Hyperspectral Unmixing Via Sparse Regression Optimization Problems and Algorithms

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Outline

- Introduction to hyperspectral unmixing
- Hyperspectral unmixing via sparse regression.
- Recovery guarantees/convex and nonconvex solvers
- Improving hyperspectral sparse regression
  - structured sparsity
  - dictionary pruning
- Solving convex sparse hyperspectral unmixing and related convex inverse problems with ADMM
- Concluding remarks
Hyperspectral imaging (and mixing)
Linear mixing model (LMM)

Incident radiation interacts only with one component (checkerboard type scenes)

\[ y = \sum_{i=1}^{p} s_i \rho_i \]

\[ \rho_i = \begin{bmatrix} \rho_{1i} \\ \rho_{2i} \\ \vdots \\ \rho_{mi} \end{bmatrix} \]

\[ y = Ms \]

\[ M \equiv [\rho_1, \rho_2, \rho_3] \]

\[ s \equiv \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} \]

Hyperspectral linear unmixing

Estimate \( M, s \)
Hyperspectral unmixing

AVIRIS of Cuprite, Nevada, USA

R – ch. 183 (2.10 μm)
G – ch. 193 (2.20 μm)
B – ch. 207 (2.34 μm)  

VCA [Nascimento, B-D, 2005]
NIR tablet imaging

[Lopes et al., 2010]
Spectral linear unmixing (SLU)

Given $N$ spectral vectors of dimension $m$: $Y = [y_1, \ldots, y_N]$

Subject to the LMM: $Y = MS + N$, $S \geq 0$, $1^T_p S = 1_N$

ANC: abundance nonnegative constraint
ASC: abundance sum-to-one constraint

Determine:
- The mixing matrix $M$ (*endmember spectra*)
- The *fractional abundance* vectors $S$

$\Rightarrow$ SLU is a blind source separation problem (BSS)
Geometrical view of SLU

\[ M = [m_1, \ldots, m_p] \]

\[ \sum_{j=1}^{p} \alpha_j = 1, \quad \alpha_j \geq 0 \]

probability simplex \((S_I)\)

\[ S_M = \{ \mathbf{x} \in \mathbb{R}^p : \mathbf{x} = M \alpha, \alpha \in S_I \} \rightarrow (p-1) - \text{simplex} \]

Inferring \( M \) \iff inferring the vertices of the simplex \( S_M \)
Minimum volume simplex (MVS)

Pure pixels

\[ \hat{m}_2 = m_2 \]
\[ \hat{m}_3 = m_3 \]
\[ \hat{m}_1 = m_1 \]

MVS works

No pure pixels

\[ \hat{m}_2 = m_2 \]
\[ \hat{m}_3 = m_3 \]
\[ \hat{m}_1 = m_1 \]

MVS works

No pure pixels

\[ \hat{m}_2 \neq m_2 \]
\[ \hat{m}_3 \neq m_3 \]
\[ \hat{m}_1 \neq m_1 \]

MVS does not work

[B-D et al., IEEE JSTATRS, 2012]
Sparse regression-based SLU

- **Key observation**: Spectral vectors can be expressed as linear combinations of a few pure spectral signatures obtained from a (potentially very large) spectral library.

  \[ y = \sum_{i \in S} a_i x_i = Ax \]

  [Iordache, B-D, Plaza, 11, 12]

- **Unmixing**: given \( y \) and \( A \), find the sparsest solution of

  \[ y = Ax \]

- **Advantage**: sidesteps endmember estimation

- **Disadvantage**: Combinatorial problem !!!
Sparse reconstruction/compressive sensing

Key result: a sparse signal is exactly recoverable from an underdetermined linear system of equations in a computationally efficient manner via convex/nonconvex programming

\[ \text{[Candes, Romberg, Tao, 06]} \quad \text{[Candes, Tao, 06]} \quad \text{[Donoho, Tao, 06]} \quad \text{[Blumensath, Davies, 09]} \]

Let \( \mathbf{A} \in \mathbb{R}^{m \times n}, \ m < n, \) and \( \mathbf{x}^*, \) such that \( \mathbf{A}\mathbf{x}^* = \mathbf{y} \)

\( \mathbf{x}^* \) is the unique solution of \( \mathbf{A}\mathbf{x} = \mathbf{y} \) if \( 2\|\mathbf{x}^*\|_0 < \text{spark}(\mathbf{A}) \)

\( \mathbf{x}^* \) is the solution of the optimization problem

\[
(P_0) \quad \min_{\mathbf{x}} \|\mathbf{x}\|_0 \quad \text{subject to} \quad \mathbf{A}\mathbf{x} = \mathbf{y}
\]

NP-hard
Sparse reconstruction/compressive sensing

Optimization strategies to cope with $P_o$ NP-hardness

**Convex relaxation**

(BP – Basis Pursuit) \[ \min_x \|x\|_1 \quad \text{st: } Ax = y \]

(BPDN – BP denoising)

\[ \min_x \|x\|_1 \quad \text{st: } \|Ax - y\|_2 \leq \delta \]

(LASSO) \[ \min_x (1/2)\|Ax - y\|_2^2 + \lambda \|x\|_1 \]

**Approximation algorithms**

Bayesian CS \[ \text{[Ji et al., 2008]} \]

CoSaMP – Compressive Sampling Matching Pursuit \[ \text{[Needell, Tropp, 2009]} \]

IHT – Iterative Hard Thresholding

\[ \text{[Blumensath, Davies, 09]} \]

GDS - Gradient Descent Sparsification

\[ \text{[Garg Khandekar, 2009]} \]

HTP – Hard Thresholding Pursuit

\[ \text{[Foucart, 10]} \]

MP - Message Passing \[ \text{[Villa Schniter, 2012]} \]
Exact recovery of sparse vectors

Recovery guarantees: linked with the restricted isometric property (RIP)

Restricted isometric constant: \( \delta_p(A), \quad A \in \mathbb{R}^{m \times n}: \)

\[
(1 - \delta) \|x\|_2 \leq \|Ax\| \leq (1 + \delta) \|x\|_2, \quad \|x\|_0 \leq p
\]

Many SR algorithms ensure exact recovery provided that:

\[ \delta_t(A) \leq \delta_*, \text{ for some } t \text{ and } \delta_* \]

This condition is satisfied for random matrices provided that

\[
m \simeq c \frac{t}{\delta_*^2} \log(n/t)
\]

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>BP</th>
<th>HTP</th>
<th>CoSaMP</th>
<th>GDP</th>
<th>IHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_t &lt; \delta_* )</td>
<td>( \delta_{2s} &lt; 0.465 )</td>
<td>( \delta_{3s} &lt; 0.577 )</td>
<td>( \delta_{4s} &lt; 0.384 )</td>
<td>( \delta_{2s} &lt; 0.333 )</td>
<td>( \delta_{3s} &lt; 0.555 )</td>
</tr>
<tr>
<td>Ratio ( t/\delta_*^2 )</td>
<td>9.243</td>
<td>9</td>
<td>27.08</td>
<td>18</td>
<td>12</td>
</tr>
</tbody>
</table>

(from [Foucart, 10])
Example: Gaussian matrices; signed signals

\( \mathbf{A} \in \mathbb{R}^{m \times n}; \) \( m = 200, n = 400; \) \( \mathcal{N}(0, 1); \) iid; \( x_i \sim \mathcal{N}(0, 1) \)

Algorithms:

\[
\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{st:} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \delta
\]

Signal to Reconstruction Error

- **BPDN – SUnSAL** [B-D, Figueiredo, 2010]
- **EM-GM-AMP** [Villa Schniter, 2012]
Example: Gaussian matrices; non-negative signals

\[ A \in \mathbb{R}^{m \times n}; \quad \mathcal{N}(0, 1) \text{iid} ; \quad m = 200, n = 400; \quad x_i \sim \text{UD in the simplex} \]

Algorithms:

\[
\begin{align*}
&\text{(BPDN – SUnSAL)} \\
&\min_x \|x\|_1 \quad \text{st: } \|Ax - y\|_2 \leq \delta, \ x \geq 0
\end{align*}
\]

EM-BG-AMP

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**Signal to Reconstruction Error**

![Graph showing Signal to Reconstruction Error vs. \(\|x\|_0\) for SNR = 100 dB and SNR = 25 dB.](image)

- **BPDN**: Blue line
- **EM-GM-AMP**: Green line

SNR = 100 dB

SNR = 25 dB
Hyperspectral libraries exhibit poor RI constants

(Mutual coherence close to 1 \cite{Iordache, B-D, Plaza, 11, 12})

Illustration: \( \mathbf{A} \in \mathbb{R}^{m \times n}; \) subset from USGS; \( m = 200, n = 400 \)
Example: Hyperspectral library; non-negative signals

Illustration: \( \mathbf{A} \in \mathbb{R}^{m \times n} \); subset from USGS; \( m = 200, n = 300 \)

Algorithms:

\[
\begin{align*}
\min_{x} & \quad \|x\|_1 \\
\text{st:} & \quad \|\mathbf{A}x - \mathbf{y}\|_2 \leq \delta, \ x \geq 0
\end{align*}
\]

(\text{BPDN – SUnSAL})

\[
\theta_{\min} \geq 10^\circ
\]

\[
\theta_{\min} \leq 4^\circ
\]

Significant to Reconstruction Error

- **BPDN - 100 dB**
- **EM-GM-AMP 100 dB**
- **BPDN - 30 dB**
- **EM-GM-AMP 30 dB**
Phase transition curves

\( \ell_1, -/\ell_0 \) equivalence for non-negative signals \((A \in \mathbb{R}^{m \times n})\)

\[ \delta = \frac{m}{n} \quad \text{undersampling factor} \quad \rho = \frac{\|x\|_0}{m} \quad \text{fractional sparsity} \]

\[ \rho = 0.025 \quad \text{(enough for many hyperspectral applications)} \]

\[ \text{SNR = 30 dB} \]

\[ \text{SNR = } \infty \]

\[ \text{USGS} \]

A - orthogonal projector

Results linked with the k-neighborliness property of polytopes

[Donoho, Tanner, 2005]
Real data – AVIRIS Cuprite

[Iordache, B-D, Plaza, 11, 12]
Sparse reconstruction of hyperspectral data: Summary

Bad news: Hyperspectral libraries have poor RI constants

Good news: Hyperspectral mixtures are highly sparse, very often $p \leq 5$

Surprising fact: Convex programs (BP, BPDN, LASSO, …) yield much better empirical performance than non-convex state-of-the-art competitors

Directions to improve hyperspectral sparse reconstruction

- Structured sparsity + subspace structure (pixels in a give data set share the same support)
- Spatial contextual information (pixels belong to an image)
Beyond $l_1$ pixelwise regularization

**Rationale:** introduce new sparsity-inducing regularizers to counter the sparse regression limits imposed by the high coherence of the hyperspectral libraries.

Let’s rewrite the LMM as

\[ Y = AX + N \]

where

\[ Y \in \mathbb{R}^{m \times N} \]
\[ A \in \mathbb{R}^{m \times n} \]
\[ X \in \mathbb{R}^{n \times N} \]

- $X$ is the image band
- $x_j$ are the regression coefficients of pixel $j$
Constrained total variation sparse regression (CTVSR)

\[
\min_{\mathbf{X}} \frac{1}{2} \left\| \mathbf{A}\mathbf{X} - \mathbf{Y} \right\|_F^2 + \lambda_1 \left\| \mathbf{X} \right\|_1 + \lambda_2 \phi_{TV}(\mathbf{X})
\]

subject to: \( \mathbf{X} \geq 0 \)

\[
\phi_{TV}(\mathbf{X}) := \sum_{i=1}^{n} \left\| \mathbf{Lx}^i \right\|_1 = \sum_{i=1}^{n} \sum_{k=1}^{N} \sqrt{([D_h x^i]_k)^2 + ([D_v x^i]_k)^2}
\]

Related work \[\text{[Zhao, Wang, Huang, Ng, Plemmons, 12]}\]
Illustrative examples with simulated data: SUnSAL-TV

$\mathbf{A} \in \mathbb{R}^{224 \times 240}$ (from USGS library)

Original data cube

$\left( m = 224, N = 75 \times 75, k = 5 \right)$

SUnSAL estimate

Original abundance of EM5

SUnSAL-TV estimate
Constrained collaborative sparse regression (CCSR)

\[
\min_{X} \frac{1}{2} \| AX - Y \|_F^2 + \lambda \| X \|_{2,1}
\]

subject to: \( X \geq 0, \quad 1^T_n X = 1^T_N \)

[Iordache, B-D, Plaza, 11, 12]  
[Turlach, Venables, Wright, 2004]

Theoretical guarantees (superiority of multichannel): the probability of recovery failure decays exponentially in the number of channels.  

[ Eldar, Rauhut, 11 ]
Illustrative examples with simulated data: CSUnSAL

\[ \mathbf{A} \in \mathbb{R}^{224 \times 350} \text{ (from USGS library)} \quad \mathbf{x} \in \mathbb{R}^{350 \times 100} \text{ (sparsity } k = 5) \]

\[ \text{SNR} = 35 \text{dB} \quad \text{time} = 10 \text{ sec} \]
Multiple measurements

The multiple measurement vector (MMV) problem

\[
\text{minimize } \|X\|_0 \\
\text{subject to: } Y = AX
\]

\[\|X\|_0 - \text{number of non-null rows of } X\]

MMV has a unique solution iff

\[\|X\|_0 \leq \frac{\text{spark}(A) + \text{rank}(Y) - 1}{2}\]

MMV gain

If \(\text{rank}(Y) = \|X\|_0\), the above bound is achieved using the multiple signal classification (MUSIC) algorithm

[Feng, 1997], [Chen, Huo, 2006], [Davies, Eldar, 2012]
Endemember identification with MUSIC

\[ Y = AX \quad \text{(noiseless measurements)} \]

MUSIC algorithm

1) Compute \( E = [e_1, \ldots, e_p] \), the first \( p \) eigenvalues of \( R_y = YY^T/n \)

2) Compute \( \varepsilon_i = \frac{\|P_y^\perp a_i\|}{\|a_i\|}, \) for \( i = 1, \ldots, m \) and set \( M = A_S \) with \( S = \{ i : \varepsilon_i = 0, i = 1, \ldots, m \} \)

\[ \text{span}\{A_S\} = \text{span}\{E\} \]
Examples (simulated data)

$A \sim \text{USGS (} \geq 3^\circ \text{)} \ p = 10, \ N=5000, \ X \sim \text{uniform over the simplex}, \ \text{SNR} = \infty,$

\[ Y = AX + N \]

Gaussian iid noise, $\text{SNR} = 25 \text{ dB}$
Examples (simulated data)

\[ Y = AX + N \]

colored noise, SNR = 25 dB

cause of the large projection errors: poor identification of the subspace signal

\[ R_y \simeq A_S R_x A_S^T + R_n \neq \sigma^2 I \]

\[ \Rightarrow \text{span}\{E\} \neq \text{span}\{A_S\} \]

cure: identify the signal subspace
Signal subspace identification

colored noise, SNR = 25 dB (incorrect signal subspace)

colored noise, SNR = 25 dB (signal subspace identified with HySime, [BD, Nascimento, 2008])
Proposed MUSIC – Collaborative SR algorithm

**MUSIC-CSR algorithm [B-D, 2012]**

1) Estimate the signal subspace \( \text{span}\{A_S\} \) using, e.g., the HySime algorithm.

2) Compute \( \varepsilon_i = \frac{\|P_y a_i\|}{\|a_i\|} \), for \( i = 1, \ldots, m \) and define the index set \( S = [i : \varepsilon_i \leq \delta, i = 1, \ldots, m] \)

3) Solve the collaborative sparse regression optimization

\[
\min_{X} \left( \frac{1}{2} \|Y - A_S X\|^2 + \lambda \|X\|_{2,1}, \quad X \geq 0 \right)
\]

[B-D, Figueiredo, 2012]

Related work: CS-MUSIC [Kim, Lee, Ye, 2012]

\( (N < k \text{ and iid noise}) \)
MUSIC – CSR results

A – USGS ($\geq 3^\circ$), Gaussian shaped noise, SNR = 25 dB, $k = 5$, $m = 300$,

$$|S| = 11 \quad \hat{X} \ (\text{MUSIC-CSR})$$

MUSIC-CSR

- SNR = 11.7 dB
- Computation time $\simeq 10$ sec

CSR

- SNR = 0 dB
- Computation time $\simeq 600$ sec

true endmembers

acummulated abundances
Results with CUPRITE

- size: 350x350 pixels
- spectral library: 302 materials (minerals) from the USGS library
- spectral bands: 188 out of 224 (noisy bands were removed)
- spectral range: 0.4 – 2.5 um
- spectral resolution: 10 nm
- “validation” map: Tetracorder***
Results with real data

Tetracorder classification maps

Abundance maps obtained with SUnSAL ($\lambda = 0.001$) when 40 materials are retained

Note: Good spatial distribution of the endmembers

Processing times: 2.6 ms/pixel using the full library; 0.22ms/pixels using the pruned library with 40 members
Convex optimization problems in SLU

Constrained least squares (CLS)

\[ \min_{X} \|AX - Y\|_F^2 \]
subject to: \( X \geq 0 \)
\[ \|X\|_F^2 := \text{tr}\{XX^T\} \]

Fully constrained least squares (FCLS)

\[ \min_{X} \|AX - Y\|_F^2 \]
subject to: \( X \geq 0, \quad 1^T_n X = 1^T_N \)

Constrained sparse regression (CSR)

\[ \min_{X} (1/2)\|AX - Y\|_F^2 + \lambda\|X\|_1 \]
subject to: \( X \geq 0 \)
\[ \|X\|_1 := \sum_{i=1}^N \|x_i\|_1 \]
Convex optimization problems in SLU

Constrained basis pursuit (CBP)

\[
\min_{X} \|X\|_1 \\
\text{subject to: } AX = Y, \ X \geq 0
\]

CBP denoising (CBPDN)

\[
\min_{X} \|X\|_1 \\
\text{subject to: } \|AX - Y\|_F \leq \delta, \ X \geq 0
\]

Constrained collaborative sparse regression (CCSR)

\[
\min_{X} \frac{1}{2}\|AX - Y\|_F^2 + \lambda \|X\|_{2,1} \\
\text{subject to: } X \geq 0, \ 1^T_n X = 1^T_N
\]

Constrained total variation (CTV)

\[
\min_{X} \frac{1}{2}\|AX - Y\|_F^2 + \lambda_1 \|X\|_1 + \lambda_2 \phi_{TV}(X) \\
\text{subject to: } X \geq 0
\]

\[\|X\|_{2,1} := \sum_{i=1}^{n} \|x^i\|_2\]
Convex optimization problems in SLU

Structure of the optimization problems:

\[ \min_{X \in C} \sum_{i=1}^{J} g_i(X) \]

Source of difficulties: large scale \((n \times N \gtrsim 10^7)\); nonsmoothness

Line of attack: alternating direction method of multiplies (ADMM)

[Glowinski, Marrocco, 75], [Gabay, Mercier, 76]
Alternating Direction Method of Multipliers (ADMM)

Unconstrained (convex) optimization problem: \[
\min_{z \in \mathbb{R}^d} f_1(z) + f_2(Gz)
\]

ADMM [Glowinski, Marrocco, 75], [Gabay, Mercier, 76]

\[
z_{k+1} = \arg \min_z f_1(z) + \frac{\mu}{2} \| Gz - u_k - d_k \|^2
\]

\[
u_{k+1} = \arg \min_u f_2(u) + \frac{\mu}{2} \| Gz_{k+1} - u - d_k \|^2
\]

\[
d_{k+1} = d_k - (Gz_{k+1} - u_{k+1})
\]

Interpretations: variable splitting + augmented Lagrangian + NLBGS;
Douglas-Rachford splitting on the dual [Eckstein, Bertsekas, 92];
split-Bregman approach [Goldstein, Osher, 08]
Consider the problem

$$\min_{z \in \mathbb{R}^d} f_1(z) + f_2(Gz)$$

Let $f_1$ and $f_2$ be closed, proper, and convex and $G$ have full column rank.

Let $(z_k, k = 0, 1, 2, \ldots)$ be the sequence produced by ADMM, with $\mu > 0$, then, if the problem has a solution, say $\bar{z}$, then

$$\lim_{k \to \infty} z_k = \bar{z}$$

The theorem also allows for inexact minimizations, as long as the errors are absolutely summable.

Convergence rate: $O(1/\varepsilon)$ [He, Yuan, 2011]
Consider a more general problem

\[
\min_{\mathbf{z} \in \mathbb{R}^d} \sum_{j=1}^{J} g_j(\mathbf{H}^{(j)} \mathbf{z}) \quad (P)
\]

\[g_j : \mathbb{R}^{p_j} \to \bar{\mathbb{R}}\]
\[\mathbf{H}^{(j)} \in \mathbb{R}^{p_j \times d}\]

Proper, closed, convex functions

There are many ways to write \((P)\) as

\[
\min_{\mathbf{z} \in \mathbb{R}^d} f_1(\mathbf{z}) + f_2(\mathbf{G} \mathbf{z})
\]

We adopt:

\[f_1(\mathbf{z}) = 0, \quad \mathbf{G} = \begin{bmatrix} \mathbf{H}^{(1)} \\ \vdots \\ \mathbf{H}^{(J)} \end{bmatrix}, \quad f_2\left(\begin{bmatrix} \mathbf{u}^{(1)} \\ \vdots \\ \mathbf{u}^{(J)} \end{bmatrix}\right) = \sum_{j=1}^{J} g_j(\mathbf{u}^{(j)})\]

Another approach in [Goldfarb, Ma, 09, 11]
Applying ADMM to more than two functions

\[
z_{k+1} = \left[ \sum_{j=1}^{J} (H^{(j)})^* H^{(j)} \right]^{-1} \left( \sum_{j=1}^{J} (H^{(j)})^* \left( u^{(j)}_k + d^{(j)}_k \right) \right)
\]

\[
u^{(1)}_{k+1} = \arg\min_u g_1(u) + \frac{\mu}{2} \| u - H^{(1)} z_{k+1} + d^{(1)}_k \|^2 = \text{prox}_{g_1/\mu}(H^{(1)} z_{k+1} - d^{(1)}_k)
\]

\[
u^{(j)}_{k+1} = \arg\min_u g_J(u) + \frac{\mu}{2} \| u - H^{(j)} z_{k+1} + d^{(j)}_k \|^2 = \text{prox}_{g_1/\mu}(H^{(1)} z_{k+1} - d^{(j)}_k)
\]

\[
d^{(1)}_{k+1} = d^{(1)}_k - (H^{(1)} z_{k+1} - u^{(1)}_{k+1})
\]

\[
d^{(j)}_{k+1} = d^{(j)}_k - (H^{(j)} z_{k+1} - u^{(j)}_{k+1})
\]

Conditions for easy applicability:

- inexpensive proximity operators
- inexpensive matrix inversion
Constrained sparse regression (CSR)

Problem \[ \min_X (1/2)\|AX - Y\|_F^2 + \lambda\|X\|_1, \quad \text{subject to: } X \geq 0 \]

Equivalent formulation \[ \min_X (1/2)\|AX - Y\|_F^2 + \lambda\|X\|_1 + \text{indicator of the first orthant} \]

Template: \[ \min_X \sum_{j=1}^{J=3} g_j(H^{(j)}X) \]

Mapping:

\[
\begin{align*}
H^{(1)} &= A, & g_1(Z) &= (1/2)\|Z - Y\|_F^2, \\
H^{(2)} &= I, & g_2(Z) &= \lambda\|Z\|_1, \\
H^{(3)} &= I, & g_3(Z) &= \nu_{\mathbb{R}^+}(Z)
\end{align*}
\]

Proximity operators:

\[
\begin{align*}
\text{prox}_{g_1/\mu}(W) &= \frac{Y + \mu W}{1 + \mu}, \\
\text{prox}_{g_2/\mu}(W) &= \text{soft}(W, \lambda/\mu), \\
\text{prox}_{g_3/\mu}(W) &= \max\{0, W\}
\end{align*}
\]

Matrix inversion can be precomputed \( (\text{typical sizes } 200\sim300 \times 500\sim1000) \)

\[
\left[ \sum_{j=1}^{J} (H^{(j)\ast} H^{(j)}) \right]^{-1} = \left[ A^T A + 2I \right]^{-1}
\]

Spectral unmixing by split augmented Lagrangian (SUnSAL) [B-D, Figueiredo, 2010]

Related algorithm (split-Bregman view) in [Szlam, Guo, Osher, 2010]
Concluding remarks

- Sparse regression framework, used with care, yields effective hyperspectral unmixing results.

- Critical factors
  - High mutual coherence of the hyperspectral libraries
  - Non-linear mixing and noise
  - Acquisition and calibration of hyperspectral libraries

- Favorable factors
  - Hyperspectral mixtures are highly sparse

- ADMM is a very flexible and efficient tool solving the hyperspectral sparse regression optimization problems


References


