Statistical Signal Processing Workshop

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A Copula based Framework for Distributed Inference

Pramod K. Varshney

EECS Department, Syracuse University | varshney@syr.edu

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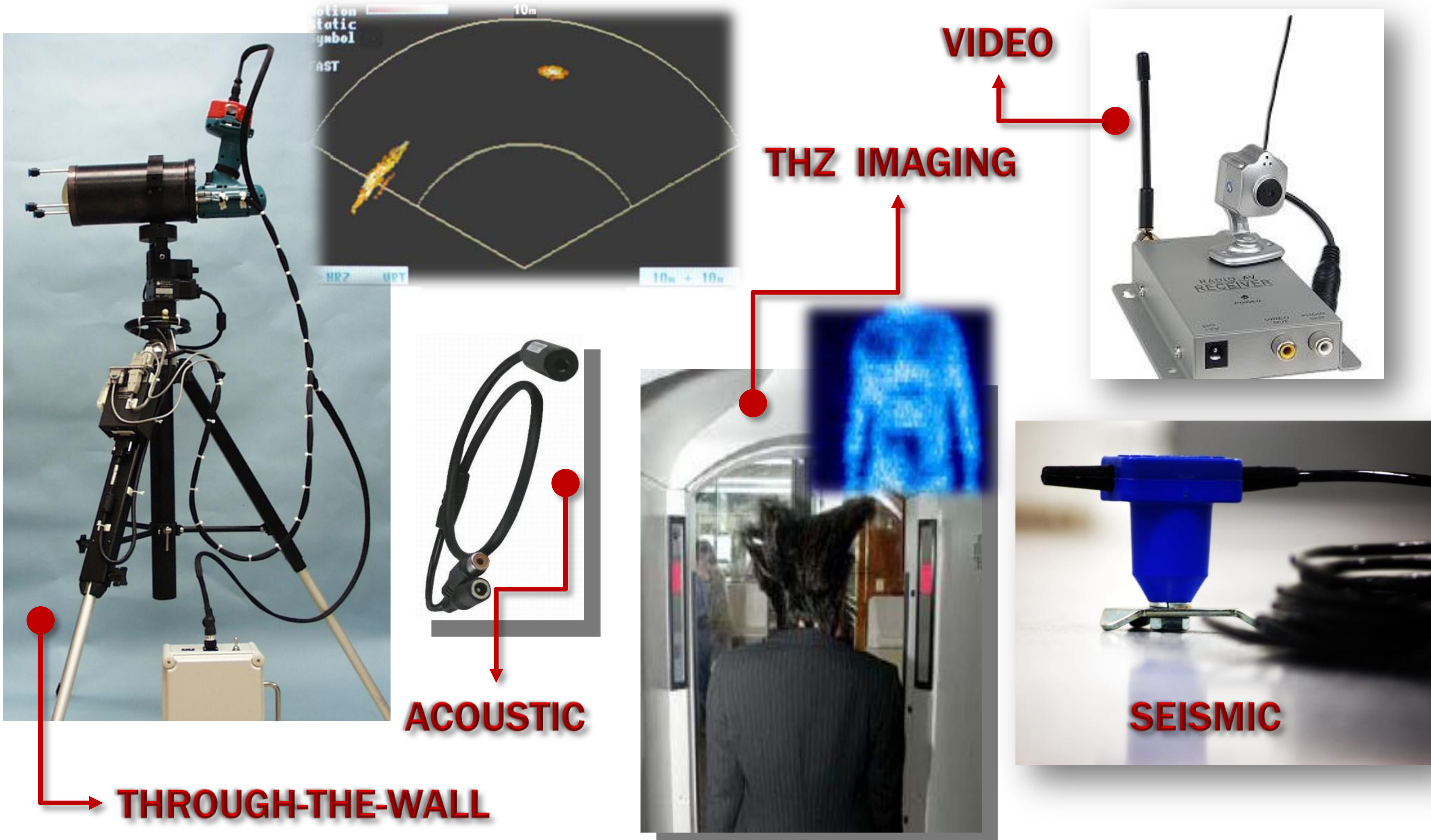
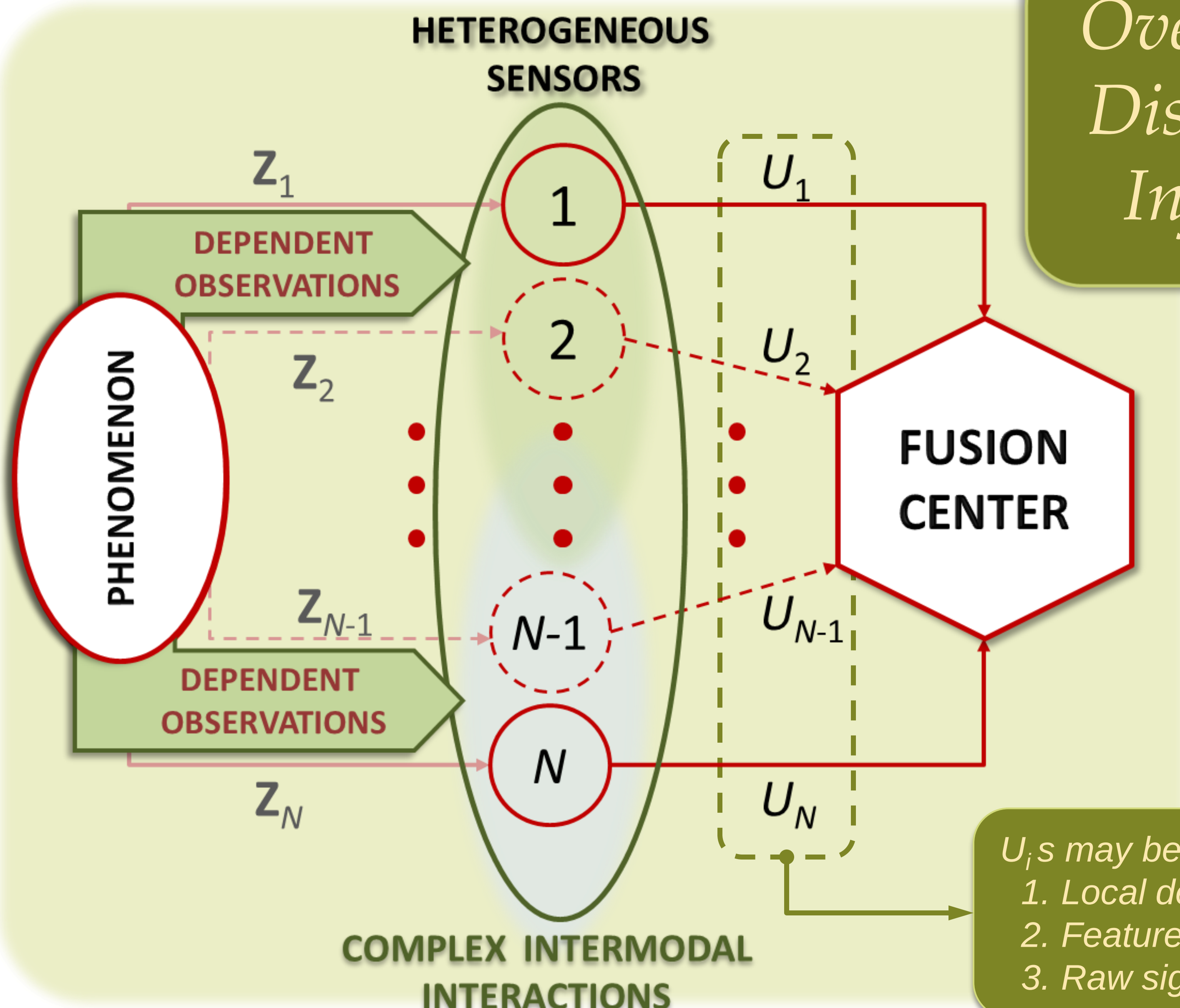
Different Sensors, Diverse Information



Overview of Distributed Inference



U_is may be

1. *Local decisions*
2. *Features*
3. *Raw signal*

Outline

- Introduction
- Heterogeneity and Dependence
- Copula theory
- Signal Detection Using Copulas
- Copula-based Parameter Estimation (Localization)
- Classification using copulas
- *Applications in finance are not considered!*
- Conclusion

Previous Work: Signal Processing Using Dependent Observations

- Different characterizations of dependence exist, e.g.,
  - Correlation coefficient – Linear measure of dependence
  - Information theoretic, e.g., mutual information – Computational difficulties
- Initial work on *distributed inference* assumed independence for tractability
  - Distributed detection with dependent observations is an NP-complete problem [*Tsitsiklis & Athans, 1985*]
- Decision fusion strategies to incorporate correlation among sensor decisions
  - [*Drakopolous & Lee, 1991*] Assumes correlation coefficients are known
  - [*Kam et al. 1992*] Bahadur-Lazarsfeld expansion of PDF’s
  - Both approaches assume prior knowledge of joint statistics

nference with dependent observations: difficult probler

Proposed solutions: largely problem specific

Inference Under Dependent Observations: Different Approaches

- Non-parametric, learning-based
  - ***HMMs & other graphical models***
    - M. J. Beal *et al.*, "A Graphical Model for Audio-Visual Object Tracking," *Trans. PAMI*, July 2003, Vol. 25, No. 7, pp. 828-836.
    - M. R. Siracusa and J. Fisher III, "Dynamic dependency tests: analysis and applications to multi-modal data association," in *Proc. AI Stats*, 2007.
  - ***Manifold learning***
    - S. Lafon, Y. Keller, R. R. Coifman, "Data fusion and multicue data matching by diffusion maps," *IEEE Trans. PAMI*, vol. 28, no. 11, pp. 1784--1797, Nov. 2006
- General ***information theoretic*** framework for multimodal signal processing
  - T. Butz and J. Thiran, "From error probability to information theoretic (multi- modal) signal processing," *Elsevier: Signal Processing,* vol. 85, May 2005.

Heterogeneity and Dependence

Heterogeneous Random Vectors

Definition

A random vector $\mathbf{Z} = [Z_1, Z_2, \cdots, Z_N]$ *governing the joint statistics of an N-variate data set can be termed as multimodal or heterogeneous if the marginals* Z_n $(n = 1, \cdots, N)$ *are non-identically distributed.*

- For example Z_1 and Z_2 may represent acoustic and video signals/features, respectively
- Definition is general
  - Includes independent and identically distributed (iid) marginals

Models of Dependence

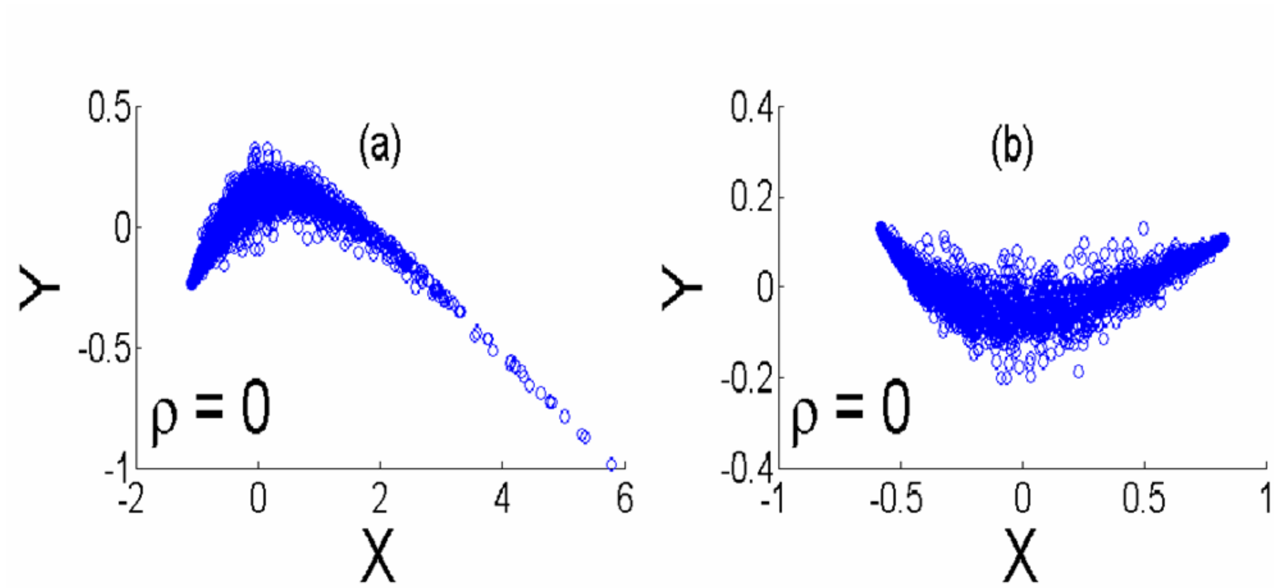
- Product distribution

$$\widehat{\mathbf{f}_{\mathbf{Z}}}(\mathbf{z}) = \prod_{n=1}^{N} \mathbf{f}_{Z_n}(z_n) = \mathbf{f}_{\mathbf{Z}}^{\mathbf{m}}(\mathbf{z})$$

  - Accounts for disparate marginals but not statistical dependence

- Multivariate Gaussian
  - Cannot model disparate marginals
  - Models dependence through Pearson's ρ
  - ρ measures only *linear* relationship

Why is ρ insufficient?



- Dependence between *X* and *Y* evident from scatter plot
- Correlation coefficient is unable to capture this: $\rho = 0$

Measures of Dependence

- Rank-based (nonparametric) measures: Kendall's τ and Spearman's ρ^s quantify *concordance*
- For a bivariate random vector (X, Y) and its realizations (X_1, Y_1) and (X_2, Y_2)

$$\tau_{X,Y} = \underbrace{P\left[(X_1 - X_2)(Y_1 - Y_2) > 0\right]}_{concordance} - \underbrace{P\left[(X_1 - X_2)(Y_1 - Y_2) < 0\right]}_{discordance}$$

- $-1 \leq \tau \leq 1$

Measures of Dependence

- Relative entropy: "distance" from product distribution
  - Multi-information is the multivariate extension of mutual information

$$\mathcal{I}(Z_1; \ldots ; Z_N) = \int_{\mathbf{z}} \mathbf{f_Z}(\mathbf{z}) \log \left(\frac{\mathbf{f_Z}(\mathbf{z})}{\prod_{i=1}^{N} \mathbf{f}_{Z_i}(z_i)} \right) d\mathbf{z}$$

- Normalized measure

$$\delta^* = \sqrt{1 - \exp(-2.\mathcal{I})}$$

$$0 \leq \delta^* \leq 1$$

H. Joe, "Relative entropy measures of multivariate dependence," *Journal of the American Statistical Association*, vol. 84, no. 405, pp. 157-164, 1989

M. Studeny and J. Vejnarova, "The multiinformation function as a tool for measuring stochastic dependence," in *Learning in Graphical Models (M. I. Jordan ed.)* Kluwer, Dordrecht 1998, pp. 261-298

Measures of Dependence

- Wyner's Common Information

$$C(X,Y) = \min_{X \to W \to Y} I(XY;W)$$

- Gács and Körner's Common Randomness
  - For random sequences X^n, Y^n
    - Let $W_1 = f_n(X^n)$ and $W_2 = g_n(Y^n)$. Define $\epsilon_n = \Pr(W_1 \neq W_2)$.

$$K(X,Y) = \lim_{n\to\infty, \epsilon_n \to 0} \sup \frac{1}{n} H(W_1)$$

$$K(X,Y) \leq I(X;Y) \leq C(X,Y)$$

A. D. Wyner, "The common information of two dependent random variables," *IEEE Trans. Inf. Theory*, vol. 21, no. 2, pp. 163-179, March 1975

Copula Theory

Motivation

Concepts

Motivation: Why Copulas?

- Multimodal sensors provide information diversity: *fusing heterogeneous information is a challenge*
  - Incommensurate modalities ⇒ Disparate marginal distributions: e.g. audio-video
  - Complex intermodal interactions → ρ is not sufficient
- A *parametric* probabilistic basis for fusion: Joint distribution of sensor observations as an explicit function of parameters

Copula-based approach attempts to address these issues

- Pr
  - Learning-based
    - Non-parametric, e.g., Hidden Markov Models, Neural Nets, Bayesian Nets
    - Scenario dependent performance and analysis is difficult
    - Curse of dimensionality
  - Simple models are assumed, e.g., independence, joint normality

Copula Theory

- Copulas are functions that *couple* marginals to form a joint distribution
- Sklar's Theorem is a key result – existence theorem

Sklar's Theorem

The joint cumulative distribution function (CDF) $F_{\mathbf{Z}}(z_1, z_2, \cdots, z_N)$ *of random variables* $Z_1, Z_2 \cdots, Z_N$ *are joined by a copula function* $C(\cdot)$ *to the respective marginal distributions* $F_{Z_1}(z_1), F_{Z_2}(z_2), \cdots, F_{Z_N}(z_N)$ *as*

$$F_{\mathbf{Z}}(z_1, z_2, \cdots z_N) = C(F_{Z_1}(z_1), F_{Z_2}(z_2), \cdots, F_{Z_N}(z_N))$$

Further, if the marginals are continuous, $C(\cdot)$ *is unique.*

Copula Theory

- Differentiate the joint CDF to get the joint PDF

$$f(z_1, \ldots, z_N) = \left(\prod_{i=1}^{N} f(z_i) \right) c(F_1(z_1), \ldots, F_N(z_N))$$

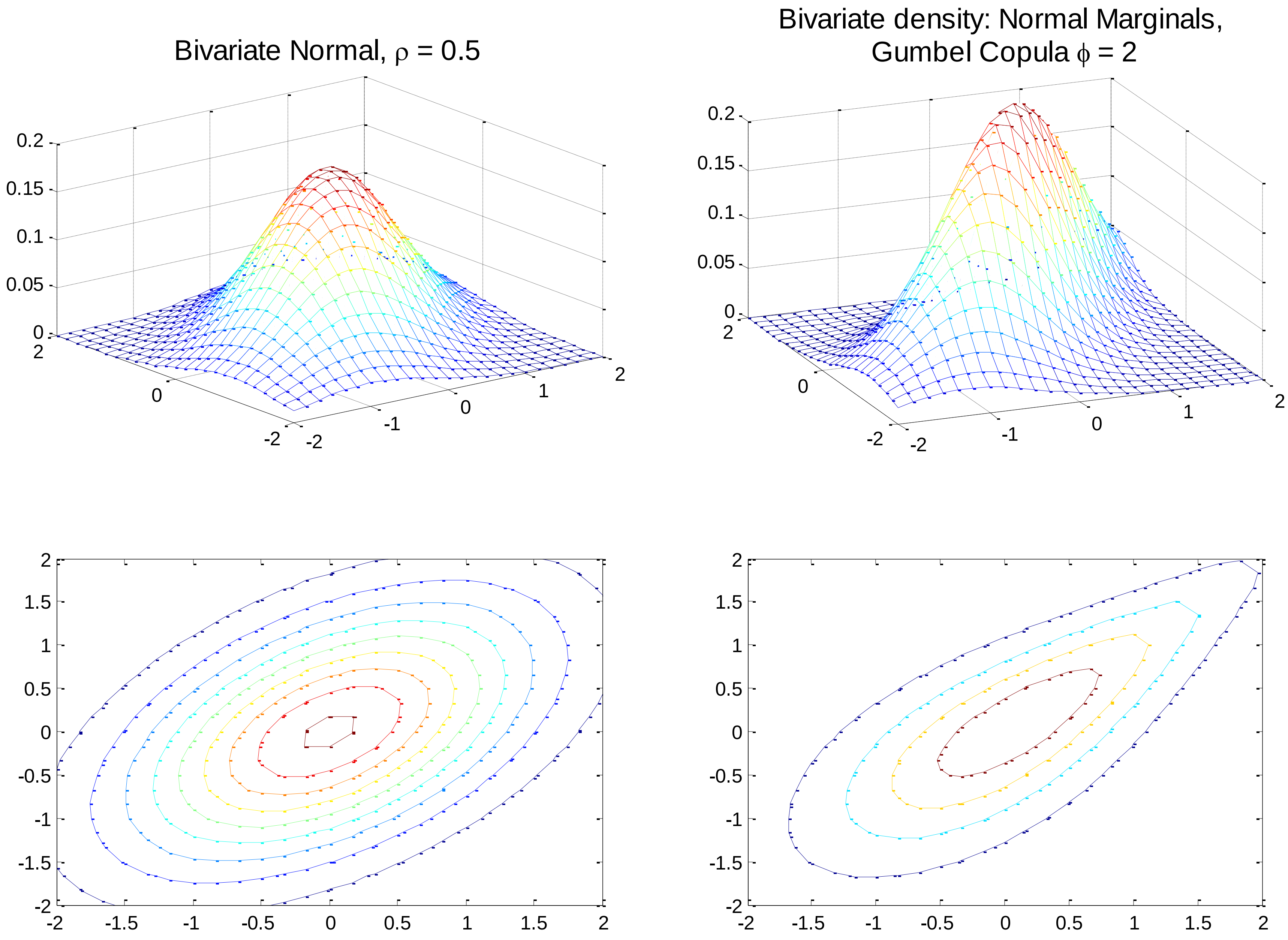
Product density → Independence

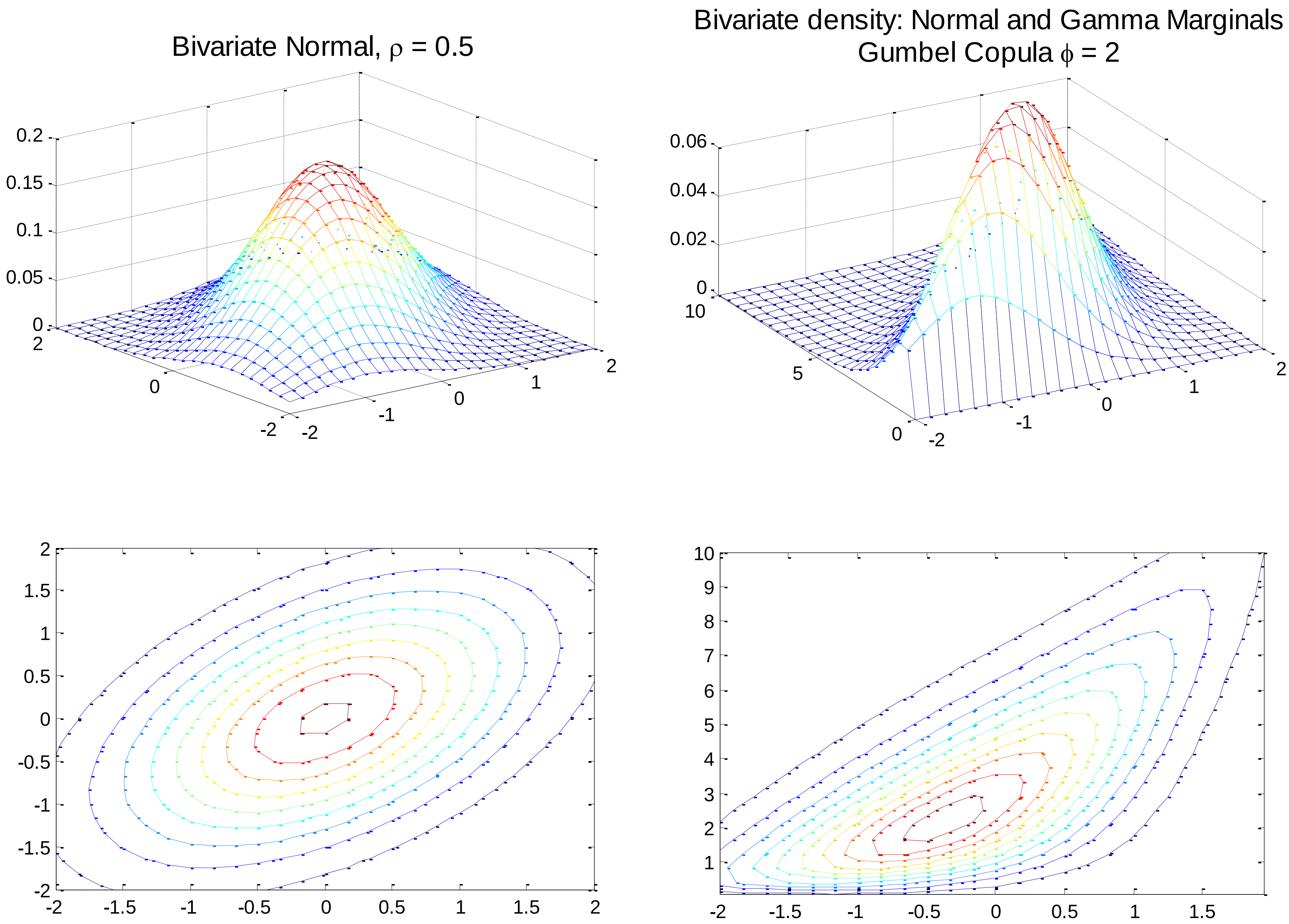
Copula density

N marginals (E.g., from *N* sensors) → Uniform random variables!

Copula Theory

- Several copulas have been proposed
  - R. Nelsen, *An Introduction to Copulas,* Springer 1999
  - Archimedean copulas & Elliptical copulas
- Widely used in econometrics
  - David Li pioneered the use of the Gaussian Copula
  - Blamed for the meltdown on Wall Street
  - Highlights dangers of applying theory without understanding the implications
- A pictorial example
  - Copulas can characterize skewed dependencies
  - Copulas can express dependency between marginals that do not share the same support (e.g. Normal and Gamma)


Bivariate Normal, ρ = 0.5
Bivariate density: Normal Marginals,
Gumbel Copula ϕ = 2
0.2
0.15
0.1
0.05
0
-2
-1.5
-1
-0.5
0.5
1
1.5
2


Bivariate Normal, ρ = 0.5
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1.5

Summary of Copula Functions

- Copulas are typically defined as a CDF
- Elliptical copulas: derived from multivariate distributions

$$C^G(\mathbf{k}|\Sigma) = \Phi_\Sigma(\Phi^{-1}(k_1), \ldots, \Phi^{-1}(k_m))$$ Gaussian copula

$$C^t(\mathbf{k}|\Sigma, \nu) = t_{\nu,\Sigma}(t_\nu^{-1}(k_1), \ldots, t_\nu^{-1}(k_m))$$ *t*-copula

- Archimedean Copulas

Copula	Generator Function	Parametric Form
Clayton	$\frac{1}{\phi}\left(k^{-\phi}-1\right)$	$\left(\sum_{i=1}^m k_i^{-\phi}-1\right)^{-\frac{1}{\phi}}, \quad \phi \in [-1,\infty)\backslash\{0\}$
Frank	$\frac{\exp^{-\phi}-1}{\exp^{-\phi k}-1}$	$-\frac{1}{\phi}\ln\left(1+\frac{\prod_{i=1}^m(\exp^{-\phi k_i}-1)}{\exp^{-\phi}-1}\right), \phi \in \mathbb{R}\backslash\{0\}$
Gumbel	$-\ln k^\phi$	$\exp\left\{-\left(\sum_{i=1}^m(-\ln k_i)^\phi\right)^{\frac{1}{\phi}}\right\}, \quad \phi \in [1,\infty)$
Independent	$-\ln k$	$\prod_{i=1}^m k_i$

Dependence Through Copulas

- Amount of dependence is characterized by the parameter vector of the copula functions
  - Σ, ν for the elliptical copulas
  - ϕ for the Archimedean copulas

  } Collectively denoted as $\boldsymbol{\phi}$
- Typically $\boldsymbol{\phi}$ is unknown: Estimated using
  - Kendall's τ: For random variables A, B and copula C

$$\begin{aligned} k_\tau(A,B) &= 4\mathbb{E}\{C_{AB}\} - 1 \\ &= \int\int_{I^2} C_{AB}(u,v)dC_{AB}(u,v) - 1 \end{aligned}$$

  - Maximum likelihood

$$\hat{\boldsymbol{\phi}} = \arg\max \sum_i \log c(\mathbf{u}_i|\boldsymbol{\phi})$$

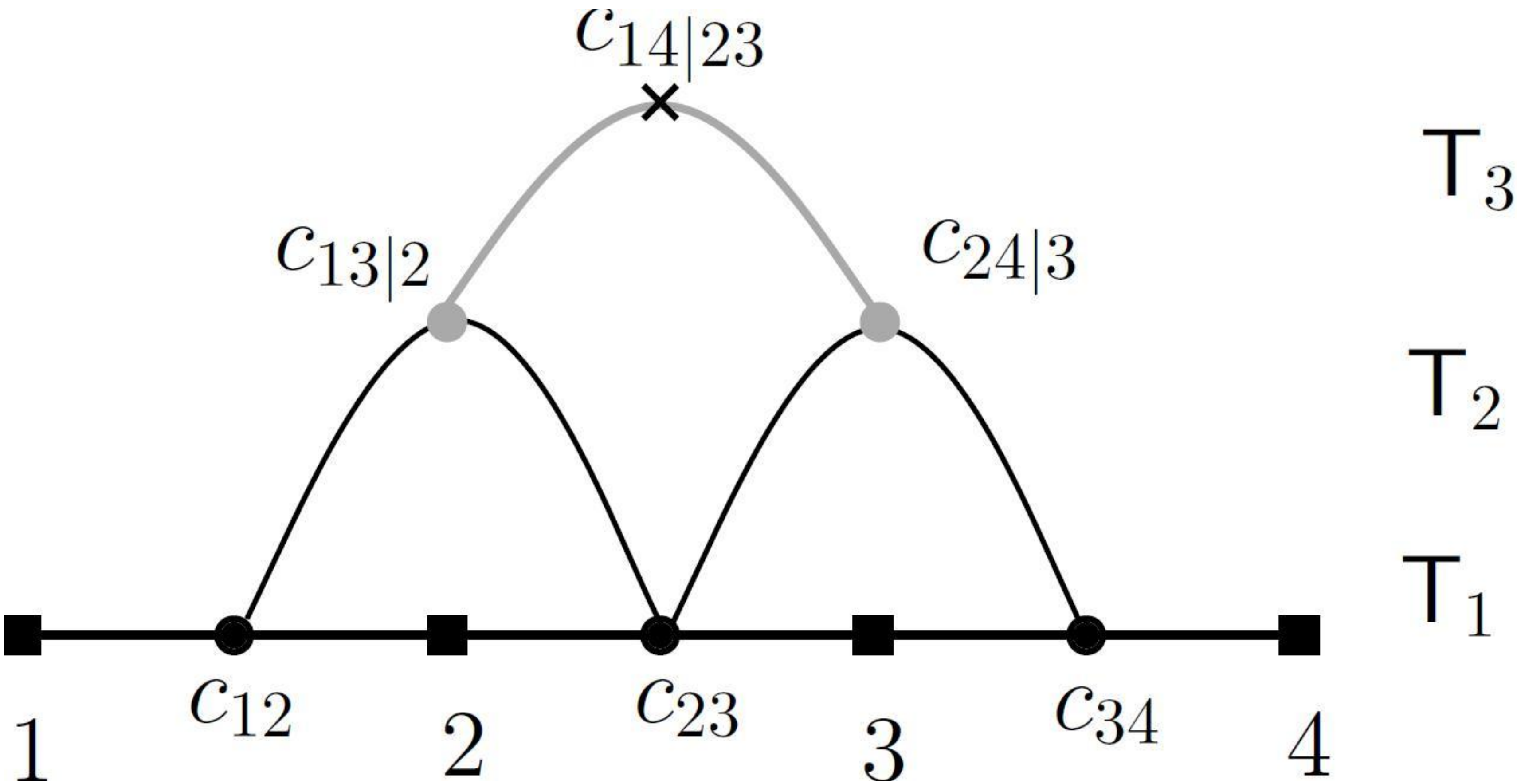
Multivariate Copulas

- A copula density, c, is defined on $\mathbb{R}^N, \mathrm{N} \geq 2$, however,
  - Closed-forms are difficult to obtain from the copula CDF
  - Archimedean copulas: only 1 dependence parameter for $N > 2$
- Archimedean and Elliptical copulas
  - Exchangeability condition
  - Symmetry

  } Limitations
- Multivariate distribution using *vines*
  - A vine is a nested set of trees, where the edges of a the k-th tree are the nodes of the $(k+1)$-th tree
- We consider a class called *D-vines*

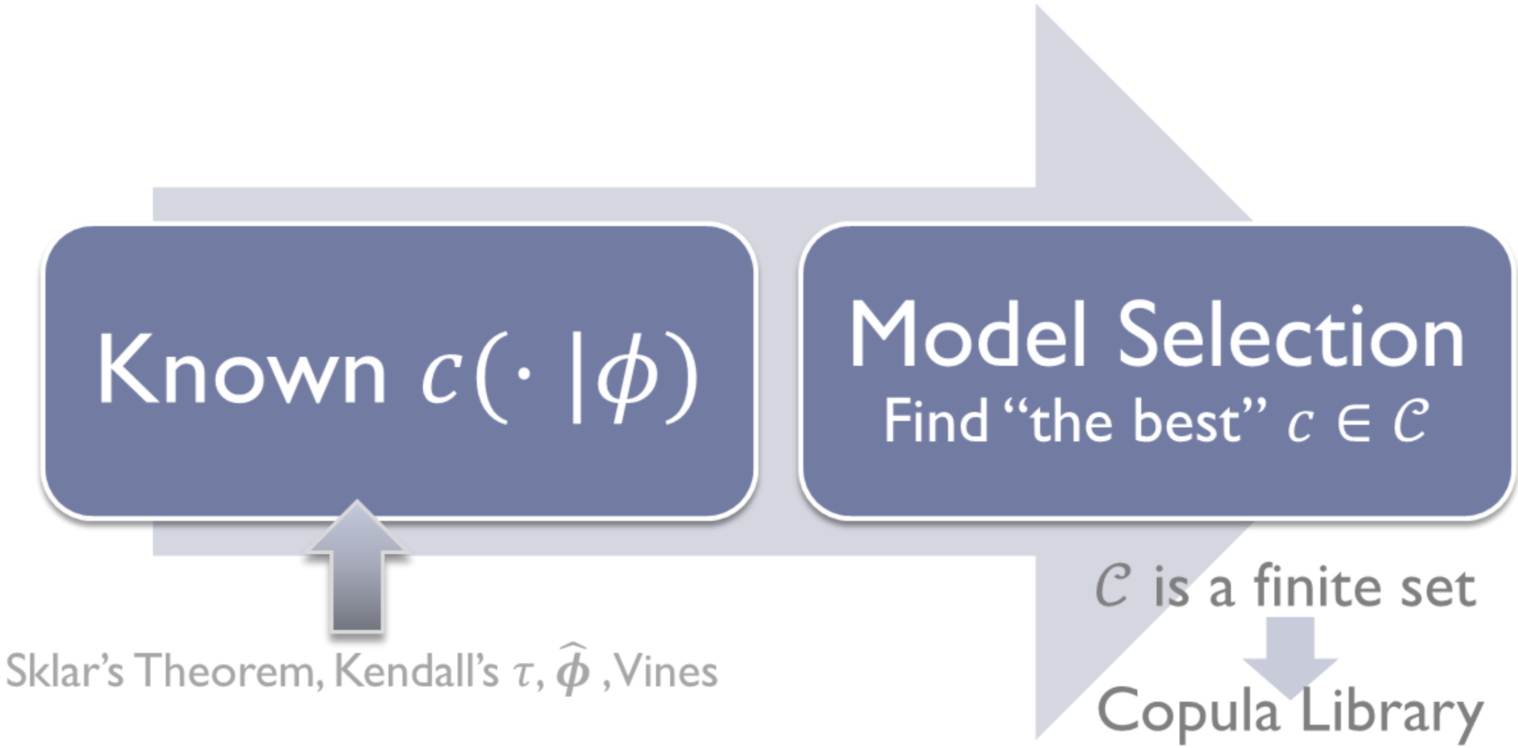
A. Subramanian, A. Sundaresan and P. K. Varshney, “Fusion for the detection of dependent signals using multivariate copulas,” in *Proc. 14th International Conf. on Information Fusion*, to be published

Construction of a Multivariate Copula

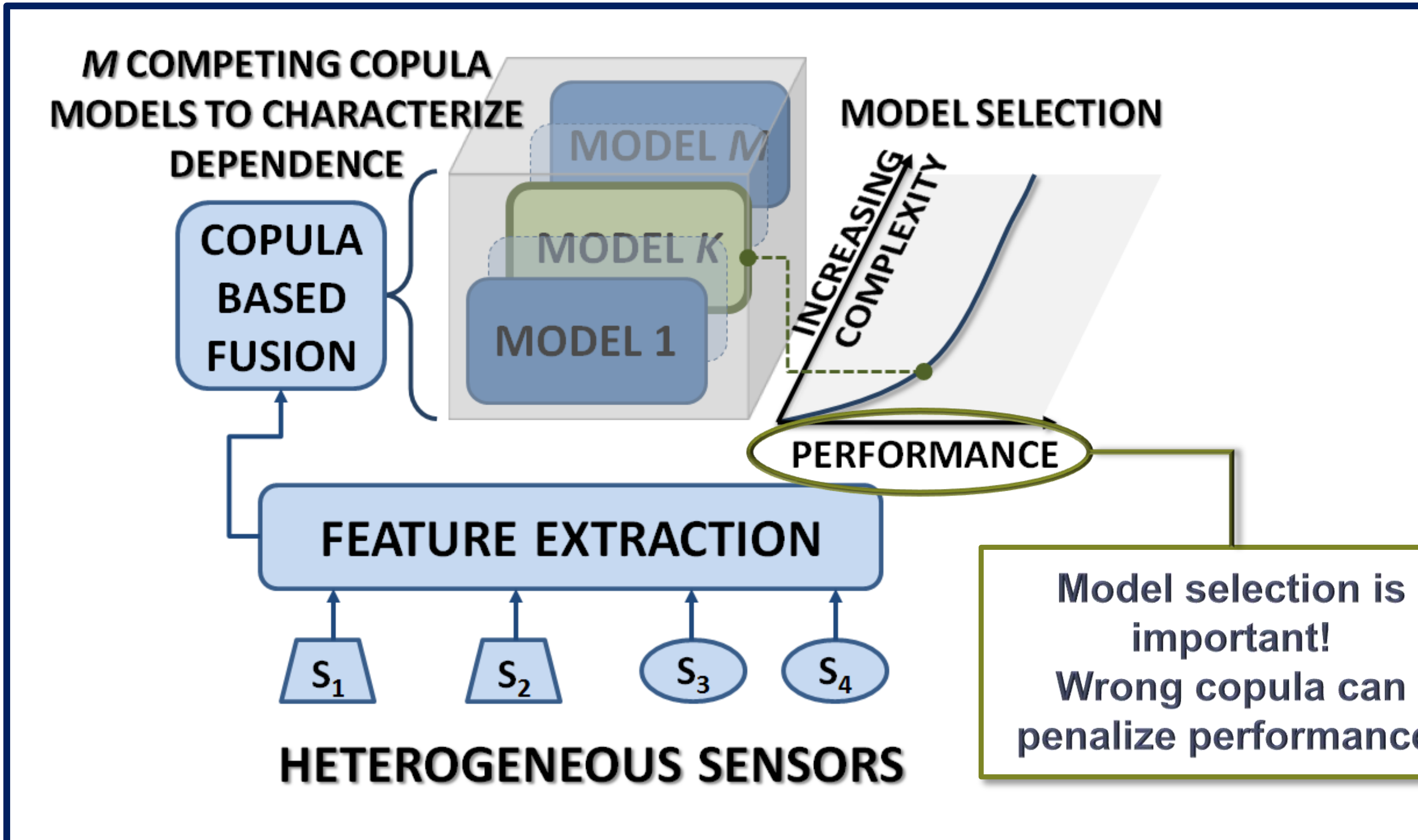


Vines: 4 sensor example

$$
\begin{aligned}
f(\mathbf{x}) = \left[\prod_{i=1}^{4} f(x_i)\right] & c_{12}(F_1(x_1), F_2(x_2)) \\
& \cdot c_{23}(F_2(x_2), F_3(x_3))\, c_{34}(F_3(x_3), F_4(x_4)) \\
& \cdot c_{13|2}(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)) \\
& \cdot c_{24|3}(F_{2|3}(x_2|x_3), F_{4|3}(x_4|x_3)) \\
& \cdot c_{14|23}(F_{1|23}(x_1|x_2, x_3), F_{4|23}(x_4|x_2, x_3))
\end{aligned}
$$


Known $c(\cdot|\phi)$
Model Selection
Find "the best" $c \in \mathcal{C}$
$\mathcal{C}$ is a finite set
Copula Library
Sklar's Theorem, Kendall's τ, $\widehat{\phi}$, Vines

Copula-based Inference: Framework



Model selection is important!
Wrong copula can penalize performance!

Copula Selection: MDL-based Approach

- Criteria based on Minimum Description Length principles
  - Akaike Information Criterion (AIC)
  - Bayesian Information Criterion (BIC)
  - Stochastic Information Criterion (SIC)
  - Normalized Maximum Likelihood (NML)
- Onl

$$I_k = -2\log(L_k(\hat{\phi})) + q_k$$

Model index

Joint likelihood

ML Estimate of Copula parameter

Penalty term proportional to model complexity

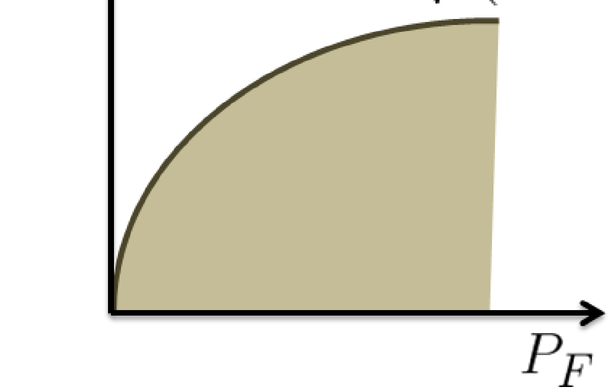
Copula Selection: AUC-based Approach

- Area Under (receiver operating) Curve
  - Application specific approach
  - *Best possible* detector from the available library of models
  - ROC is best for assessing detector performance → AUC is easier to evaluate
- Offline approach – training/testing paradigm

$$\text{AUC} = \int_0^1 P_D(P_F)\,dP_F,$$

$$0 \leq \text{AUC} \leq 1$$

$$P_e \leq \sqrt{(1 - \text{AUC})/2}$$



Copula Theory Applied to Inference

Signal Detection
Localization Estimation
Classification

Signal Detection

- Binary hypothesis testing problem

$$H_1 : f(\mathbf{z}|\boldsymbol{\theta}_1, \phi_1) = \left[\prod_i f_i(\cdot|\theta_{1i})\right] c_1(\mathbf{F}(\cdot|\boldsymbol{\theta}_1)|\phi_1)$$

$$H_0 : g(\mathbf{z}|\boldsymbol{\theta}_0, \phi_0) = \left[\prod_i g_i(\cdot|\theta_{0i})\right] c_0(\mathbf{G}(\cdot|\boldsymbol{\theta}_0)|\phi_0)$$

- General formulation
  - All distribution parameters are unknown
  - Estimated using MLE

Generalized likelihood ratio test

$$\Lambda(\mathbf{z}) = \left[\sum_i \log \frac{f_i(\cdot|\hat{\theta}_{1i})}{g_i(\cdot|\hat{\theta}_{0i})}\right] + \log \frac{c_1(\cdot|\hat{\phi}_1)}{c_0(\cdot|\hat{\phi}_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

GLR under independence

Dependence term

- Copula based test-statistic decouples marginal and dependency information
- Information theoretic analysis of copula mismatch and AUC-based results*

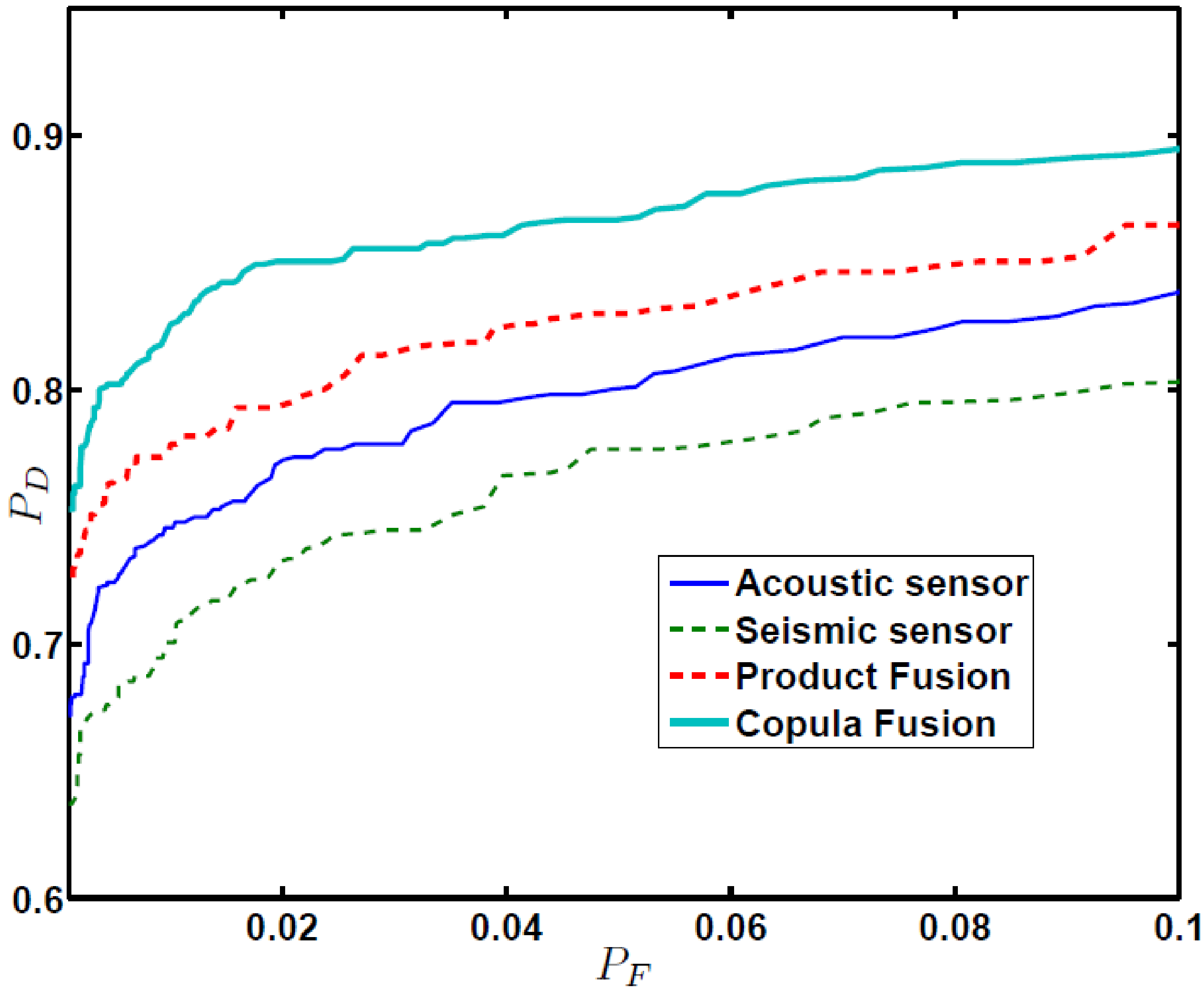
* S. Iyengar, P. K. Varshney, and T. Damarla, "A parametric copula based framework for hypotheses testing using heterogeneous data," *IEEE Trans. Signal Process.*, Vol 59, No. 5, May 2011, pp. 2308 - 2319

Indoor Activity Detection

Indoor Activity Detection

- Signals are preprocessed using short-time Fourier Transform (STFT)
- Canonical Correlation Analysis (CCA) on STFT coefficients
  - Inter-modal correlation is *emphasized*
  - Dimensionality reduction: $\arg\max_{\mathbf{a},\mathbf{b}} \mathrm{Corr}(\mathbf{u} = \mathbf{a}^T\mathbf{X}, \mathbf{v} = \mathbf{b}^T\mathbf{Y})$
- Marginal distributions fitted using generalized Gaussian
- Marginal parameters under H_0 assumed known
- Dependence under H_0 modeled using Gaussian copula

Results: Seismic-acoustic Fusion



Distributed Detection: Fusion of correlated local decisions

- Binary hypothesis testing problem
  - Sensors make local decisions
  - Local decisions are fused at a fusion center
- No prior knowledge of joint distribution of sensor observations
- Design problem
  - Find individual sensor threshol τ_i
  - Design optimal fusion rule $\Lambda(\mathbf{u})$
- Neyman-Pearson (N-P) framework
- Temporal independence assumed

Distributed Detection: Preliminaries

- 2 sensor case
  - Over *N* time instants from sensors 1and 2 respectively,

$$H_0 : \text{Source absent} \longrightarrow f(z_{in}|H_0) \quad i = 1, 2; n = 1, \ldots, N$$

$$H_1 : \text{Source present} \longrightarrow f(z_{in}|H_1) \quad i = 1, 2; n = 1, \ldots, N$$

Sensor Observations

$$u_{in} = \mathbb{Q}(z_{in}) = \begin{cases} 0 & \text{if} \quad -\infty < z_{in} \leq \tau_i \\ 1 & \text{if} \quad \tau_i \leq z_{in} < \infty \end{cases}$$

Local Sensor Decisions

Sensor Threshold

$$\mathbf{u}_1 = [u_{11}, \ldots, u_{1N}]^{\mathrm{T}} \quad \mathbf{u}_2 = [u_{21}, \ldots, u_{2N}]^{\mathrm{T}}$$

Distributed Detection: Dependent Observations

- $P_{ij} = \Pr(u_{1n} = i, u_{2n} = j | H_1)$
- $Q_{ij} = \Pr(u_{1n} = i, u_{2n} = j | H_0)$

For binary quantizers $i, j \in \{0,1\}$

- For example,

$$P_{00} = \int_{z_{1n}=-\infty}^{\tau_1} \int_{z_{2n}=-\infty}^{\tau_2} f(z_{1n}, z_{2n} \mid H_1) dz_{1n} dz_{2n}$$

$$Q_{00} = \int_{z_{1n}=-\infty}^{\tau_1} \int_{z_{2n}=-\infty}^{\tau_2} f(z_{1n}, z_{2n} \mid H_0) dz_{1n} dz_{2n}$$

*Observations **may not** be conditionally independent*

$$\left\{ \left[\prod_i f(z_{in} | H_m) \right] c_m(F(z_{1n}), F(z_{1n}) | \phi) \right. \quad m = 0,1$$

Copula term

Distributed Detection:
Fusion Statistic (Likelihood Ratio)

$$\log \Lambda(\mathbf{u}) = C_1 \sum_{n=1}^{N} u_{1n} + C_2 \sum_{n=1}^{N} u_{2n} + C_3 \sum_{i=n}^{N} u_{1n} u_{2n}$$

Chair-Varshney Fusion Statistic
Conditional independence term

Cross-product Term
Accounts for correlated observations

$$C_1 = \log \frac{P_{10} Q_{00}}{P_{00} Q_{10}} \quad C_2 = \log \frac{P_{01} Q_{00}}{P_{00} Q_{01}} \quad C_3 = \log \frac{P_{00} P_{11} Q_{01} Q_{10}}{P_{01} P_{10} Q_{00} Q_{11}}$$

Distributed Detection: Remarks

- Test statistic: Asymptotically normal
  - Used for calculating P_D and optimal thresholds
- L-sensor case is similarly solved*
- Discussion assumed known ϕ, use MLE if unknown*

*A. Sundaresan, P. K. Varshney, and N. S. V. Rao, "Copula-based fusion of correlated decisions," *IEEE Trans. Aerosp. Electron. Syst*., vol. 47, no. 1, pp. 454–471, 2011

Distributed Detection Example: Radiation Detection

- Two models
  1. Poisson model in Gaussian noise
  2. Hierarchical Poisson-Gamma model

$$s_{in}^{j} \mid \lambda_{ij} \sim \text{Poisson}(\lambda_{ij})$$

$$\lambda_{ij} \mid \alpha_{ij}, \beta_{ij} \sim \text{Gamma}(\alpha_{ij}, \beta_{ij})$$

- Binary hypothesis testing

$$H_0 : z_{in} = s_{in}^{0} + w_{in}$$

$$H_1 : z_{in} = s_{in}^{1} + w_{in}$$

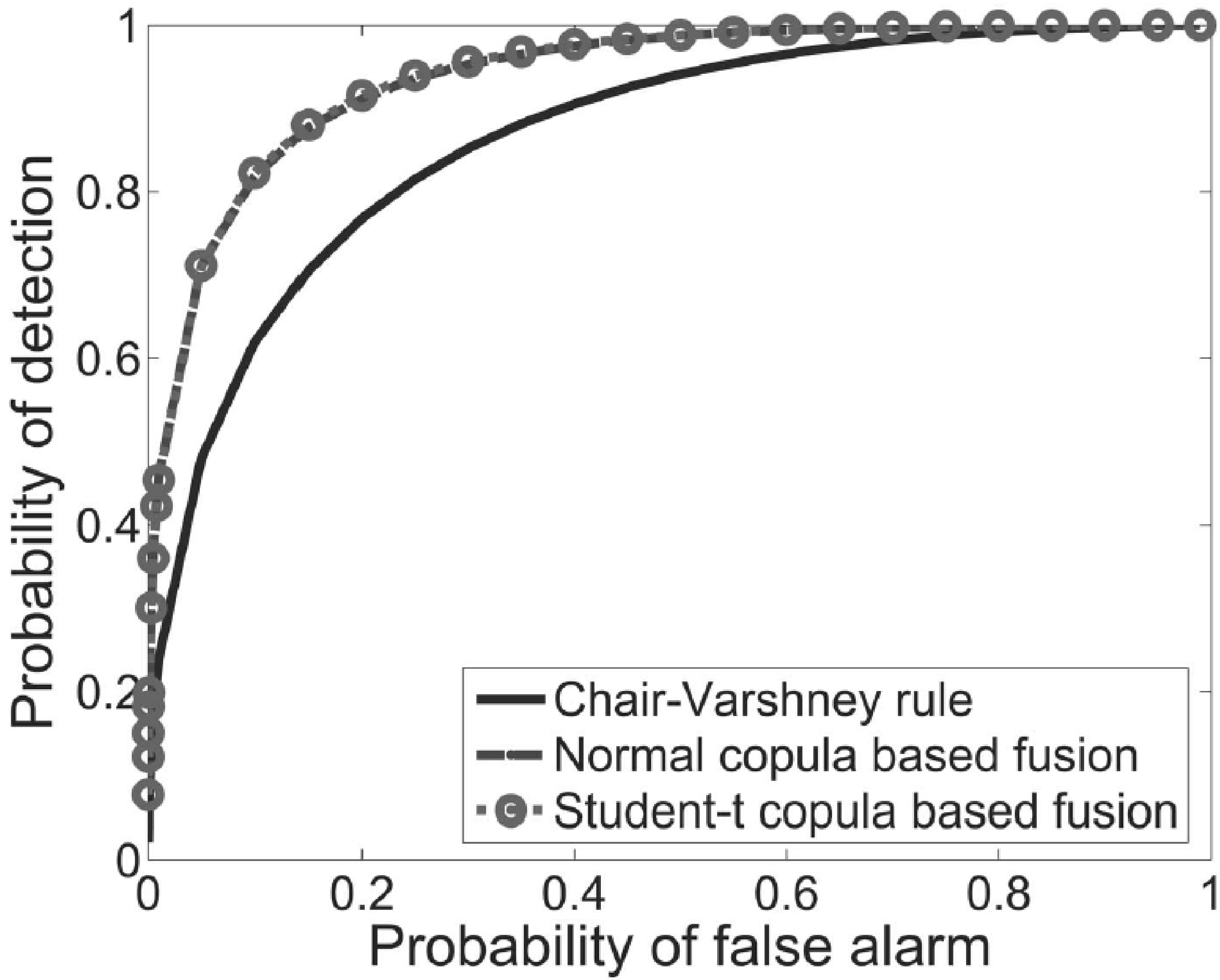
$$i = 1, 2; n = 1, \ldots, N$$

Background radiation

Signal from radioactive material

- $s_{in}^{j} \sim$ Count model $\Rightarrow z_{in} \sim$ Infinite Gaussian Mixture

Distributed Detection Example: Radiation Detection – ROC (Poisson Model)



Copula-based Location Estimation

- Given the intensity of a radiating source A_0 and its location (x_0, y_0),

$$\boldsymbol{\theta} = [A_0, x_0, y_0]$$

- Find,

$$\widehat{\boldsymbol{\theta}} = \arg\max \left[\sum_{n=1}^{N} \sum_{l=1}^{L} \log(f(v_{in}|\boldsymbol{\theta})) + \sum_{n=1}^{N} \log c(\mathbf{F}(\cdot|\boldsymbol{\theta})|\boldsymbol{\phi}) \right]$$

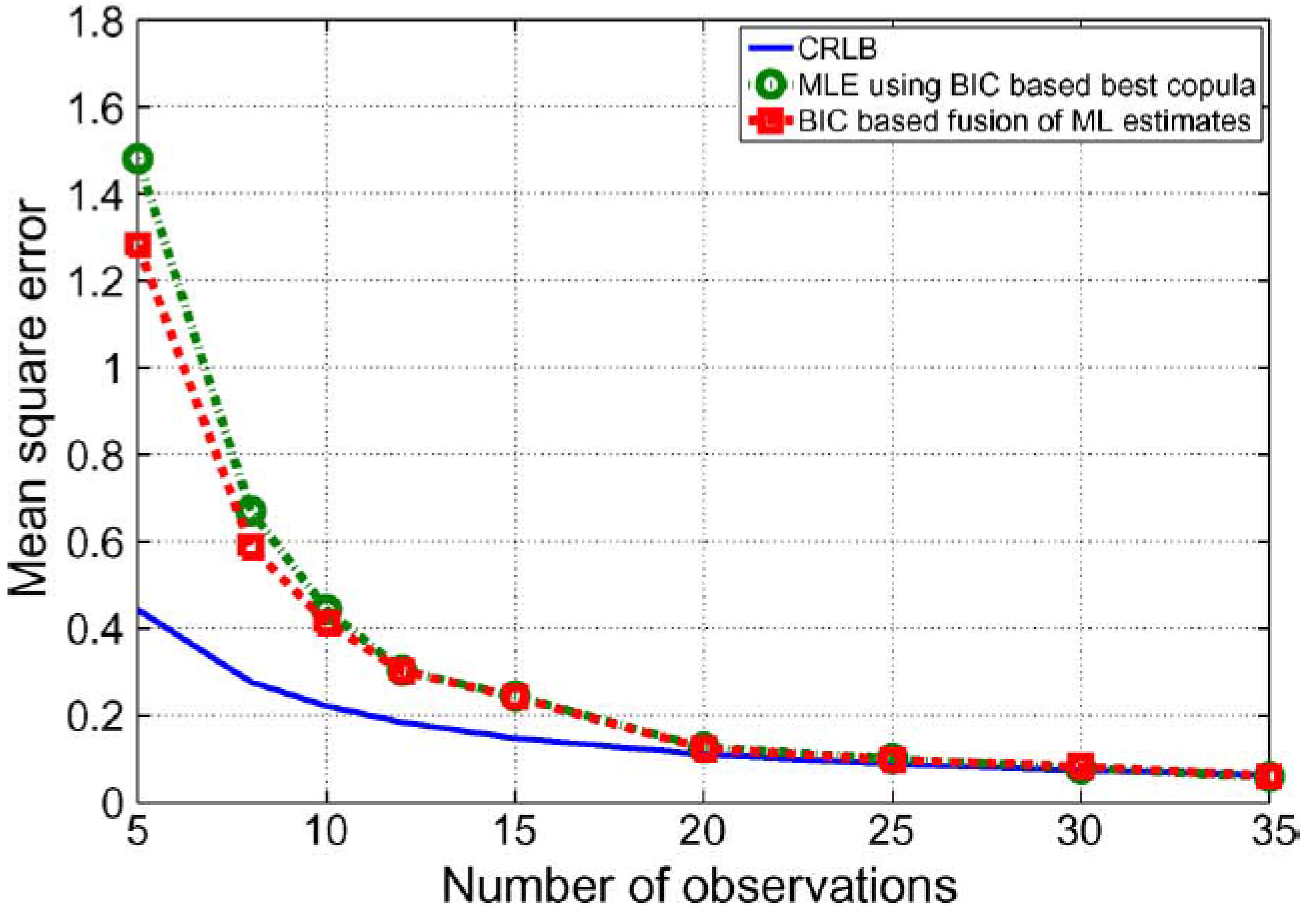
- Copula parameter is estimated as a nuisance parameter

A. Sundaresan and P. K. Varshney, "Location estimation of a random signal source based on correlated sensor observations," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 787–799, 2011.

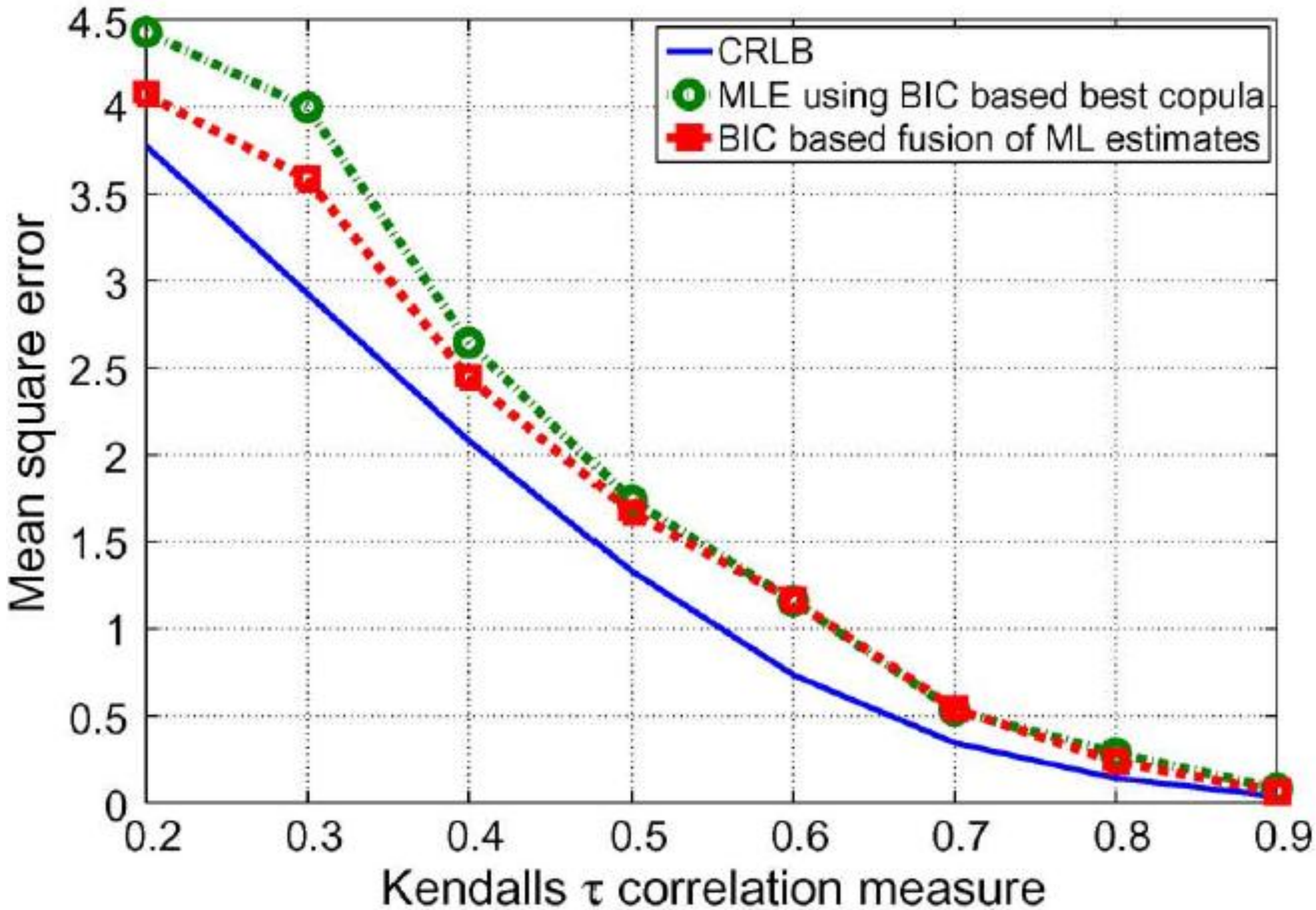
Source Localization: Model Fusion

- Using the same data for statistical inference and model selection leads to *selection bias*
- Model fusion can reduce selection bias
- Uses weighted sum of all K models, including those rejected

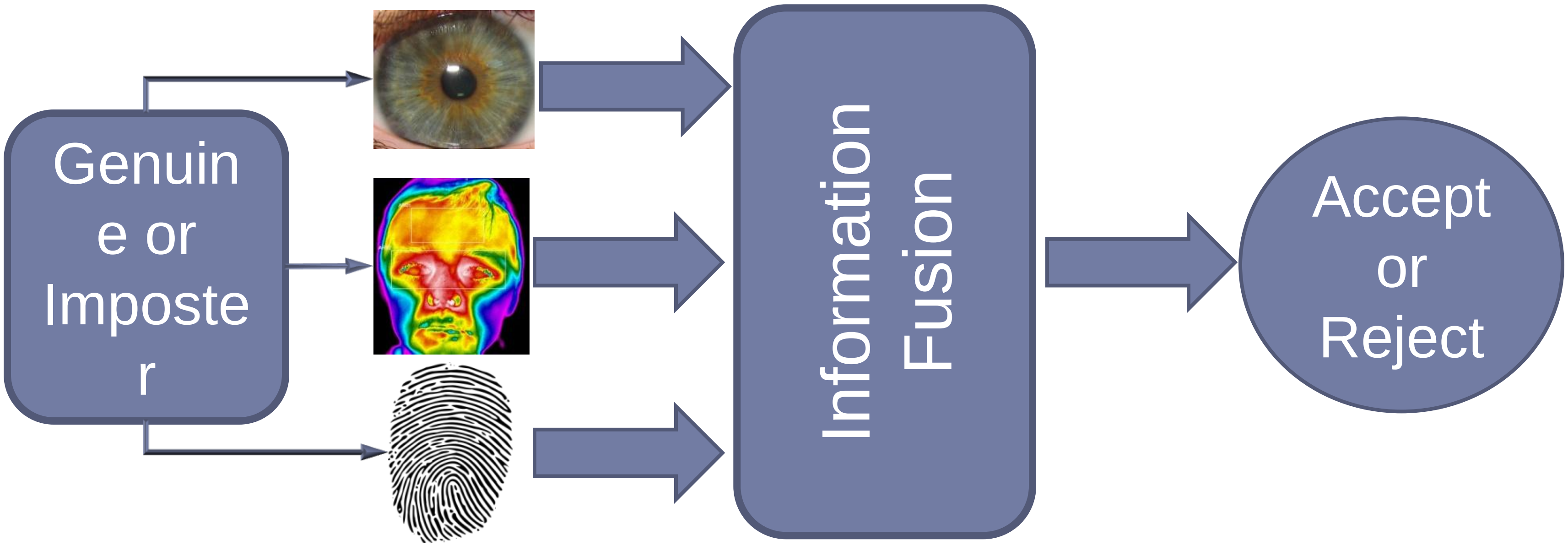
Results:
Localization of an isotropic radiating source



Results:
Localization of an isotropic radiating source



Classification: Application to Multi-biometrics



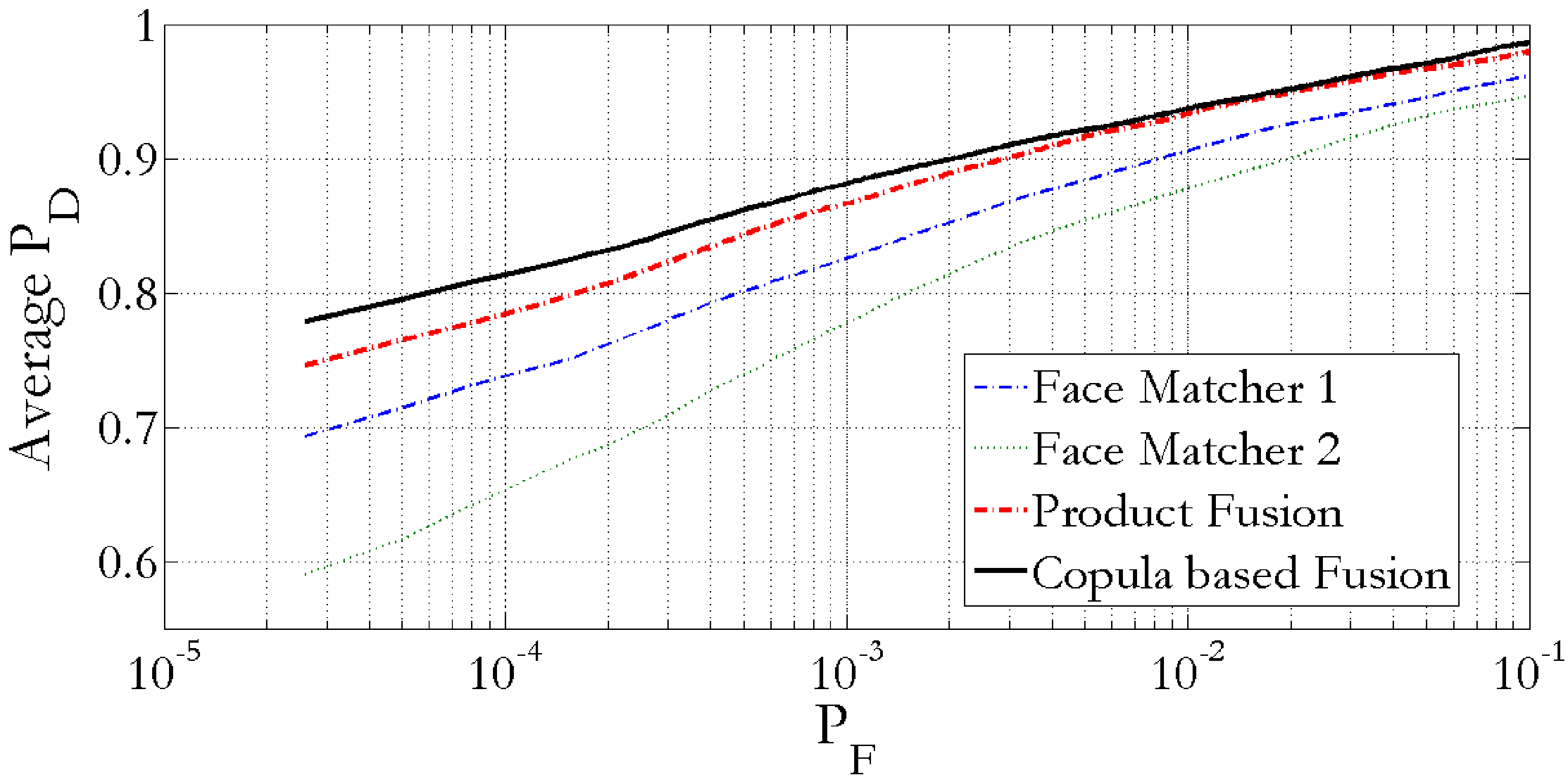
- Data from NIST
  - Two face-matchers with different performance and statistical properties
- Data partitioning: Randomized test/train partitions
- Fusion of algorithms

Iyengar et al., *IEEE Trans. Signal Process.*, Vol 59, No. 5, pp. 2308 – 2319, 2011.

Also see S. G. Iyengar, P. K. Varshney and T. Damarla, "Biometric Authentication: A Copula Based Approach," in *Multibiometrics for Human Identification*, B. Bhanu and V. Govindraju, Eds. Cambridge Univ. Press

Multi-biometrics: Results

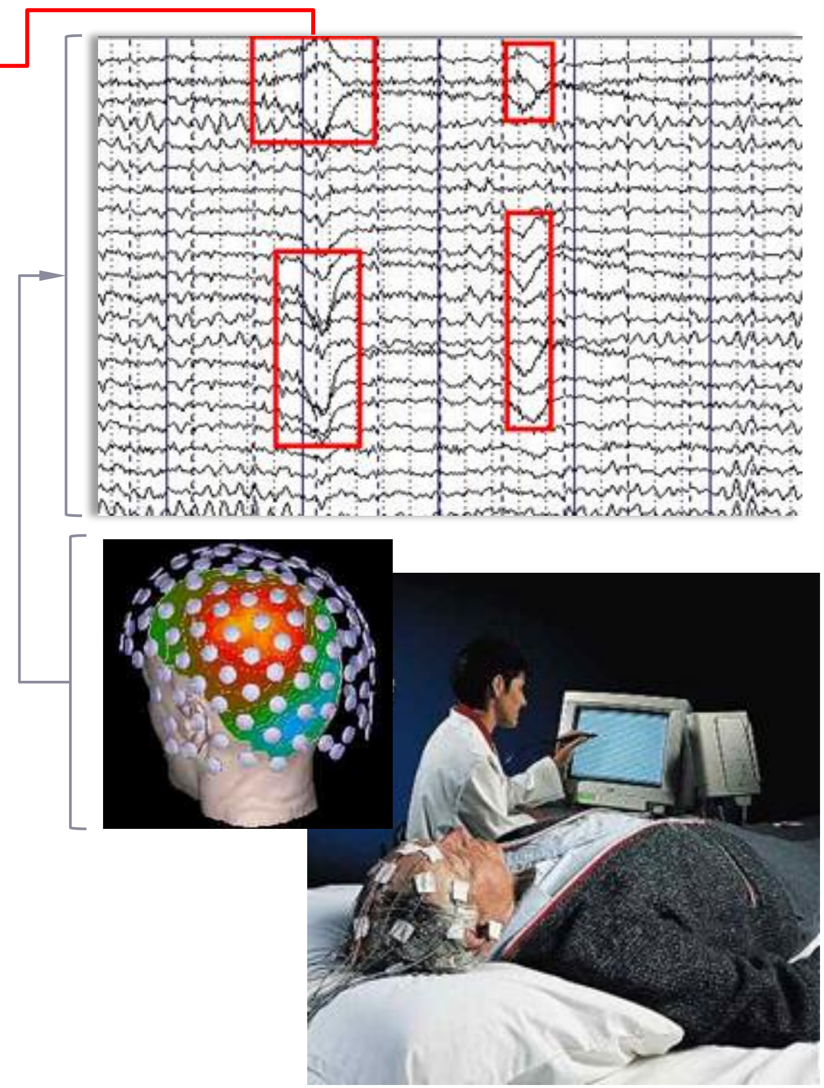
Copula selected using AUC based methodology



Receiver Operating Characteristic (ROC)

Quantification of Neural Synchrony

- Neural synchrony: co-movement of neural activity
- Why do we care?
  - Suggestive of neurophysiological disorders such as Alzheimer's Disease and epileptic seizures
  - Useful for studying brain connectivity and neural coding
- How do we quantify synchrony?
- Limitations of existing measures
  - Existing measures such as Granger causality measure only the linear relationship
  - Information theoretic measures such as mutual information are constrained to be bivariate
- Copula based multi-information developed to alleviate both limitations



Copula-based Multi-information

- Multi-information is used as a global synchrony measure
- Joint density estimated using copula

$$\mathcal{I} \; = \; \int f \log \frac{f^p c(\cdot)}{f^p} = \mathbb{E}_f \log c(\cdot)$$

- Copula based estimate (for 2 sensors, and a time frame of *m* samples → model order *m*)

$$\hat{\mathcal{I}}_h \; = \; \frac{1}{L} \sum_k \log h(u_{1k}, \ldots, u_{1(k-m)}, u_{2k}, \ldots, u_{2(k-m)})$$

  - $h(\cdot)$ is the copula density chosen *a priori*

S. G. Iyengar, J. Dauwels, P. K. Varshney and A. Cichocki, "EEG Synchrony quantification using Copulas," *Proc. IEEE ICASSP*, 2010

Early Diagnosis of Alzheimer's Disease

- Neural synchrony can be used as a feature for classification
  - Drop in neural synchrony indicates possibility of Alzheimer's Disease (AD)
  - Increased neural synchrony → Epilepsy
- 25 patients with Mild Cognitive Impairment (MCI) vs. 38 age-matched control subjects
- All 25 patients developed mild AD later
- Inclusion of copula-based feature improves classification performance

Conclusion

Summary

References

Summary

- Copula based inference has diverse applicability
  - Fusion of multimodal sensors *and* homogeneous sensors
  - *Multi-algorithm* Fusion – Approach discussed for multi-biometrics falls under this category
  - *Multi-classifier* Fusion – Fusing different classifiers
- A theory for signal inference from dependent observations
  - Inclusive theory: independence is a limiting case
  - Signal Detection
  - Signal Classification
  - Parameter estimation
- Copula-based approach shows significant improvement over previously proposed techniques on real datasets

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