Gravity tests in the solar system and the Pioneer anomaly

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Tests PPN : γ

Deflection of light

$$\theta = \frac{1+\gamma}{2}\theta_{\rm standard}$$

Shapiro time delay

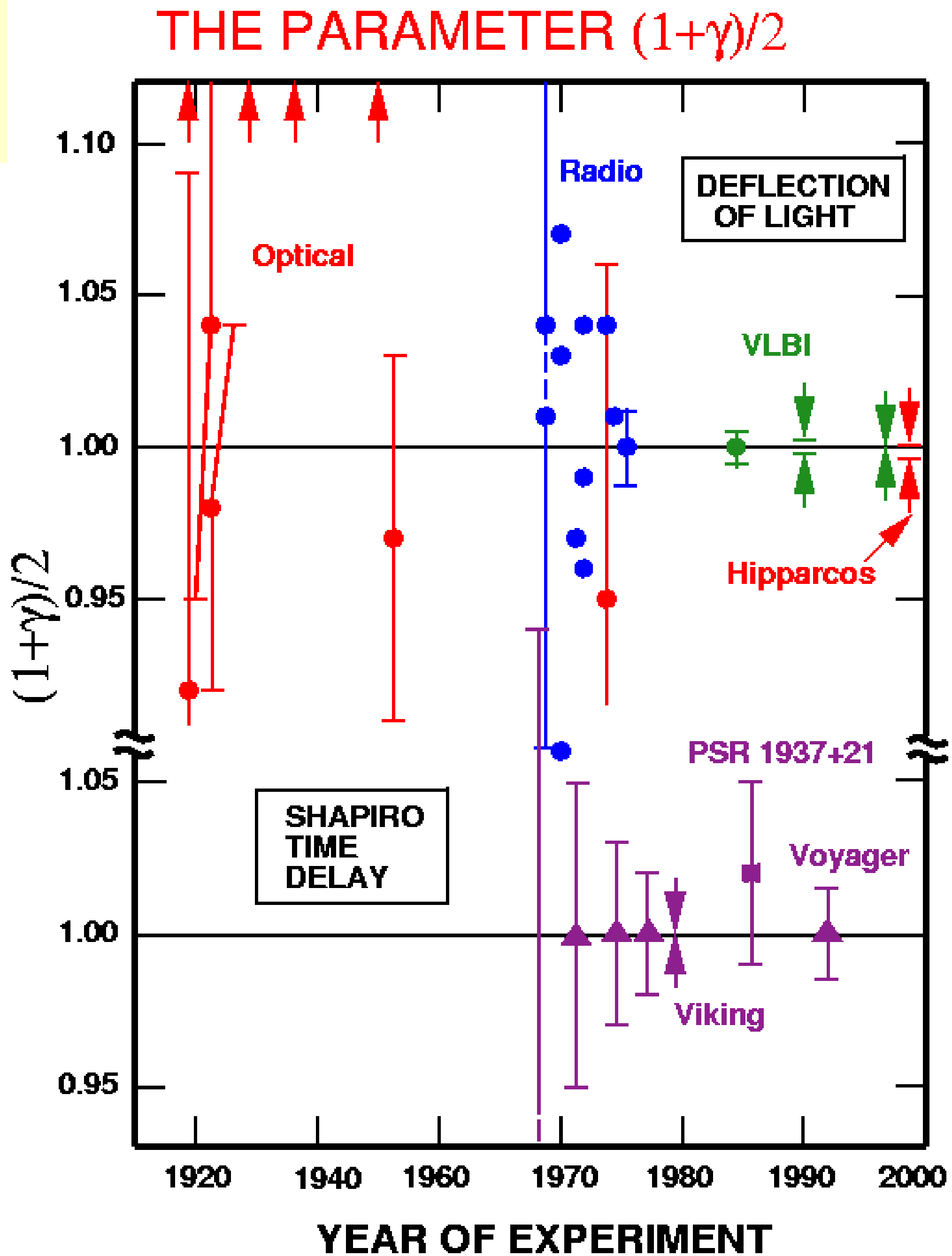
$$\tau = \frac{1+\gamma}{2}\tau_{\rm standard}$$

C. Will
Living Reviews (2001)
www.livingreviews.org

After Cassini

$$|1-\gamma| \lesssim \text{a few } 10^{-5}$$

B. Bertotti *et al, Nature (2004)*



Tests : Newton law

Yukawa correction
to the Newton law

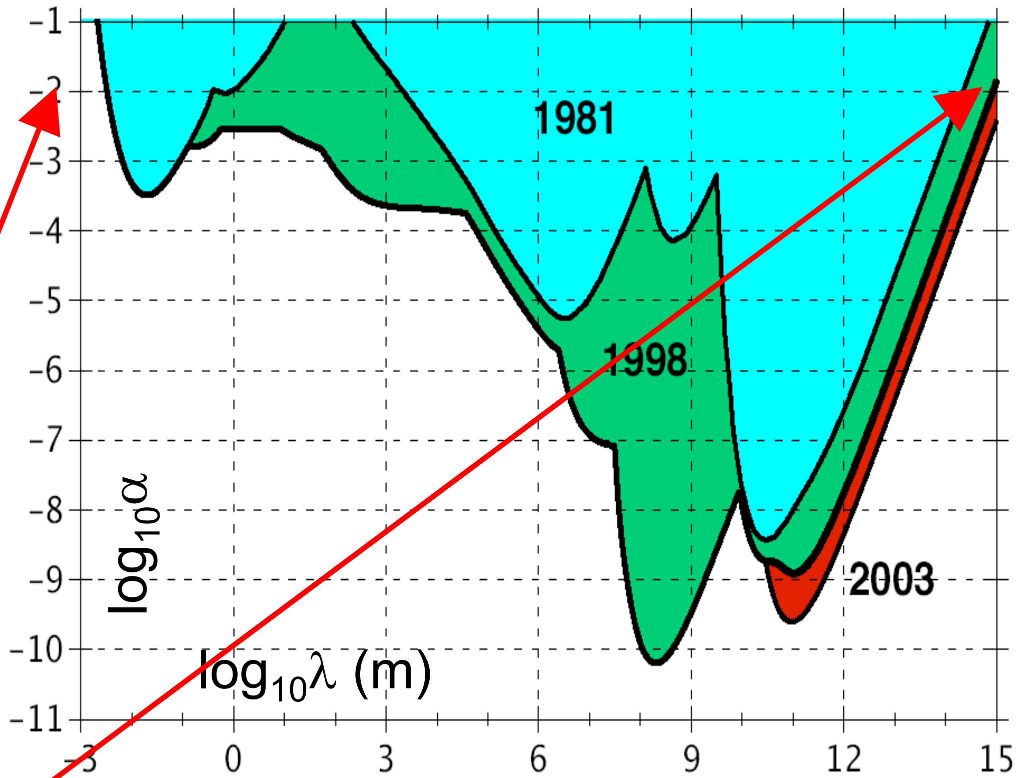
$$V(r) = -\frac{GMm}{r}\left(1 + \alpha e^{-\frac{r}{\lambda}}\right)$$

Windows remain
open for deviations
at short ranges

$\lambda < 1\,\mathrm{mm}$

or long ranges

$\lambda > 10^{16}\,\mathrm{m}$



Update : **J. Coy, E. Fischbach, R. Hellings, C. Talmadge and E. M. Standish (2003)**

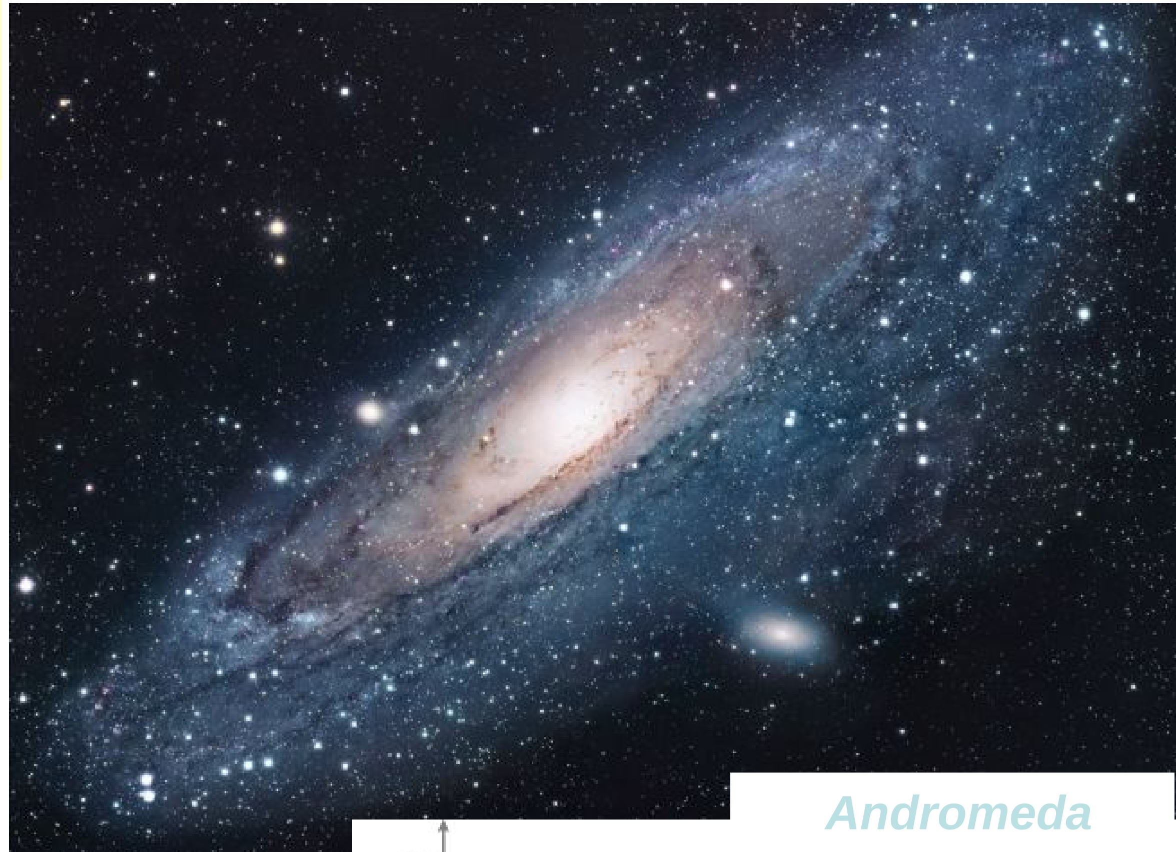
The Search for Non-Newtonian Gravity, **E. Fischbach and C. Talmadge (1998)**

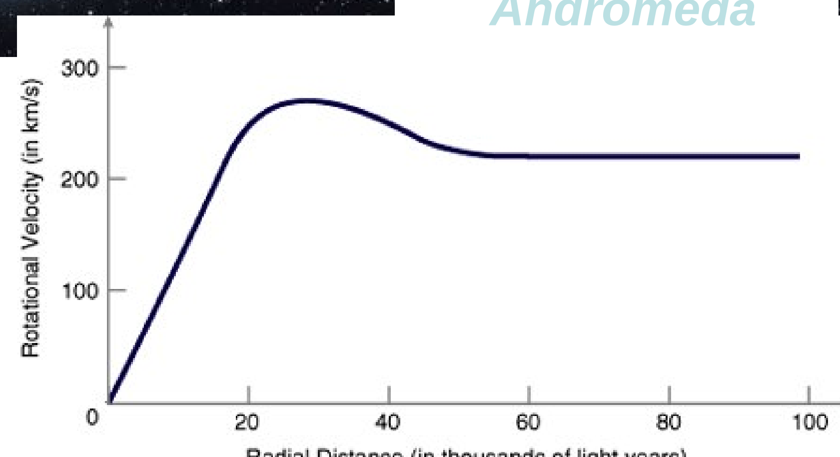
Astrophysical anomalies

Galaxy rotation curves

show deviations

from Newton law





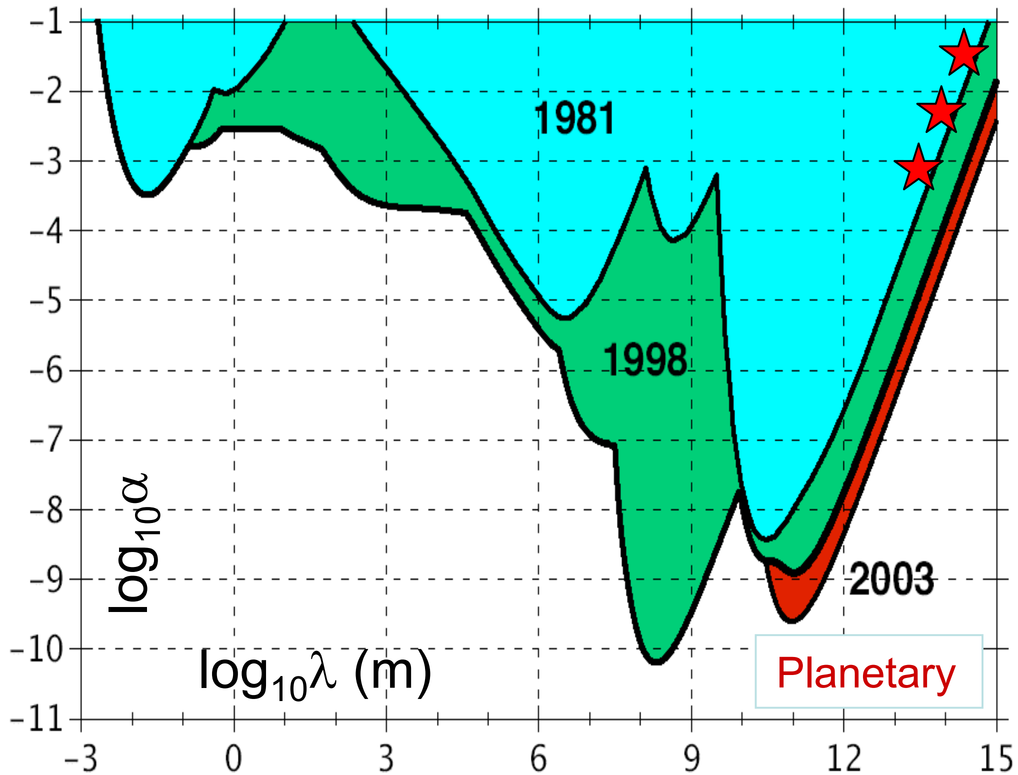
An observation of Dark Matter and / or a modification of gravity laws at large distances?

The Pioneer anomaly

J.D. Anderson *et al*,
PRD 65 (2002) 082004

An anomaly already observed at scales of the order of the size of the solar system ?

But an explanation from a long range modification of Newton law is forbidden by planetary tests !



A key question :
is the Pioneer anomaly compatible (or not) with other gravity tests in the solar system ?

Theoretical motivations for an extension of Einstein gravity theory

The geometric features of Einstein theory will be left unchanged

- metric theory
- gauge invariance
- geodesic motion …

Equivalence principle preserved

The Einstein relation between curvature and stress tensor will be modified

Metric changed in the solar system

Simplifying assumptions in a first approach

- linearized theory
- stationarity and isotropy of the gravity source

Description valid in the outer part of the solar system

Basic definitions

Same basic equations as
in linearized general relativity
expressed in the Fourier domain

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} \ll 1$$

$$\frac{\partial}{\partial x^\mu} \rightarrow \imath k_\mu$$

Curvature tensors

$$R_{\mu\rho\nu\sigma} = \frac{1}{2}\left(k_\mu k_\nu h_{\rho\sigma} - k_\mu k_\sigma h_{\rho\nu} - k_\rho k_\nu h_{\mu\sigma} + k_\rho k_\sigma h_{\mu\nu}\right)$$

$$R_{\mu\nu} = \eta^{\rho\sigma} R_{\mu\rho\nu\sigma}$$

$$R = \eta^{\mu\nu} R_{\mu\nu}$$

The Einstein tensor $E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} R$ is transverse $k^\nu E_{\mu\nu} = 0$

as the stress tensor $k^\nu T_{\mu\nu} = 0$

In standard theory, they are merely
proportional to each other

$$E_{\mu\nu} = \frac{8\pi}{c^4} G_N T_{\mu\nu}$$

Generalized gravity equation (1)

More general linear relation between the two tensors

$$E_{\mu\nu}[k] = \chi_{\mu\nu}{}^{\rho\sigma}[k]\, T_{\rho\sigma}[k]$$

$$k^{\nu}\chi_{\mu\nu}{}^{\rho\sigma}[k] = 0$$

Allows for different linear responses in the two sectors of traceless and traced components

$$E_{\mu\nu} = E_{\mu\nu}^{(0)} + E_{\mu\nu}^{(1)}$$

$$E_{\mu\nu}^{(0)} = \left(\frac{\pi_{\mu}^{\rho}\pi_{\nu}^{\sigma} + \pi_{\mu}^{\sigma}\pi_{\nu}^{\rho}}{2} - \frac{\pi_{\mu\nu}\pi^{\rho\sigma}}{3} \right) E_{\rho\sigma}$$

$$E_{\mu\nu}^{(1)} = \frac{\pi_{\mu\nu}\pi^{\rho\sigma}}{3} E_{\rho\sigma} \qquad \pi_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}$$

Momentum-dependent linear response functions χ with different behaviors in the two sectors are naturally produced by quantum corrections to general relativity

Generalized gravity equation (2)

When the Sun is described as a stationary and isotropic source,

$$T_{\rho\sigma}(x) = \eta_{\rho 0}\eta_{\sigma 0} M c^2 \, \delta(\mathbf{x})$$

$$T_{\rho\sigma}[\mathbf{k}] = \eta_{\rho 0}\eta_{\sigma 0} M c^2 \qquad k_0 \equiv 0$$

The generalized equation of gravity may be written in terms of two running constants

$$E_{\mu\nu}[\mathbf{k}] = \chi_{\mu\nu}{}^{00} M c^2$$

$$= \left(\frac{\pi_\mu^0 \pi_\nu^0 + \pi_\mu^0 \pi_\nu^0}{2} - \frac{\pi_{\mu\nu}\pi^{00}}{3} \right) \widetilde{G}^{(0)}[\mathbf{k}] \frac{8\pi M}{c^2}$$

$$+ \frac{\pi_{\mu\nu}\pi^{00}}{3} \widetilde{G}^{(1)}[\mathbf{k}] \frac{8\pi M}{c^2}$$

The running constants depend on momentum

The running constants differ in the two sectors

Standard Einstein equation is recovered when $\widetilde{G}^{(0)}[\mathbf{k}] = \widetilde{G}^{(1)}[\mathbf{k}] = G_N$

Generalized gravity equation (3)

The stationary and isotropic metric can be written in the PPN gauge

$$h_{00}(r) = 2\Phi_N(r)$$

$$h_{jk}(r) = 2\left(\Phi_N(r) - \Phi_P(r)\right)\eta_{jk}$$

The two potentials Φ_N and Φ_P are related to the two running constants :

$$-\mathbf{k}^2 \begin{pmatrix} \Phi_N[\mathbf{k}] \\ \Phi_P[\mathbf{k}] \end{pmatrix} = \frac{4\pi M}{c^2} \begin{pmatrix} \widetilde{G}_N[\mathbf{k}] \\ \widetilde{G}_P[\mathbf{k}] \end{pmatrix}$$

$$\Delta \equiv -\mathbf{k}^2$$

Laplacian

$$\begin{pmatrix} \widetilde{G}_N[\mathbf{k}] \\ \widetilde{G}_P[\mathbf{k}] \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 4 & -1 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} \widetilde{G}^{(0)}[\mathbf{k}] \\ \widetilde{G}^{(1)}[\mathbf{k}] \end{pmatrix}$$

Standard Poisson equation is recovered when $\begin{pmatrix} \widetilde{G}_N[\mathbf{k}] \\ \widetilde{G}_P[\mathbf{k}] \end{pmatrix} = \begin{pmatrix} G_N \\ 0 \end{pmatrix}$

Phenomenological consequences of the extension of gravity theory

The first potential Φ_N generalizes the Newton potential.
It has to remain close to its standard expression !

The second potential Φ_P opens free space for new phenomena.

It produces a Pioneer-like anomaly for probes with large radial velocities.

It affects propagation of light as a generalized Eddington parameter γ.

A simple version of the extension

Newton-like terms
plus terms linear in r

$$\begin{pmatrix} \Phi_N(\mathbf{x}) \\ \Phi_P(\mathbf{x}) \end{pmatrix} = -\frac{M}{rc^2}\begin{pmatrix} G_N \\ G_P \end{pmatrix} + \frac{Mr}{c^2}\begin{pmatrix} \zeta_N \\ \zeta_P \end{pmatrix}$$

Associated
running constants

$$\begin{pmatrix} \widetilde{G}_N[\mathbf{k}] \\ \widetilde{G}_P[\mathbf{k}] \end{pmatrix} = \begin{pmatrix} G_N \\ G_P \end{pmatrix} + \frac{2}{\mathbf{k}^2}\begin{pmatrix} \zeta_N \\ \zeta_P \end{pmatrix}$$

G_N effective Newton constant

ζ_N long range modification of Newton potential $\qquad \frac{\zeta_N}{G_N} \leftrightarrow \left(-\frac{\alpha}{\lambda^2}\right)$

$G_P \leftrightarrow$ Eddington parameter γ differing from unity

$\zeta_P \leftrightarrow$ long range effect for the second potential

These expressions are not exact at all distances or all momenta.
They can be considered as expansions valid for $r \ll \lambda$ or $|\mathbf{k}|\,\lambda \gg 1$
of Yukawa functions better-behaved at large distances or low momenta.

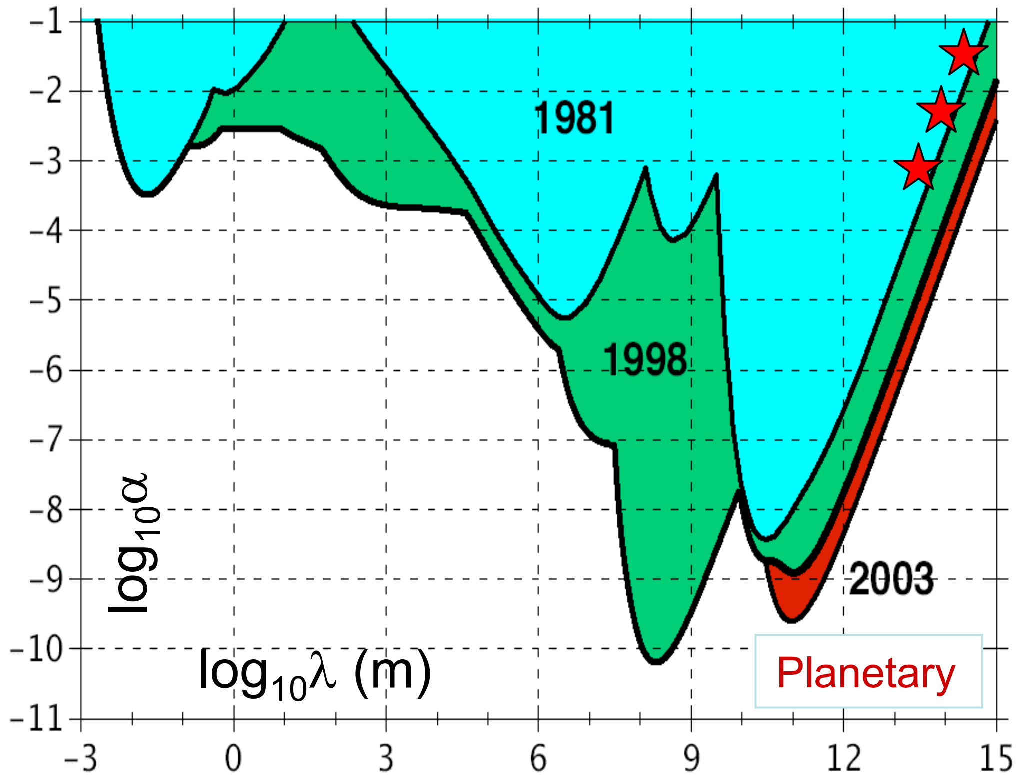
Modification of Newton potential

The value of ζ_N
needed to explain
the Pioneer anomaly
(with Φ_P =0)

$$\zeta_N M \sim 8 \times 10^{-10}\,\mathrm{ms}^{-2}$$

is much too large
to remain undetected
on planetary tests

$$|\zeta_N M| \lesssim 5 \times 10^{-13}\,\mathrm{ms}^{-2}$$



But the presence of Φ_P
opens free space
for new phenomena

In a first approach,
we evaluate its effect
with ζ_N =0

A Pioneer-like anomaly

To evaluate the effect of Φ_P (with ζ_N =0) on probes having large radial velocities :

- we evaluate the motion of the probes in the modified metric
- we take into account the perturbation of light propagation to and from the probes
- we write the result as an equivalent acceleration a
- we subtract the result of the standard calculation to obtain the anomaly $\delta a \equiv a - [a]_{\mathrm{standard}}$

We find :

υ_r radial velocity

$$\delta a \simeq 2 \frac{\mathrm{d}\Phi_P}{\mathrm{d}r} \upsilon_r^2 \simeq 2 \left(\zeta_P M + \frac{G_P M}{r^2} \right) \frac{\upsilon_r^2}{c^2}$$

A Pioneer-like anomaly constant at long distances

If we identify the constant with the observed Pioneer anomaly, we fix the unknown parameter

$$\zeta_P M \sim a_P \frac{c^2}{2\upsilon_r^2} \sim 0.25 \, \mathrm{ms}^{-2}$$

Modification of the deflection of light

The modification of Einstein gravity needed to obtain the Pioneer anomaly should not spoil its agreement with other gravity tests

A critical problem : the effect of Φ_P on the propagation of light rays

$$\delta\theta \equiv \theta - [\theta]_{\mathrm{standard}} \simeq \frac{2G_P M}{r_0 c^2} - \frac{2\zeta_P M r_0}{c^2} L\left(r_0\right)$$

If ζ_P =0, this is equivalent to the usual PPN result

$$1 - \gamma = \frac{G_P}{G_N}$$

When $\zeta_P \neq 0$, Eddington tests (and Shapiro tests) should show an anomaly depending on the distance of closest approach

This is a further motivation for high-accuracy Eddington tests (SORT, LATOR) or astrometric surveys (GAIA)

The next step : study combined effects of the two potentials

The modified equation of gravity naturally leads to combined effects of the two potentials

It is now necessary to perform new analysis of the motions of planets and probes in the solar system

Other possibilities for testing the new framework :

- check its predictions against old Pioneer data
- check its predictions against fly-by data
- look for the r_0 dependence in Eddington / Shapiro tests
- look for the υ_r dependence on new probes
- …