitation}} = \int dx^4 \sqrt{-g}\ \ W^{\alpha\beta\gamma\delta} W_{\alpha\beta\gamma\delta}$$

$$= \int dx^4 \sqrt{-g} \left\{ R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} - 2 R^{\alpha\beta} R_{\alpha\beta} + \frac{1}{3} R^2 \right\}$$

Conformal transformations

$$g_{\alpha\beta} \mapsto \chi^2(x^\mu)\ g_{\alpha\beta}$$

$$W_{\alpha\beta\gamma\delta} \mapsto \chi^2(x^\mu)\ W_{\alpha\beta\gamma\delta}$$

… leave the action invariant

Gravitation equations

$$B^{\mu\nu} \equiv R_{\alpha\beta} W^{\mu\alpha\nu\beta} + 2\, W^{\alpha\mu\beta\nu}{}_{|\alpha|\beta} = 0$$ = Bach equations

$$\ni \quad R^{\mu\nu} = 0$$ = Einstein equations

Static spherically symmetric solution

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$= \chi^2(x^\mu) \cdot \left\{ \left[1 - \frac{2V_W}{c^2} \right] c^2 dt^2 - \left[1 - \frac{2V_W}{c^2} \right]^{-1} dr^2 - r^2 \left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2 \right) \right\}$$

If $\begin{cases} \gamma_W = k_W = 0 \\ \beta_W = \dfrac{G_N M}{c^2} \end{cases}$

= Schwarzschild solution

Weyl potential

$$V_W(r) = -\frac{\beta_W}{2} \frac{(2 - 3\beta_W \gamma_W)}{r} c^2 - \frac{3}{2} \beta_W \gamma_W c^2 + \frac{\gamma_W}{2} r\, c^2 - \frac{k_W}{2} r^2\, c^2$$

= Newtonian potential

2.2 Gravitational potential

$$V_w(r) = -\frac{\beta_w}{2}\frac{(2-3\beta_w\gamma_w)}{r}c^2 - \frac{3}{2}\beta_w\gamma_w c^2 + \frac{\gamma_w}{2} r\, c^2 - \frac{k_w}{2} r^2\, c^2$$

Newtonian term (**weak field**)

constant term

linear term (**strong field**)

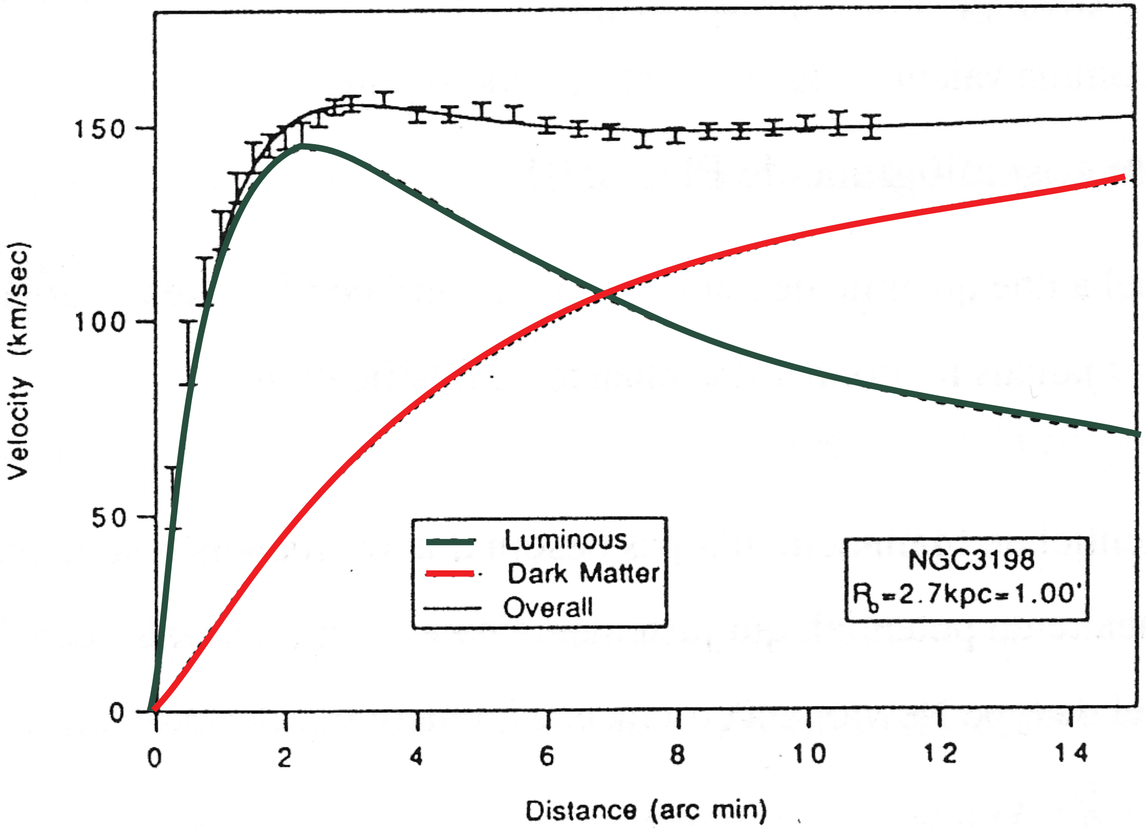
Cosmological term

Breaks conformal transformation
to asymptotically flat space.
$\beta_w\gamma_w$-term negligible.

k_w-term important only on cosmological distances.
Does not contribute to photon motion.

2.3 Mannheim-Kazanas parametrization

… to fit galactic rotation curves without dark matter ➡ $\gamma_W > 0$



$\gamma_W \approx +10^{-26}\, m^{-1}$

$\beta_W(M_{galaxy}) \approx +10^{+14}\, m^{+1}$

… but based on assumption that $\chi^2(x^\mu) = \text{cst}$

2.4 Weak- versus the strong-field limit

Weak field radius: Newtonian term dominates over the linear term

$$r_{weak\ field} <<< \frac{\pm 1 + 3\beta_w\gamma_w \pm \sqrt{1 \pm \begin{Bmatrix} +14 \\ -2 \end{Bmatrix} \beta_w\gamma_w - 3(\beta_w\gamma_w)^2}}{2\gamma_w}$$

$$\overset{(\beta_W\gamma_W)\text{-term neglected}}{\approx} \pm\frac{1}{\gamma_w}$$

$$\overset{if\ \chi^2=1\ and\ M-K\ parametrization}{\approx} +10^{+26}\,m$$

Strong field radius: linear term dominates, Newtonian term neglected

$$r_{\text{strong } field} >>> \sqrt{\frac{2\beta_W}{|\gamma_W|}}$$

$$\overset{if\ \chi^2=1\ and\ M-K\ parametrization}{\approx} \begin{cases} 2\cdot 10^{+20}\, m \ \text{ for } M = 10^{11} M_{Sun} & \text{a galaxy} \\ 2\cdot 10^{+21}\, m \ \text{ for } M = 10^{13} M_{Sun} & \\ 6\cdot 10^{+21}\, m \ \text{ for } M = 10^{14} M_{Sun} & \text{a cluster} \\ 2\cdot 10^{+22}\, m \ \text{ for } M = 10^{15} M_{Sun} & \end{cases}$$

… in this regime:

$$V_{W\ \beta_W=0}(r) = +\frac{\gamma_W}{2} r\, c^2 - \frac{k_W}{2} r^2\, c^2$$

+ conditions on the radius to insure

$$ds^2 = \underbrace{A^2(r)}_{>0} c^2 dt^2 - \underbrace{B^2(r)}_{>0} dr^2 - r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)$$

3. Light deflection in Weyl theory

A/ The geodesic equation: photons ($\mathbf{F} \equiv 0$) or massive particles ($\mathbf{F} > 0$)

$$\underbrace{\left(\frac{dr}{d\lambda}\right)^2}_{\text{“kinetic energy”}} + \underbrace{\left\{\frac{1}{r^2} + \mathbf{F}\frac{\chi^2(r)}{J^2}\right\}\left\{1 + 2\frac{V_W(r)}{c^2}\right\}}_{V_{geodesic} = \text{“geodesic potential”}} = \underbrace{\frac{E^2}{J^2}}_{\text{“total energy”}}$$

where $\frac{dr}{d\lambda} \equiv \frac{1}{r^2}\frac{dr}{d\varphi}$

$\chi^2 = 1$

$$-F_{geodesic} = \frac{dV_{geodesic}}{dr} = -\frac{2}{r^3} + \beta_W(2 - 3\beta_W\gamma_W)\left\{\frac{3}{r^4} + \frac{\mathbf{F}}{J^2 r^2}\right\}$$

$$+ 3\beta_W\gamma_W\left\{\frac{2}{r^3}\right\} \quad + \gamma_W\left\{-\frac{1}{r^2} + \frac{\mathbf{F}}{J^2}\right\}$$

$$+ k_W\left\{0 - 2\frac{\mathbf{F}r}{J^2}\right\}$$

diverges

- Photon geodesics are independent of unknown $\chi^2(x^\mu)$

➡ light deflection is a good probe for Weyl gravity

- Newtonian term = always positive (attractive)

- $\beta_w \gamma_w$ -term = $\begin{cases} \text{positive (attractive) if } \gamma_w > 0 \\ \text{negative (repulsive) if } \gamma_w < 0 \end{cases}$

function of the type of particle

- γ_w -term = if $\gamma_w > 0$,

  $\begin{cases} \text{negative (repulsive) for photons / relativistic particles} \\ \text{positive (attractive) for massive particles} \end{cases}$

  … and vice-versa if $\gamma_w < 0$

- k_w -term = $\begin{cases} \underline{\text{null for photons}} \\ \text{non-null for massive particles:} \\ \quad \begin{cases} \text{positive (attractive) if } k_w < 0 \\ \text{negative (repulsive) if } k_w > 0 \end{cases} \end{cases}$

3.2 Critical radii for photons

$$r_{\min} \approx -\frac{2}{\gamma_w}$$

$$r_{\max} \approx 3\beta_w$$

$$r_{\text{inflection}} \approx -\frac{3}{\gamma_w}$$

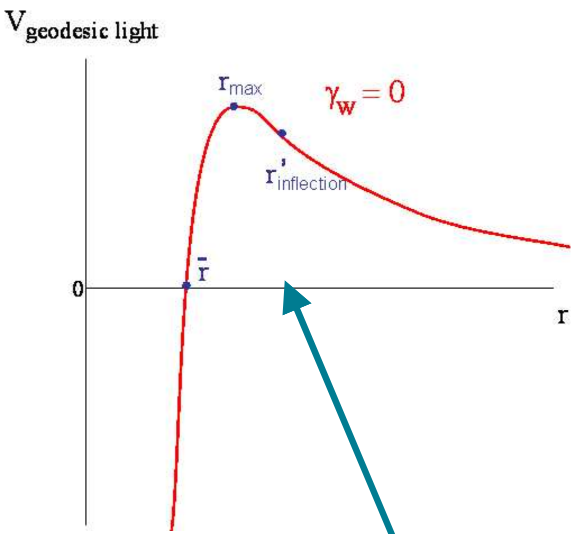
$$r_{null} \approx -\frac{1}{\gamma_w}$$

… $-1/\gamma_w$ is a particular scale

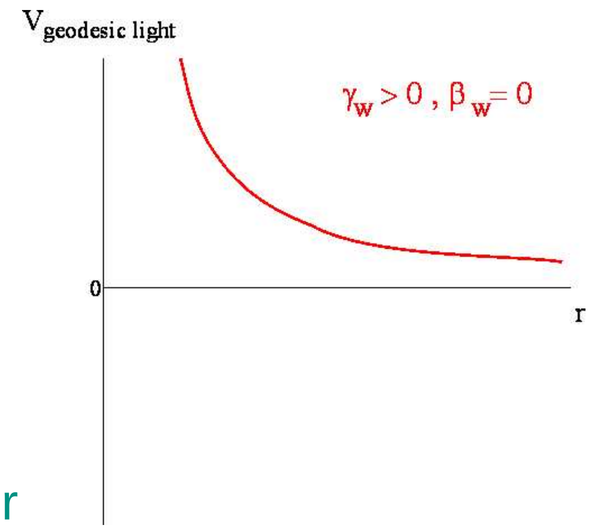
let $\beta_w \neq 0$ and $\gamma_w \neq 0$ ...

On large distance scales:
linear term dominates

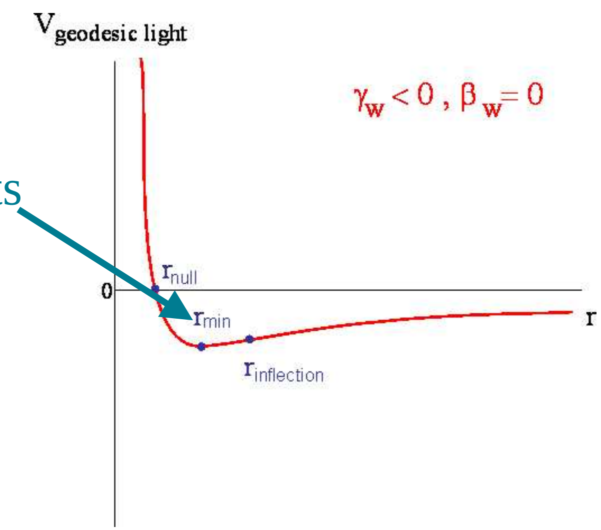
On short distance scales:
Newtonian term dominates



Schwarzchild like

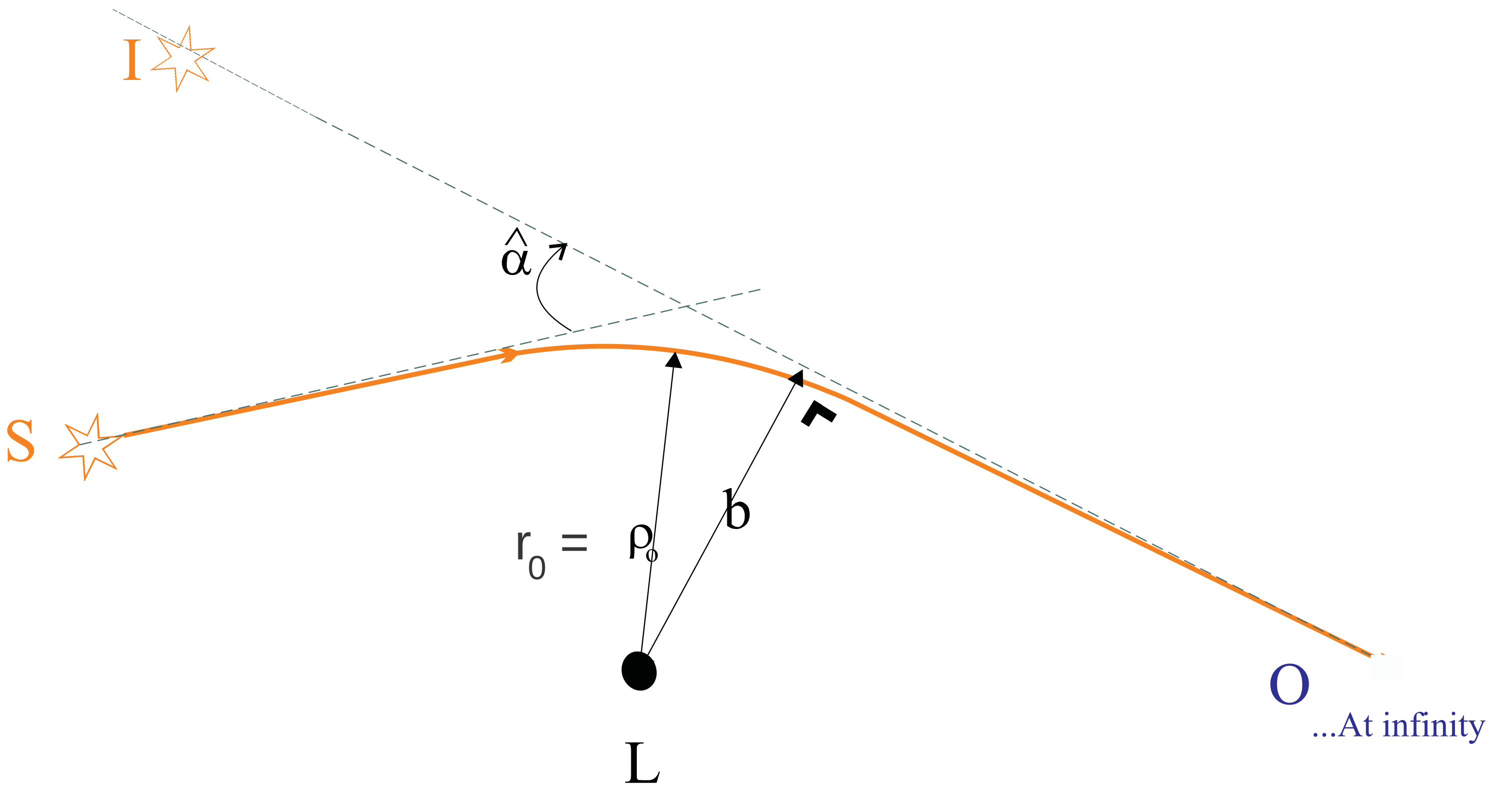


or



bound orbits

3.3 Conditions for light deflection



… unbound orbits,

… $\hat{\alpha}(r_0) > 0$ if convergent, $\hat{\alpha}(r_0) < 0$ if divergent

…in the weak field regime

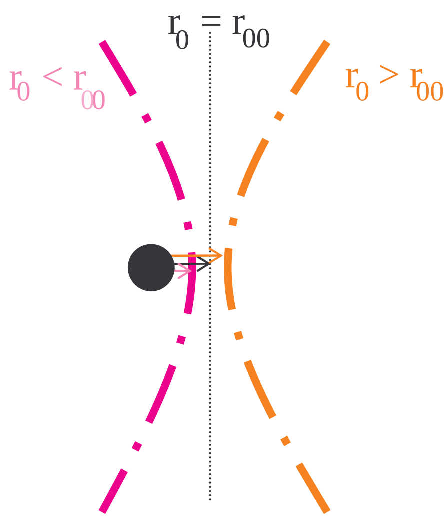
The light deflection angle

$$\hat{\alpha}_{weak\ field}(r_0) = 2\frac{\beta_W(2-3\beta_W\gamma_W)}{r_0} + \frac{3}{2}\beta_W\gamma_W\pi - \gamma_W r_0$$

Critical radius from the weak field deflection angle for $\gamma_W > 0$

$$r_{00} \approx 2\sqrt{\frac{\beta_W}{|\gamma_W|}}$$

$$\underset{\text{if } \chi^2=1 \text{ and } M-K \text{ parametrization}}{\approx} \begin{cases} 8\cdot 10^{+14}\, m \ \text{ for } M = M_{Sun} \\ 2\cdot 10^{+20}\, m \ \text{ for } M = 10^{11} M_{Sun} \ \} \text{ a galaxy} \\ 2\cdot 10^{+21}\, m \ \text{ for } M = 10^{13} M_{Sun} \\ 8\cdot 10^{+21}\, m \ \text{ for } M = 10^{14} M_{Sun} \quad \text{a cluster} \\ 2\cdot 10^{+22}\, m \ \text{ for } M = 10^{15} M_{Sun} \end{cases}$$



…in the strong field regime

Open versus closed orbits in the strong regime

$$r_{\beta_W=0} = \frac{-2/\gamma_W}{1-\frac{2+\gamma_W r_0}{\gamma_W r}\sin\left(\pm\varphi \pm \varphi_{initial}\right)}$$

with $e \equiv \left|\frac{2+\gamma_W r_0}{\gamma_W r}\right|$, the excentricity

The types of orbits allowed can be classified:

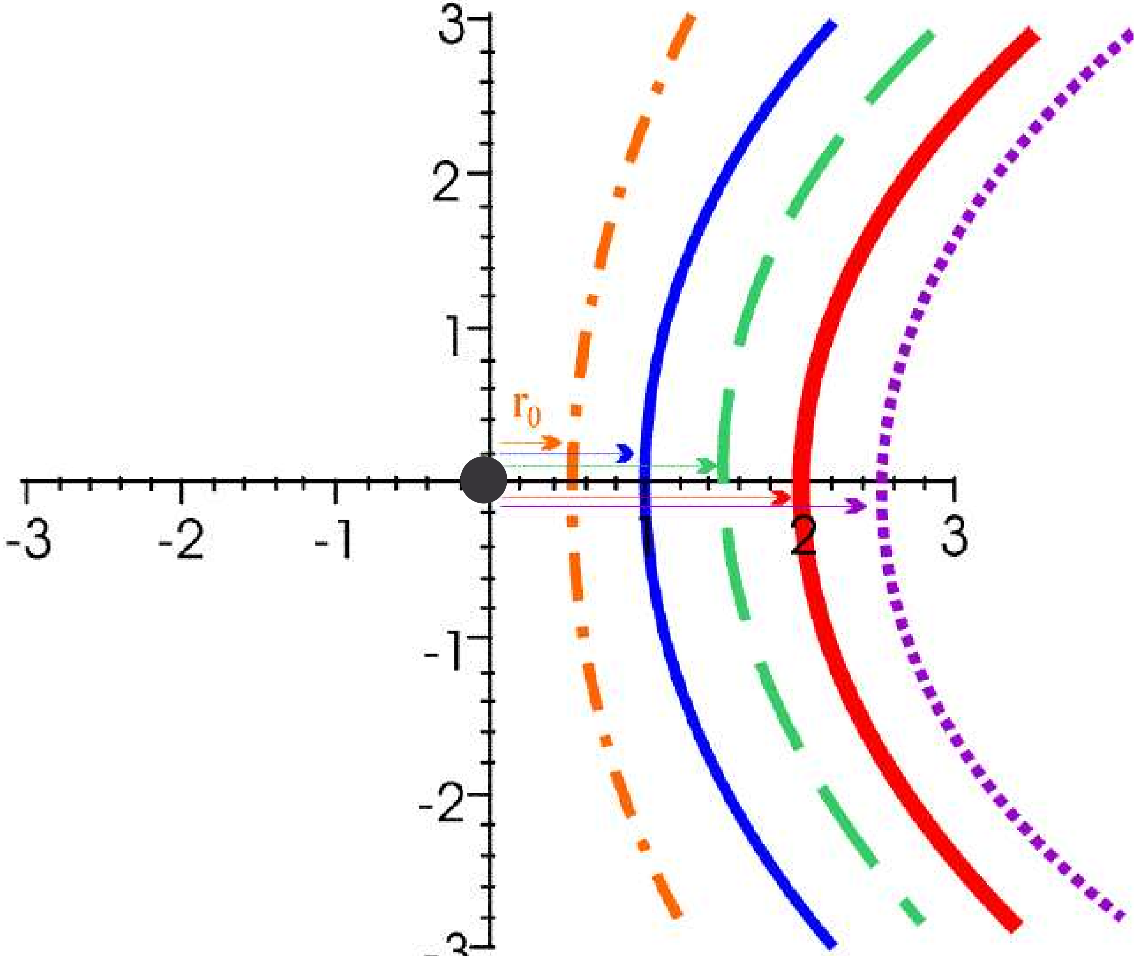
if $\gamma_W > 0 \; : \; \forall r_0$, hyperbolic ($e > 1$)

if $\gamma_W < 0 \; : \; r_0 < r_{null}$, hyperbolic ($e > 1$)

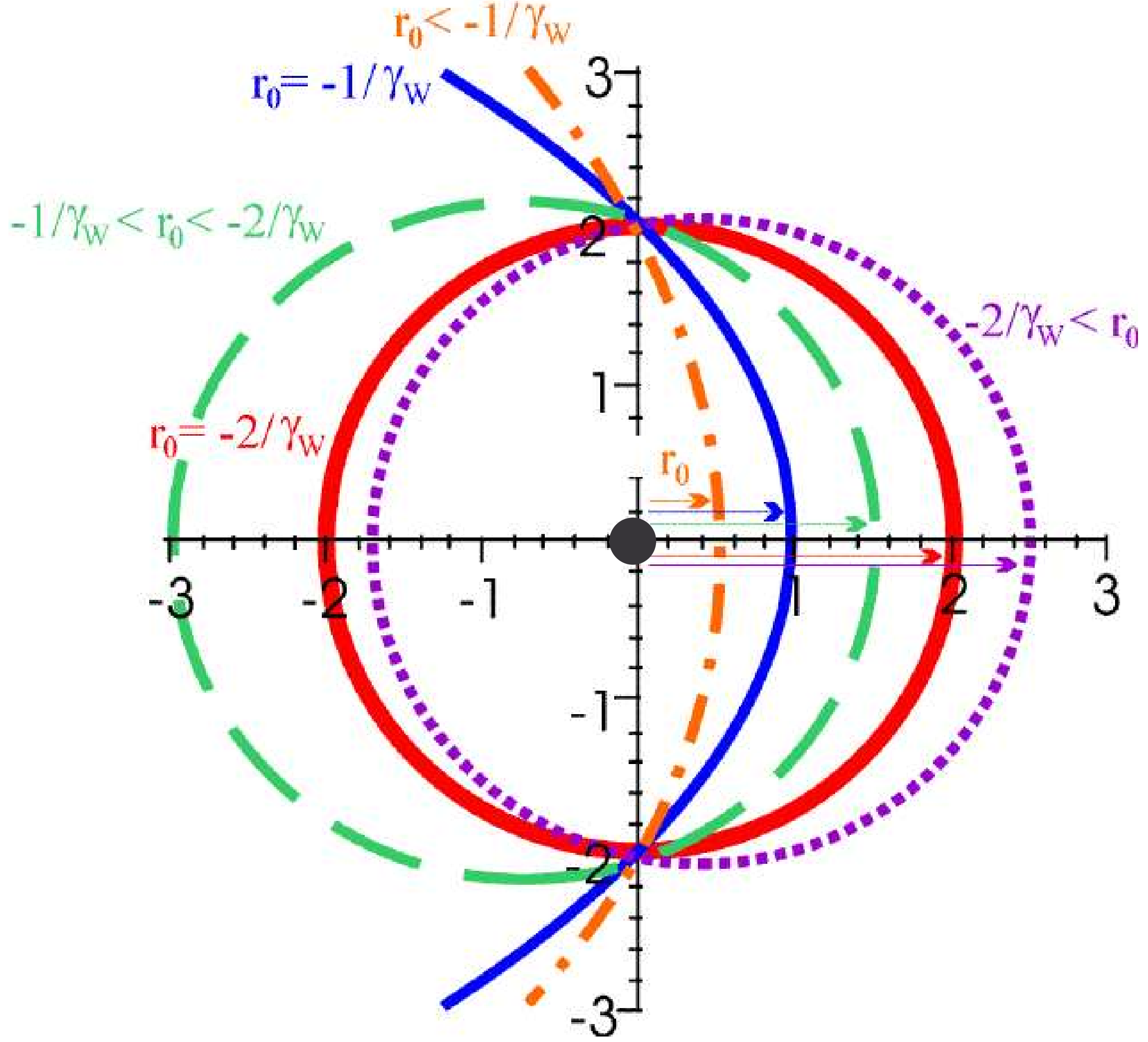
$r_0 = r_{null}$, parabolic ($e = 1$)

$r_0 > r_{null}$, elliptic ($e < 1$), circular case ($e = 0$) for $r_0 = r_{\min}$

$\gamma_w > 0$



$\gamma_w < 0$



The light deflection angle

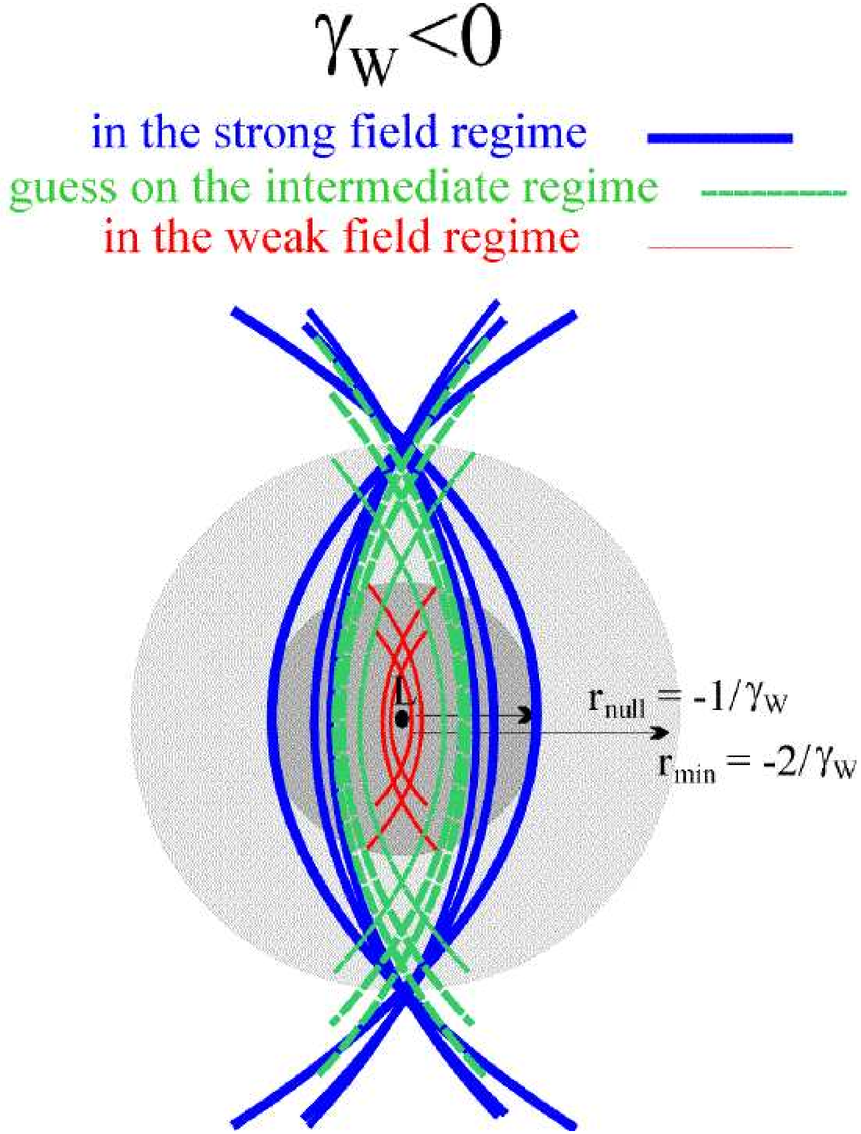
$$\hat{\alpha}_{\beta_W=0}(r_0) = -2\arcsin\left(\frac{\gamma_W r_0}{2+\gamma_W r_0}\right)$$

… recover the weak field regime

$$\hat{\alpha}_{weak\,/\,strong\;\;field}(r_0) \approx -\gamma_W r_0$$

4. Amazing features of strong field regime for a negative parameter

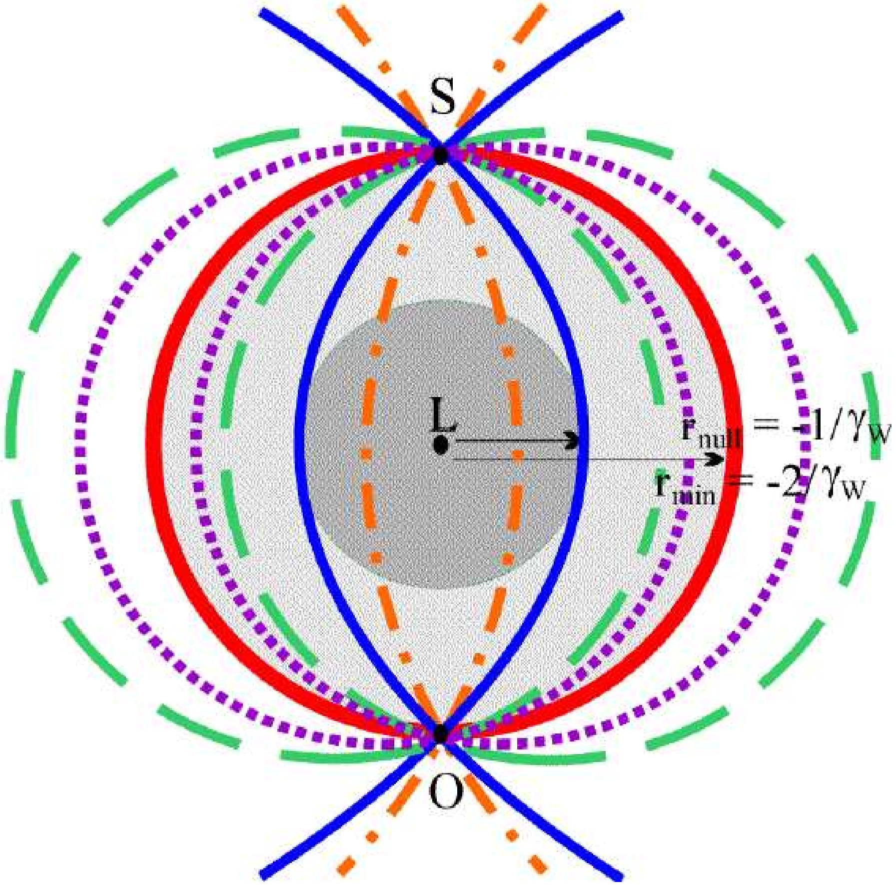
4.1 Accumulation point:



4.2 Peculiar alignment configuration:

$\gamma_W < 0$

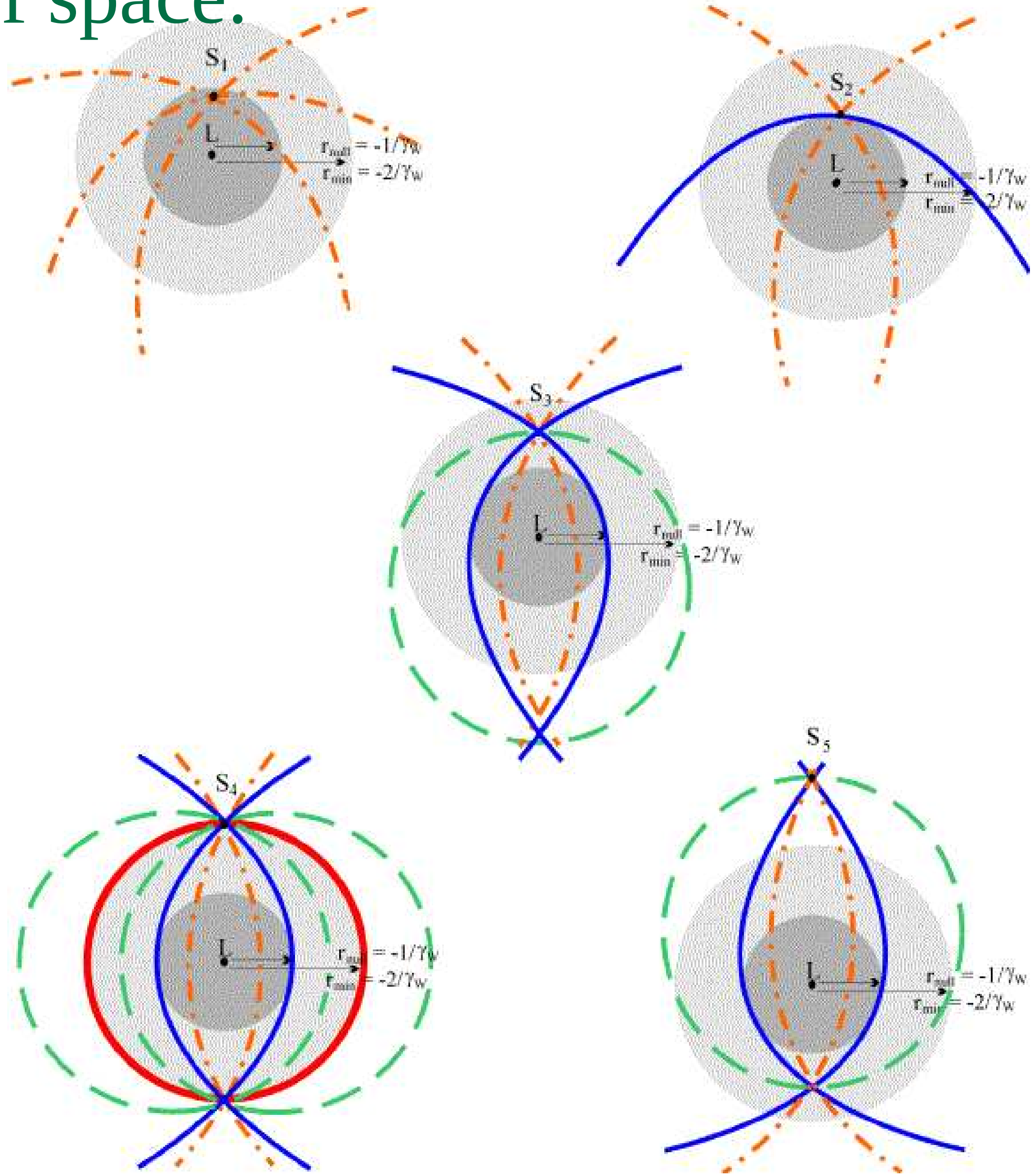
in the strong field regime



4.3 Observable regions of space:

$\gamma_W < 0$

in the strong field regime



5. Constraints on linear parameter

5.1 Solar system experiments: VLBI, CASSINI

PPN parameter γ estimate → linear parameter γ_W estimate

extrapolate at solar limb

$$\hat{\alpha}_{weak\ field}(r_0) = 2\frac{(1+\gamma)GM}{r_0}$$

$$\hat{\alpha}_{weak\ field}(r_0) \cong \frac{4\beta_W}{r_0} - \gamma_W r_0$$

$$\beta_W = \frac{G_N M_{Sun}}{c^2}$$

VLBI

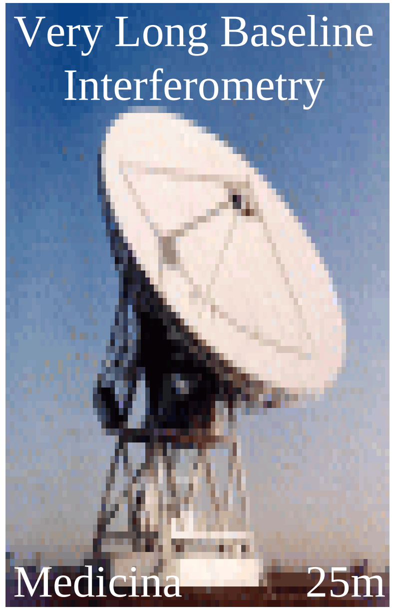
PPN parameter γ estimate

$$\gamma = 0.9996 \pm 0.0017$$

[Lebach et al. 1995]

$$\gamma = 0.99983 \pm 0.00045$$

[Shapiro et al. 2004]



extrapolate at solar limb

linear parameter γ_W estimate

$$\gamma_W \in \left[-7.9 \cdot 10^{-18}, +1.3 \cdot 10^{-17}\right] \ \mathrm{m}^{-1}$$

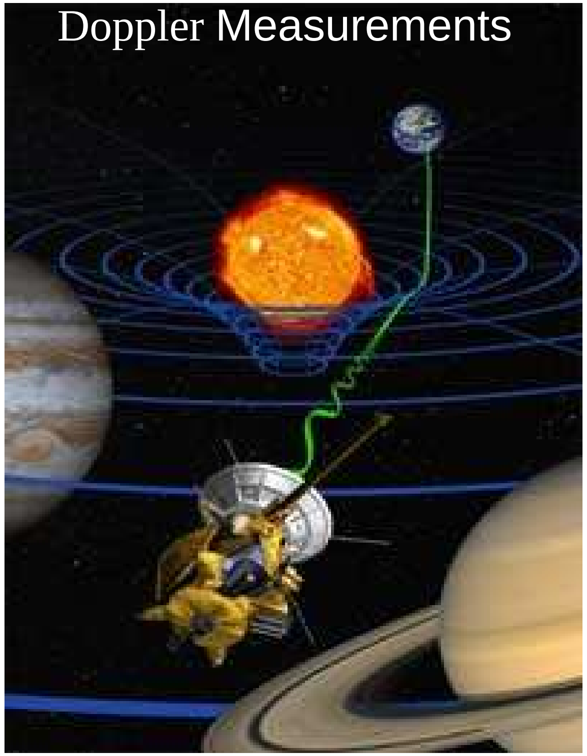
$$\gamma_W \in \left[-1.7 \cdot 10^{-18}, +1.3 \cdot 10^{-18}\right] \ \mathrm{m}^{-1}$$

CASSINI mission

PPN parameter γ estimate

$$\gamma - 1 = (-2.1 \pm 2.3) \times 10^{-5}$$

[Bertoti et al. 2003]



extrapolate at solar limb

linear parameter γ_W estimate

$$\gamma_W \in \left[-1.2 \cdot 10^{-20}, +2.7 \cdot 10^{-19}\right] \ \mathrm{m}^{-1}$$

5.2 Beyond solar system experiments: microlenses, mirages

Constraints on a negative linear parameter

If $\gamma_W < 0$, $\exists\ r_{null}$ that separates

- $r_0 > r_{null}$: bound orbits ➡ light deflection **not** possible
- $r_0 < r_{null}$: **un**bound orbits ➡ light deflection possible

➡

$$\vartheta_E \approx \frac{r_0}{D_{OL}} < \frac{r_{null}}{D_{OL}} \qquad \text{for } \gamma_W < 0$$

⬍ Hubble empirical law (D, z)

$$|\gamma_W| \lesssim \left[\frac{1}{\vartheta_E}\ h_0\ \frac{0.3}{z_L}\right] 1.7\times10^{-31}\ \text{m}^{-1} \quad \text{for } \gamma_W < 0$$

ϑ_E: arsecs

h_0:]0.55; 0.75]

Microlensing or lensing light curves

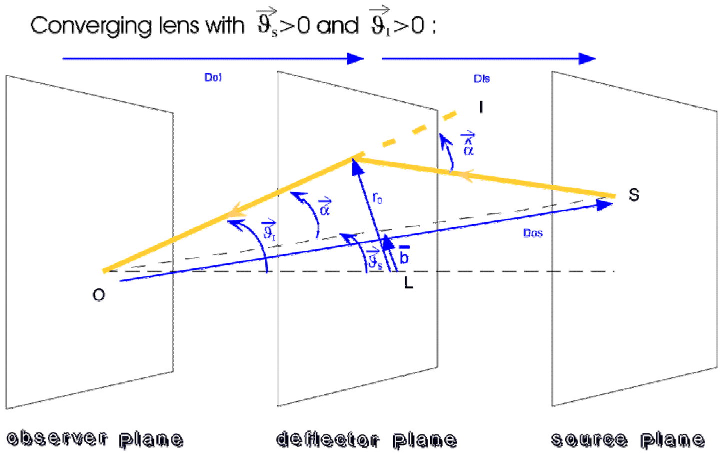
Lens equation (weak field limit):

$$\vec{\vartheta}_I^{\,2} - \frac{1}{1+n_W}\,\vec{\vartheta}_s\,\vec{\vartheta}_I - \vartheta_W^{\,2} = 0$$

with ϑ_W = angular radius of Weyl ring

$$\vartheta_W \equiv \frac{1}{\sqrt{1+n_W}}\,\vartheta_E$$

$$n_W \equiv \gamma_W \frac{D_{LS}\,D_{OL}}{D_{OS}}$$



…but corrective factor small, maybe negligible (lens statistic required)??

Summary of results

- $\exists$ critical radii function of γ_w : structure space-time (photons)

- They are physical or not according to the sign of γ_w

$\gamma_w = 0$ … General Relativity

$\gamma_w < 0$ … $\exists$ r_{null} which separates bound/unbound orbits.

$r_{weak\ field} \approx r_{null}$ ➡ light deflection always possible in THIS limit.

$\gamma_w > 0$ … light deflection always possible in ANY limit.

… $\exists$ $r_{0\,0}$ which separates convergent/divergent light deflection;

it is also function of the deflector mass.

$r_{strong\ field} \approx r_{0\,0}$ ➡ light deflection always divergent in THIS limit.

- Light deflection = good probe for Weyl theory:

$|\gamma_w| \lesssim 10^{-19}$ m^{-1} … from Solar System experiments (CASSINI)

$|\gamma_w| \lesssim 10^{-31}$ m^{-1} for $\gamma_w < 0$ … from the existence of mirages

… future missions: improve estimate of PPN γ ➡ improve estimate of γ_w

- GAIA [GAIA repport 2000] : γ at $\sim 5 \cdot 10^{-7}$
- LATOR [Turyshev et al 2004] : γ at $\sim 5 \cdot 10^{-8}$
- …

… BUT does not select between $\gamma_w = 0$, $\gamma_w < 0$ or $\gamma_w > 0$

➡ Present analysis could be refined:

amazing features?, different lens-mass models, mirage statistics …

BIBLIOGRAPHY

included in articles:

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