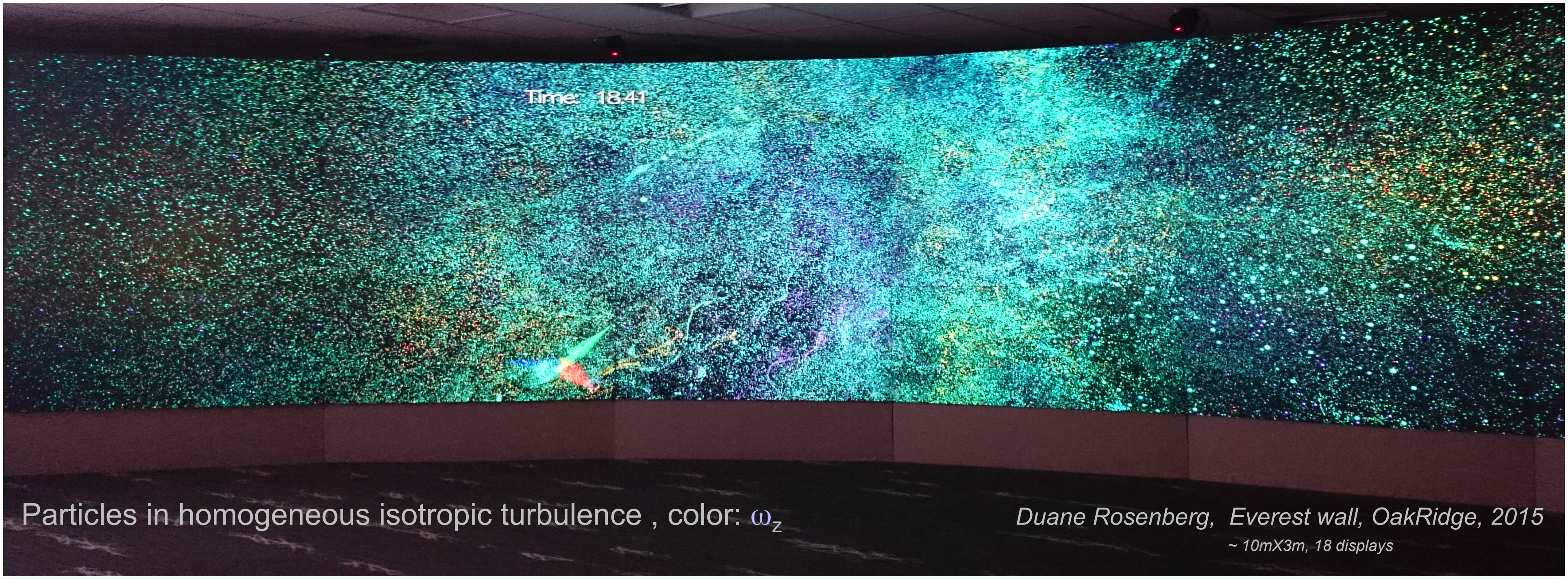


Rotating Stratified Turbulence

Annick Pouquet[1,2]
Corentin Herbert[3], Raffaele Marino[3], Pablo Mininni[4] & Duane Rosenberg[5]

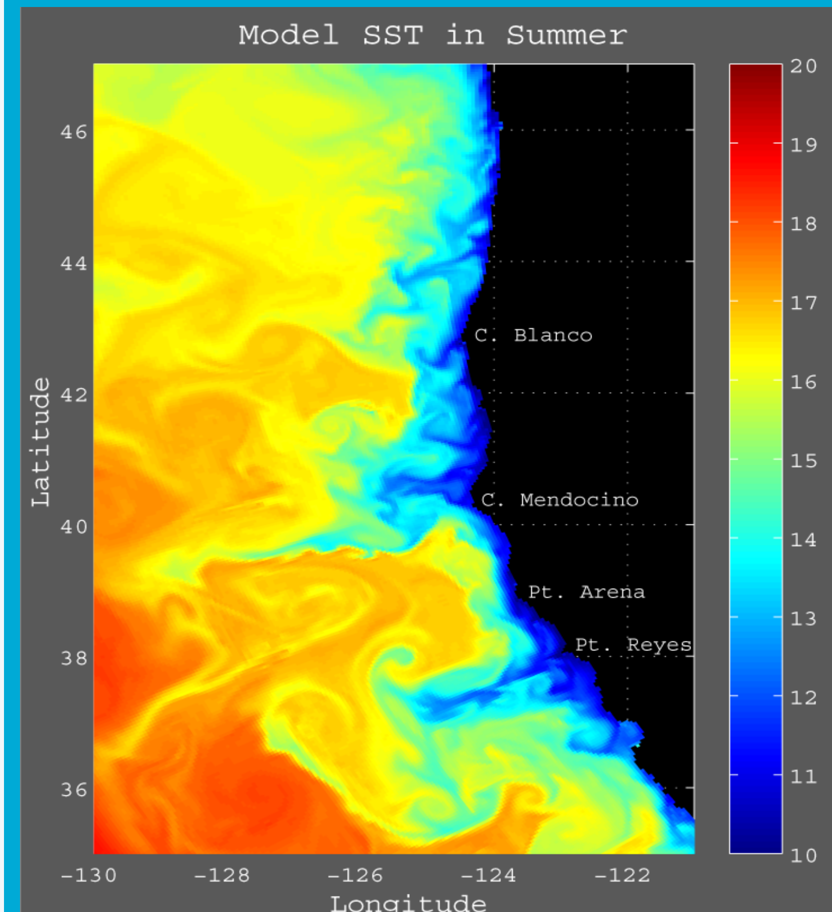
1: NCAR; 2: LASP; 3: ENS Lyon; 4: U. Buenos Aires; 5: SciTex

INCITE/DOE- Titan DE-AC05-00OR22725

NSF – kraken & NCAR - Yellowstone

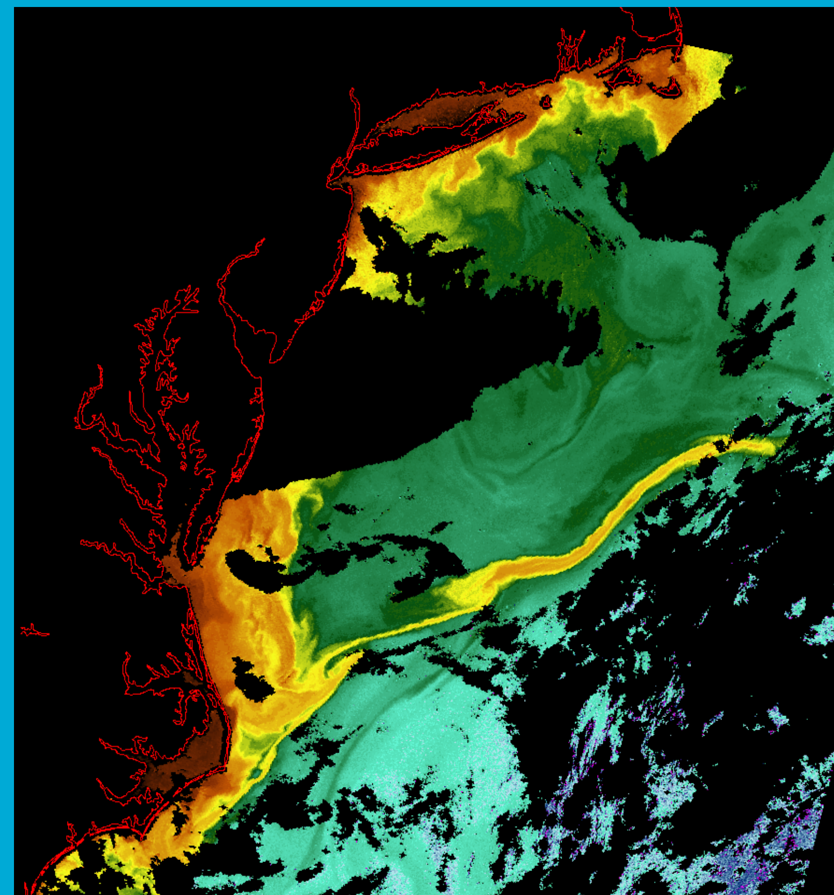
Sea Surface Temperatures (SST)

Modeled SST, West coast



McWilliams et al.

Observed SST, East coast



Davis & Yan, GRL 2004

Weather, climate and all that …

In order to progress, one needs:

^ A deeper understanding of underlying fundamental processes *(minimalist approach)*

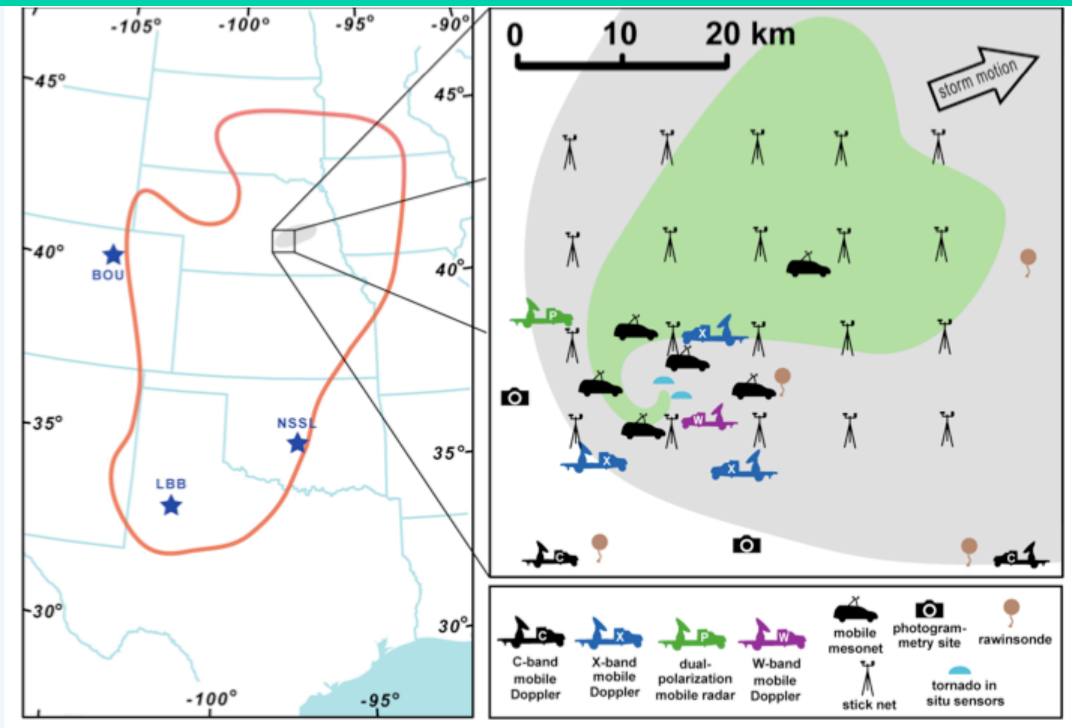
^ Combining enhanced resolutions, in space and time, observationally, experimentally & numerically *(expensive)*

^ A hierarchy of models, adapted to scale of problem

^ An added complexity in modeling *(maximalist: Physics, Chemistry, Biology, Socio-economics, …)*

VORTEX 2 (2009-2010)

Verifications of the Origin of Rotation in Tornadoes EXperiments



* How, when, and why do tornadoes form?

• Why some are violent and long lasting, while others are weak and short lived?

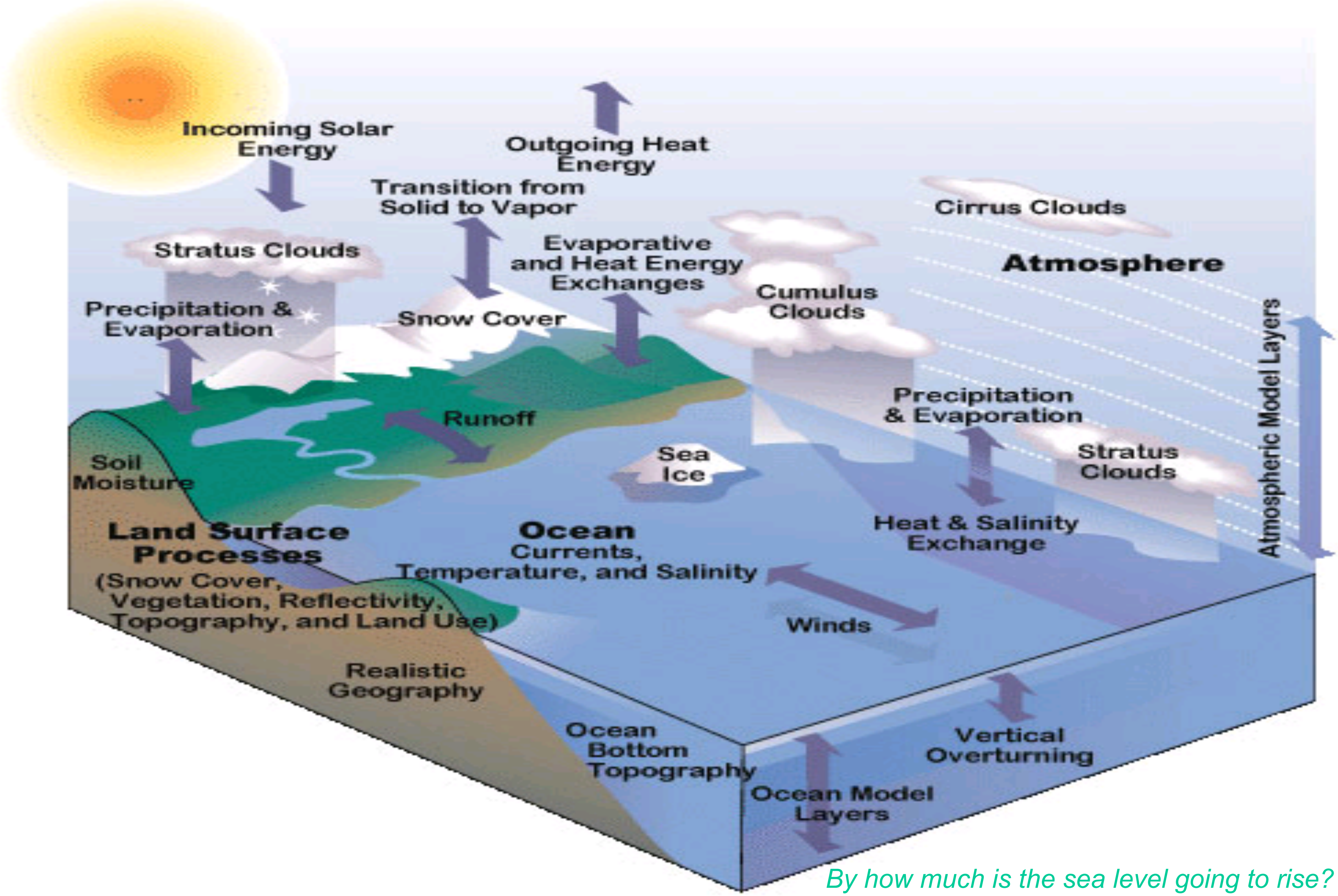
•What is their structure?

* How strong are the winds near the ground?

* How do they do damage?

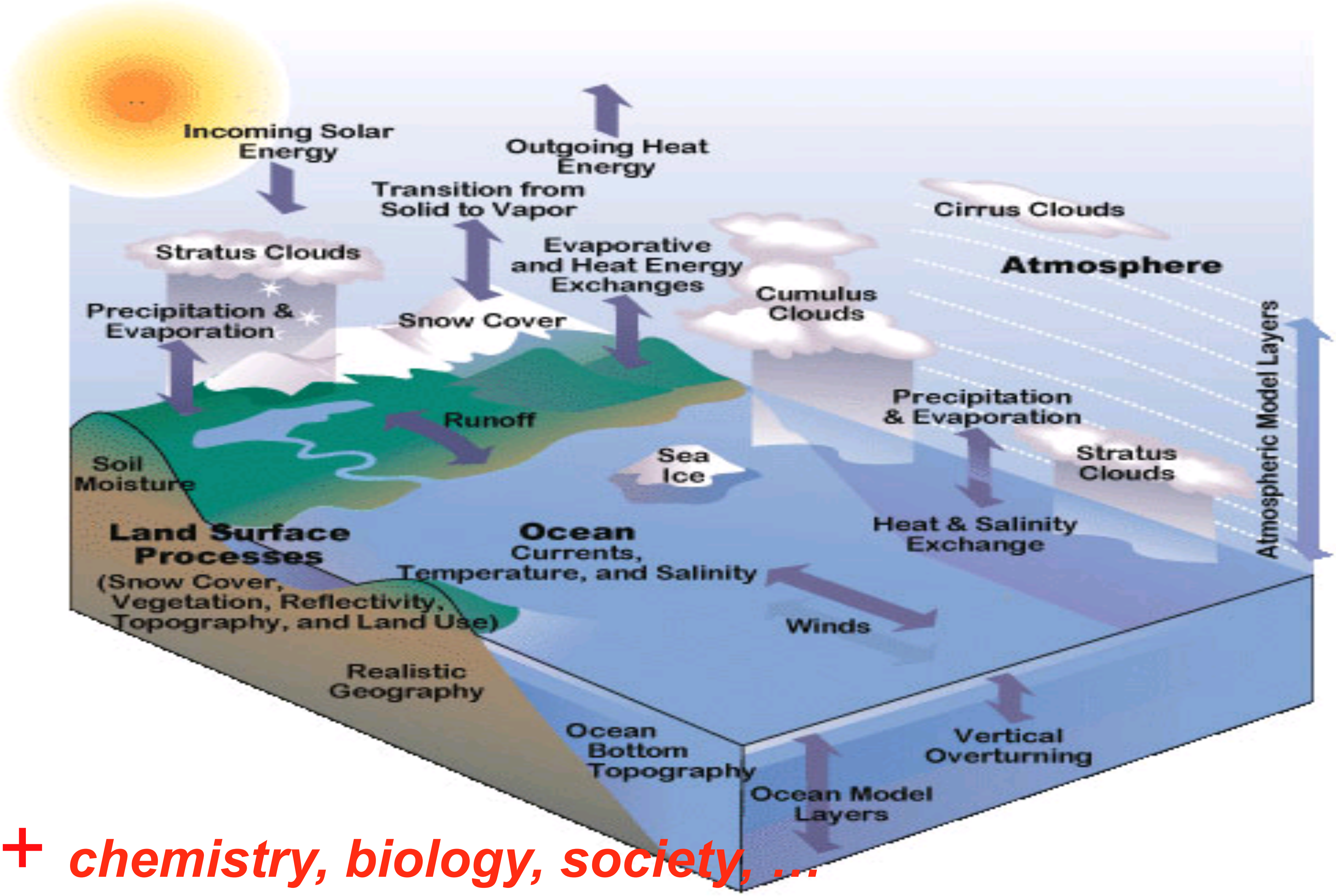
Current warnings have an only 13 minute average lead time, and a 70% false alarm rate.

Seamless predictions across scales, from hourly to decadal



By how much is the sea level going to rise?

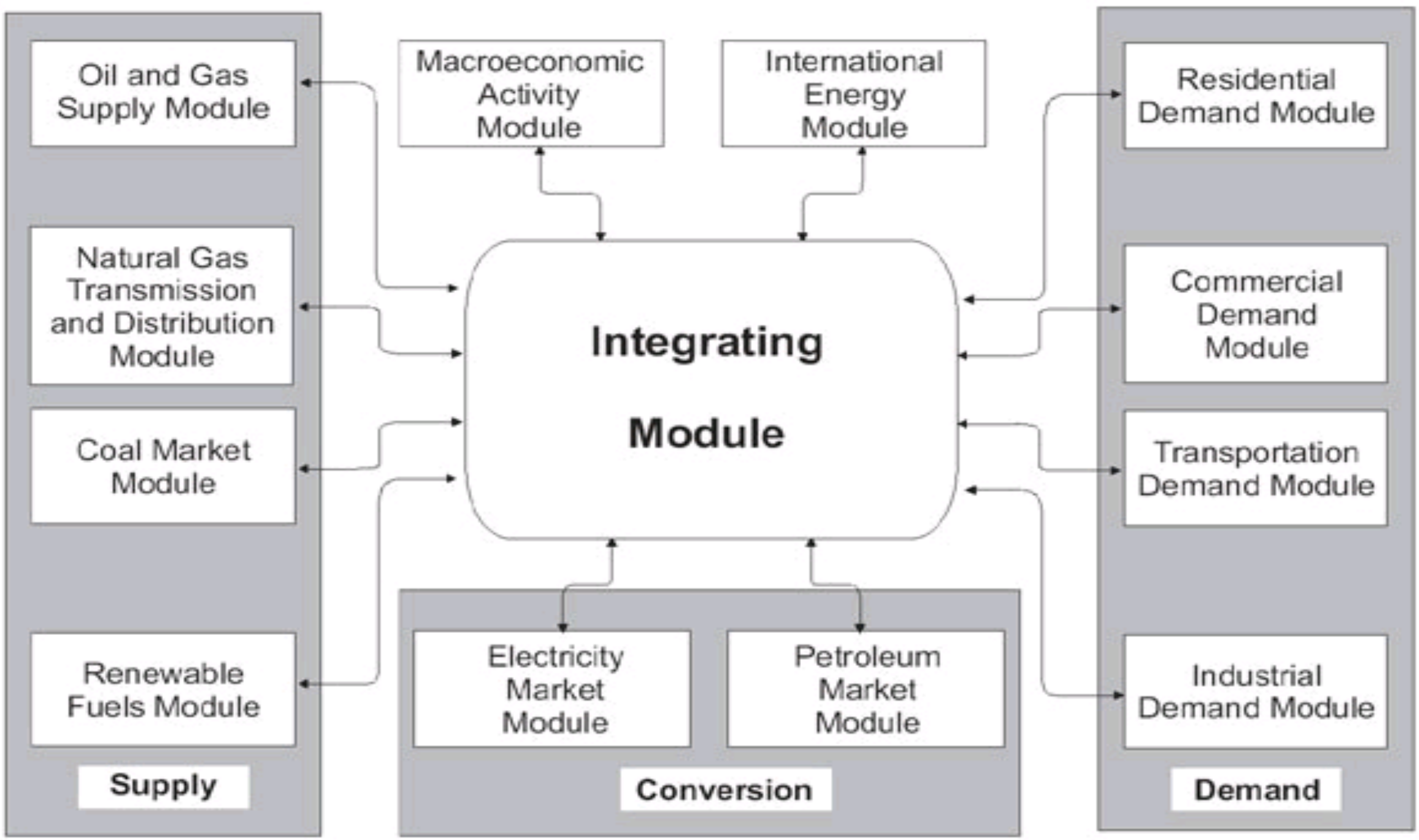
Seamless predictions across scales, from hourly to decadal



+ ***chemistry, biology, society, …***

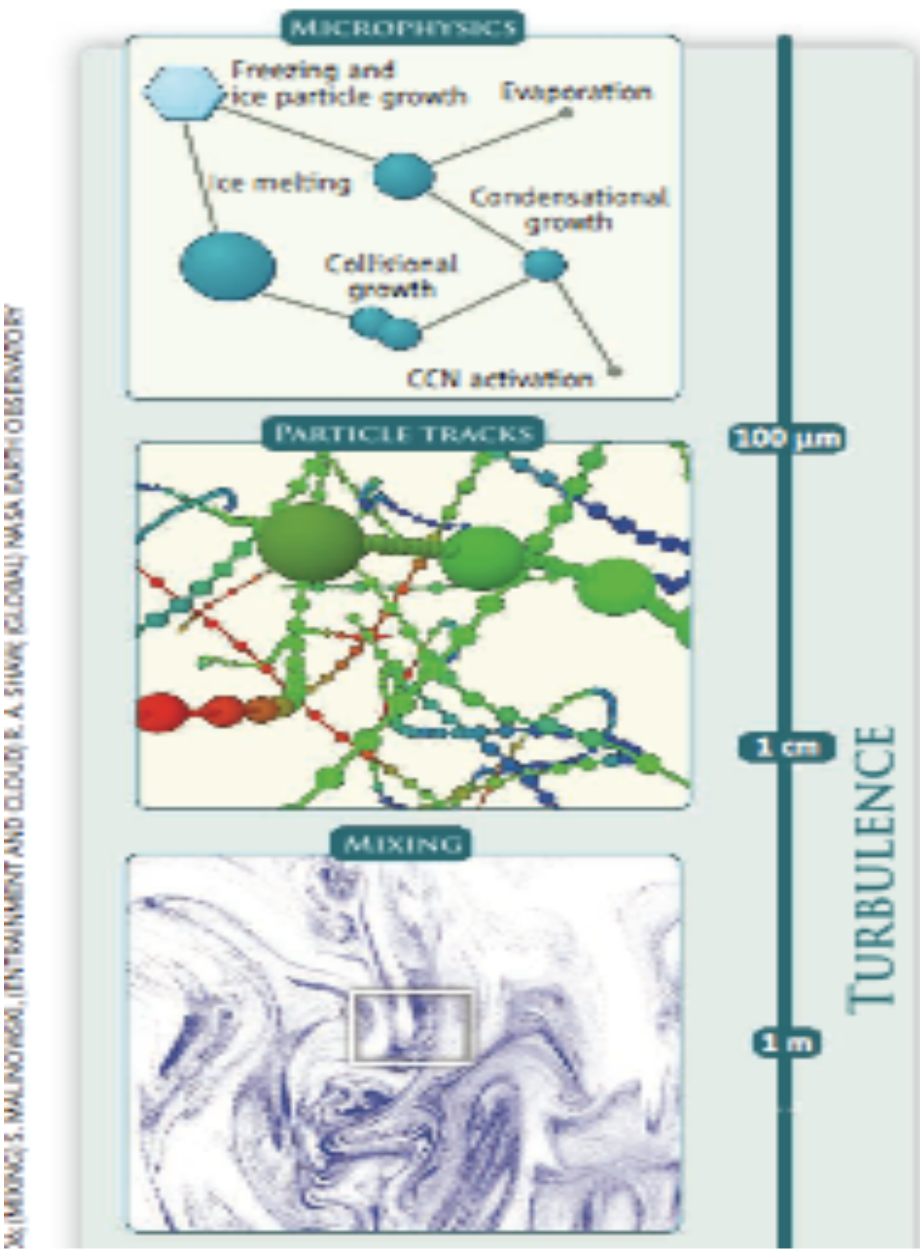
One modeling example of societal complexity: wiring diagrams

Figure 2. National Energy Modeling System



After Ian Foster, Argonne

Bodenschatz et al., Science, 2010



Can we understand clouds w/o turbulence?

Geophysical High Order Suite for Turbulence *(D. Gomez & P. Mininni)*

- Pseudo-spectral DNS, periodic BC cubic *(also 2D), single/double precision*; Runge-Kutta for incompressible Navier-Stokes, SQG & Boussinesq. Includes rotation, passive scalar(s), MHD + Hall term
- *GHOST, from laptop to high-performance, parallelizes linearly up to 130,000 processors, using hybrid MPI/Open-MP (Mininni et al. 2011, Parallel Comp. 37)*
- LES: alpha model & variants (Clark, Leray) for fluids & MHD
- Helical spectral (EDQNM) model for eddy viscosity & eddy noise
- Lagrangian particles *(w. A. Pumir)*
- Gross-Pitaevskii & Ginzburg-Landau *(PM+M. Brachet)*
- **Data, forced:** 2048^3 Navier-Stokes and 1536^3 & 3072^3 with rotation, both w. or w/o helicity. Rotating stratified turbulence w. 2048^3 grids forced at intermediate scale
- **Data, spin-down MHD**: 1536^3 random + 6144^3 ideal & 2048^3 w. T-Green symmetry
- Decaying rotating stratified flow, N/f~5, Re=5.5 10^4, 2048^3 , 3072^3 & 4096^3 grids.
- Decaying rotating stratified flow, 2.5<N/f<300, Re up to 1.8 10^4, R_B up to 10^5, 1024^3 grid.

mininni@df.uba.ar, duaner62@gmail.com, marino@ucar.edu

Some ``hero" runs in turbulence

- 4096^3 points ~6.8 10^{10} , *out of which ~ 200 millions in the last 10 Fourier shells alone in the dissipation range:* homogeneous isotropic (HIT): Japan ('03), US (PKY, '12); MHD: Germany (Homann~ '10); supersonic: Australia (Federrath ~ '13)

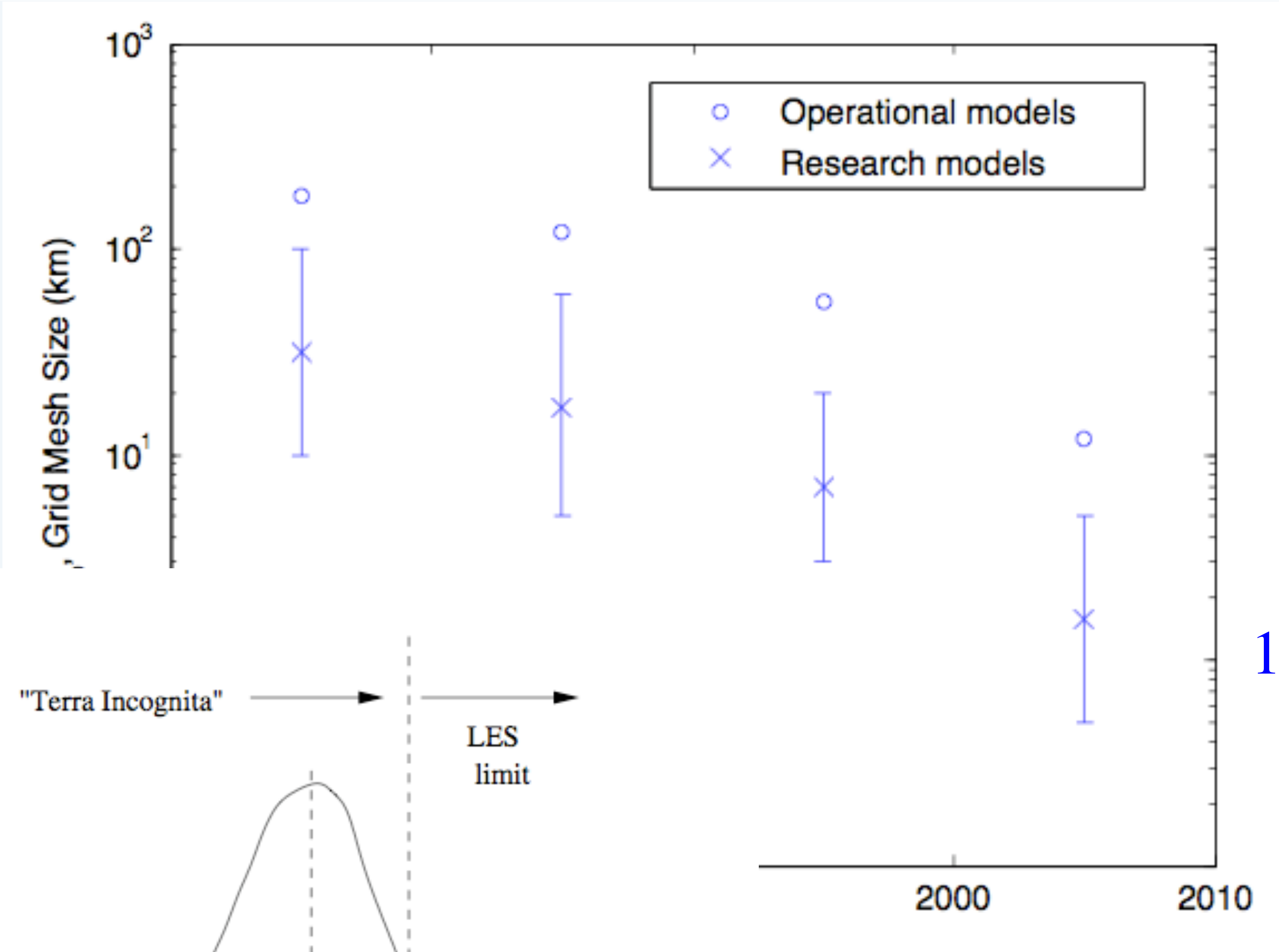
- *Purely stratified run 8192^2*4096 (ONR, deBruyn-Kops, 2015)*

- HIT, Japan K-computer (& NSF's goal for HIT): 12288^3 or ~ 1.8 10^{12} grid points ~ *\sqrt(A)*

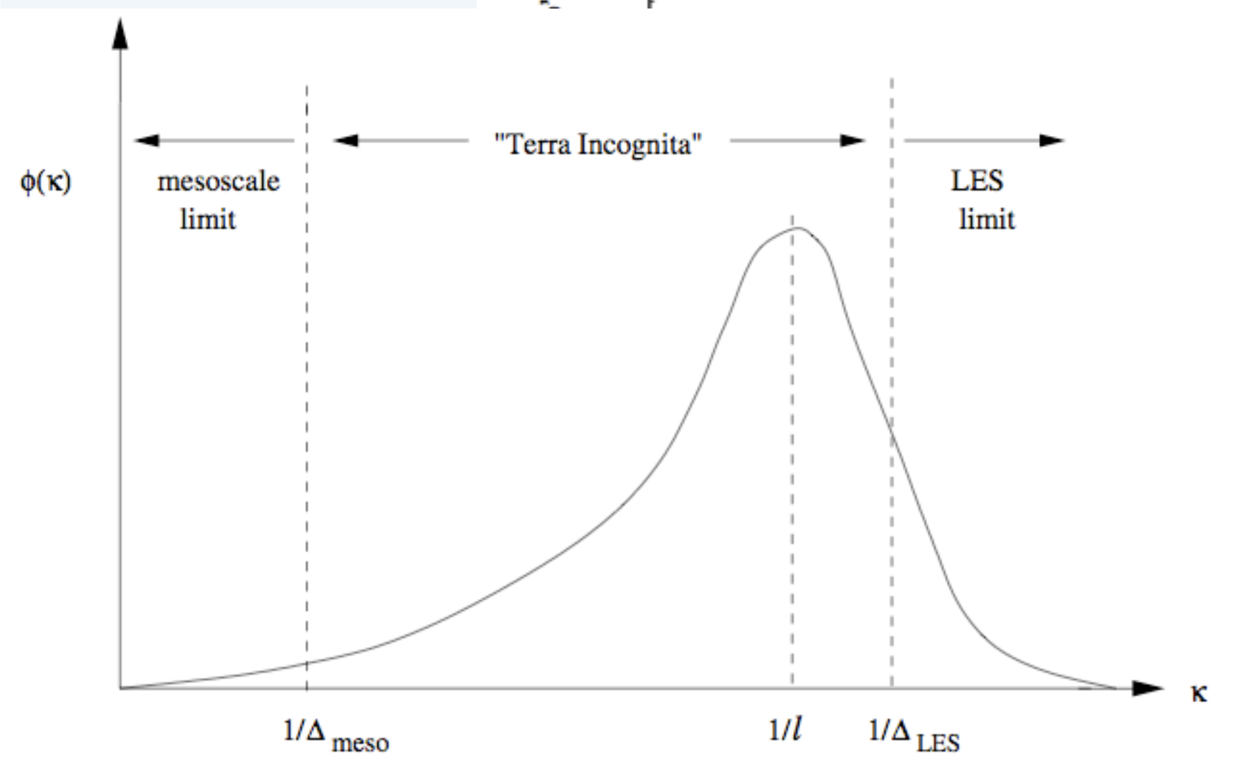
^ *4096^3 points rotation + stratification, N/f=5, R_B ~ 32 (NSF+DOE)*

A: Avogadro nb. ~ 6 10^{23}

The trend



1 km



The problem(s)

Slide after John Wyngaard

Can we go beyond Moore's law?

Doubling of speed of processors every 18 months

--> doubling of resolution for DNS in 3D every 6 years …

→ Develop models of turbulent flows (RANS, LES, closures, Lagrangian-averaged, …)

→ Improve numerical techniques

→ Develop numerical models

→ Be patient

- Is Adaptive Mesh Refinement (AMR) a solution?

If so, how do we adapt? How much accuracy do we need?

Example of 3D AMR

Hairpin vortex,
Euler case

Grauer et al. PRL 80 (1998)

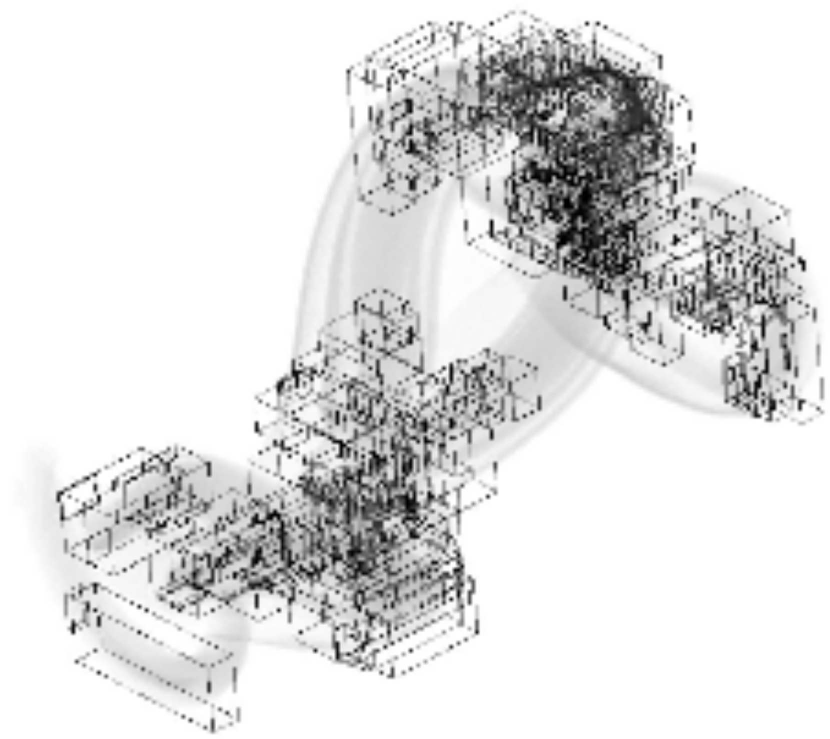


FIG. 4. Volume rendering of $|\omega|$ at time 1.32. Only level 3, 4, and 5 grids are shown.

The need for Adaptive Mesh Refinement (AMR)

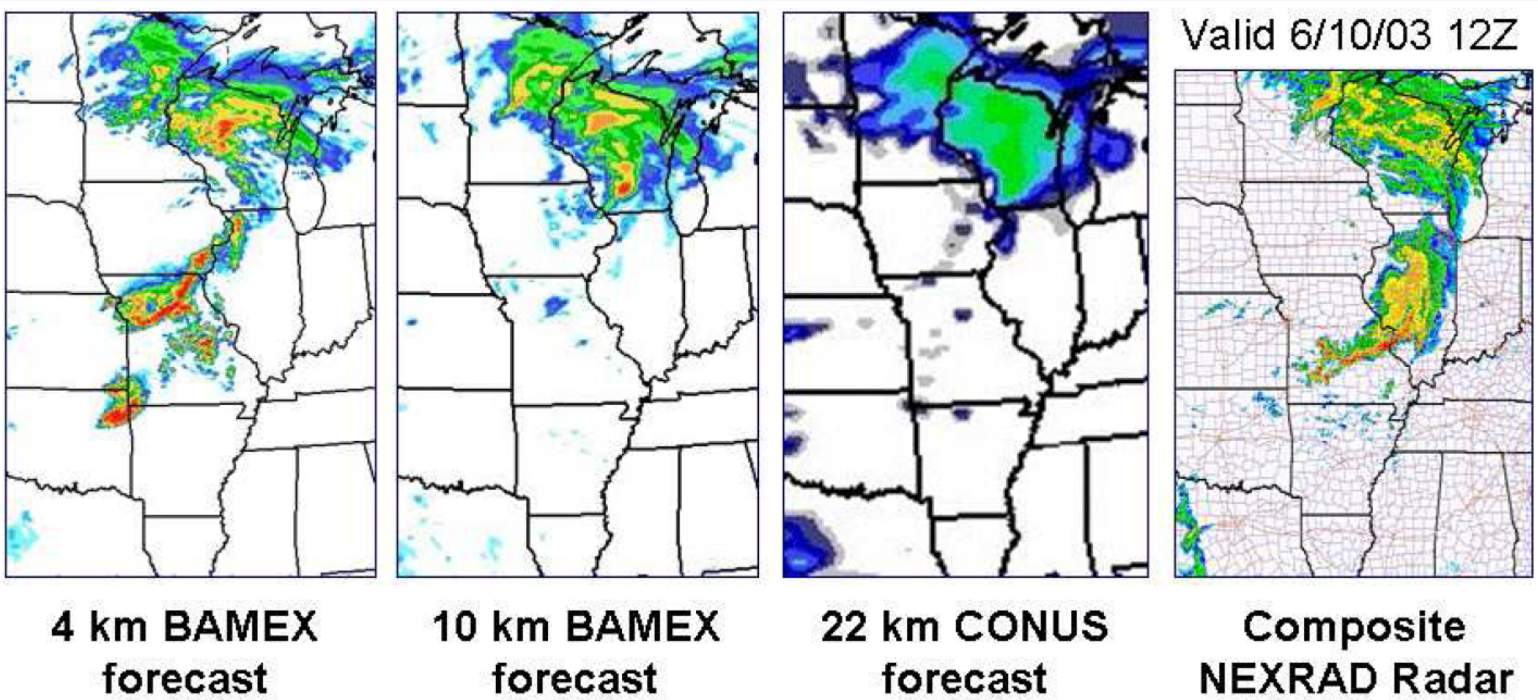


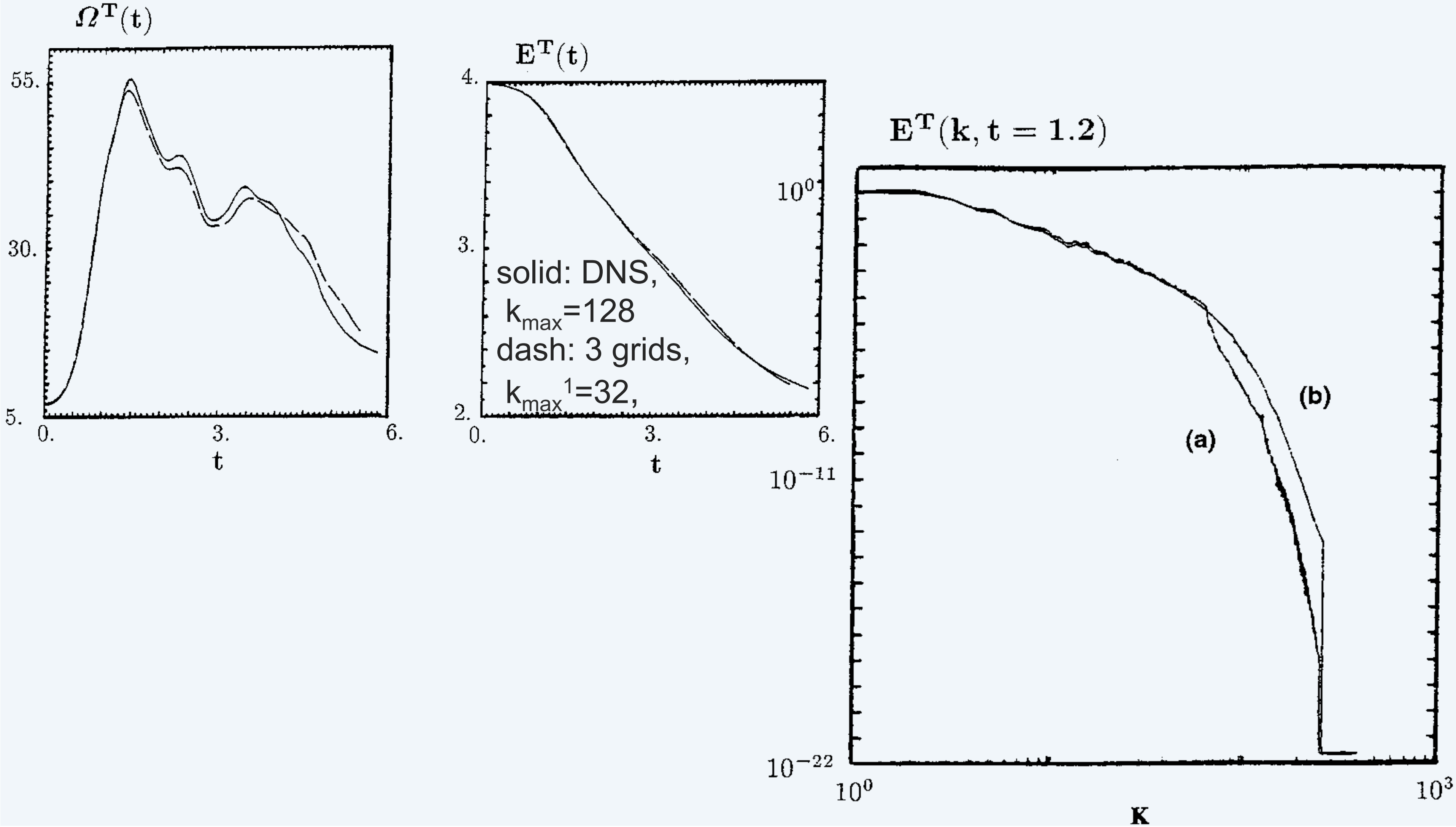
Figure 1: From Bill Skamarock, showing the lack of convergence with model resolution.

Sparse Fourier methods: decimation

Meneguzzi+ 1996

2D-MHD

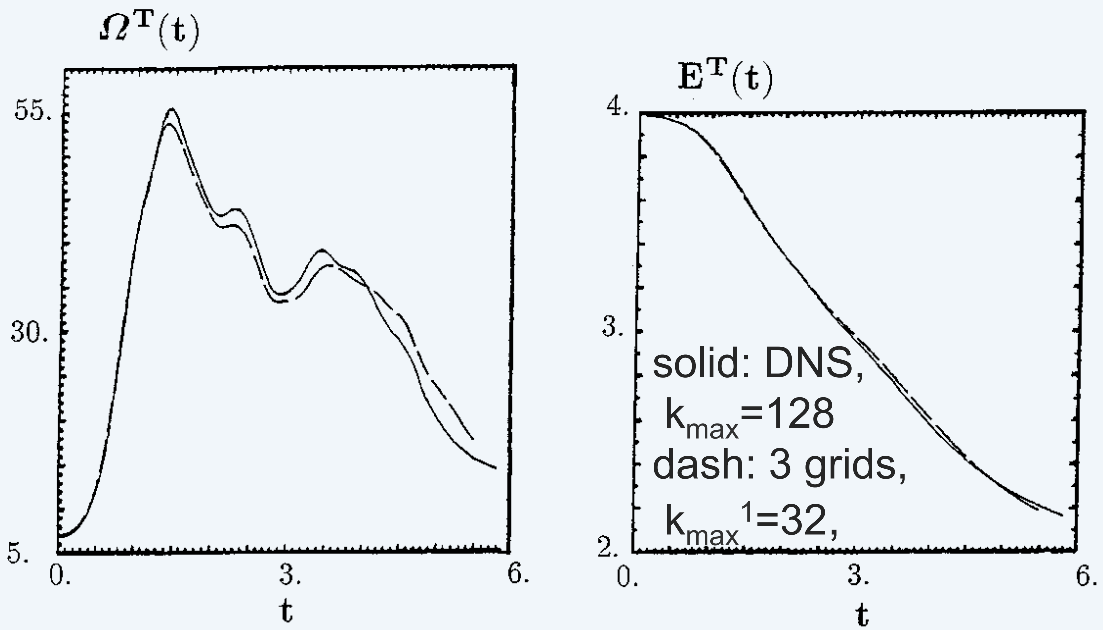
DNS, k_{max}=256 versus decimation, k_{max}^1=32, 4 grids



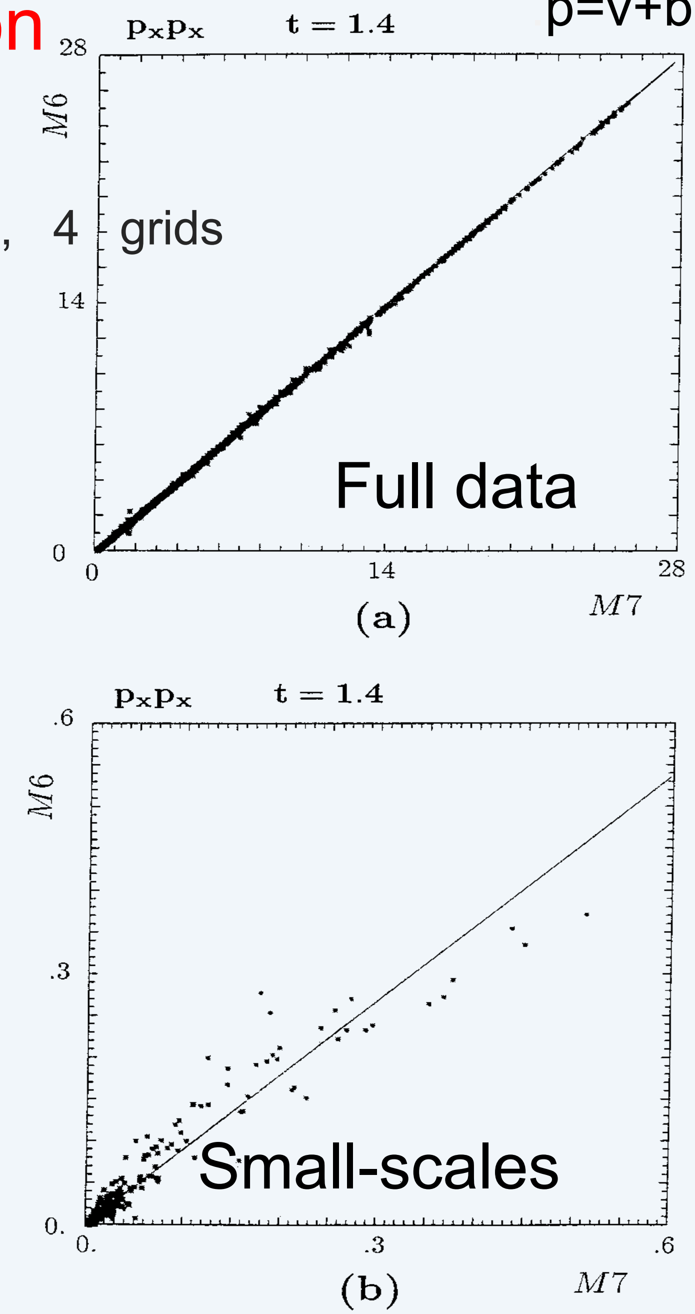
Sparse Fourier methods: decimation

2D-MHD

DNS, k_{max}=256 versus decimation, k_{max}^1=32, 4 grids



p=v+b



Meneguzzi+ 1996

Sparse Fourier methods: decimation

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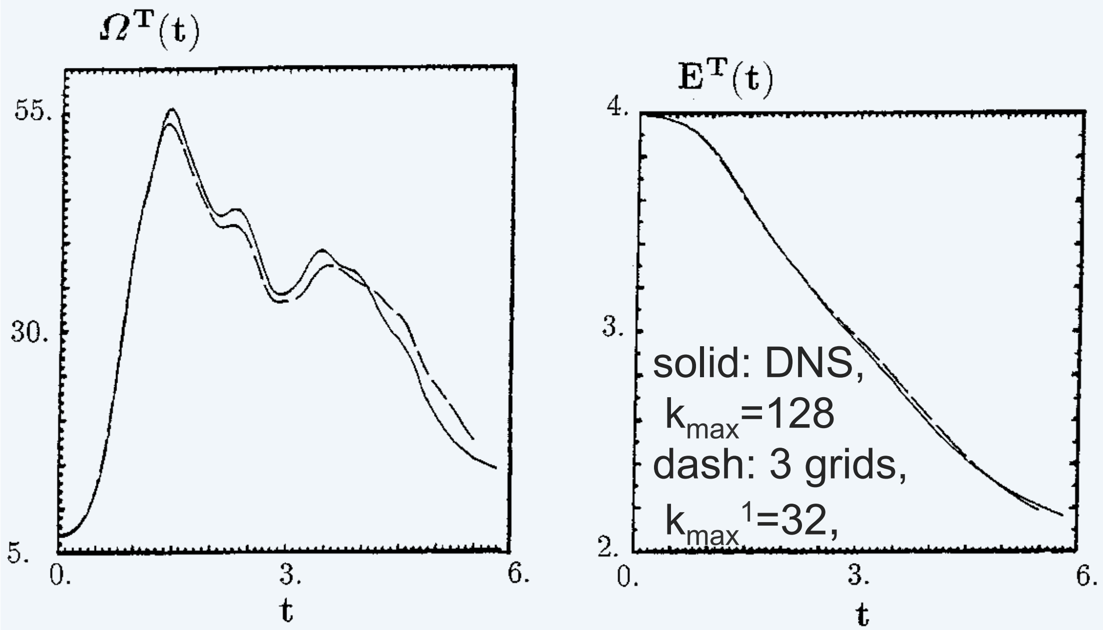
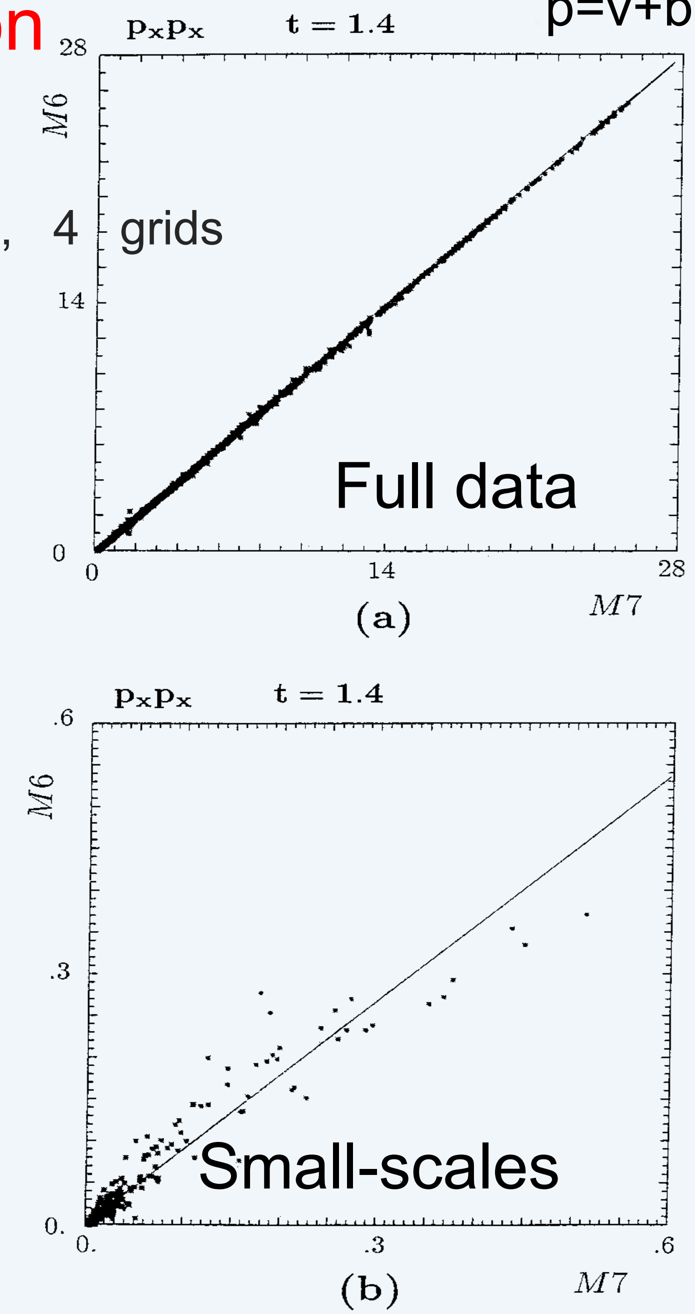




Stochastic decimation:
Number of modes in k-shell in dimension d is $C_d k^{d-1}$

(Lanotte+)

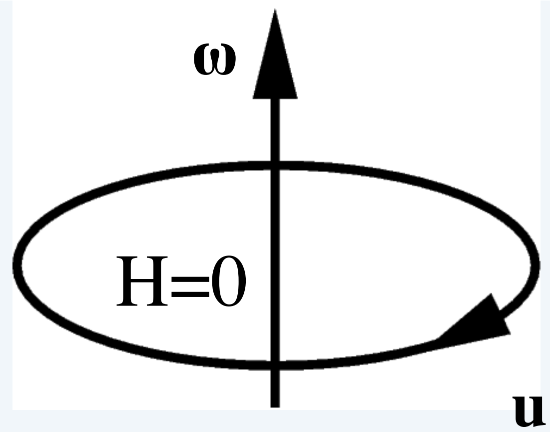
Meneguzzi+ 1996

Helicity

- Not magnetic!

Helicity

H is a pseudo (axial) scalar

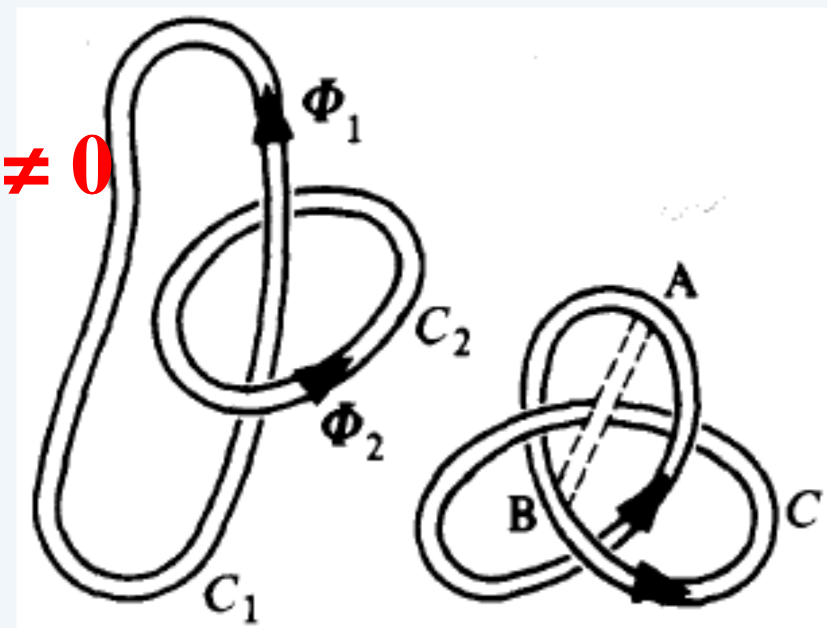


$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} \, dV$$

H (ω): off-diagonal components of the velocity gradient matrix $\partial_i u_j$

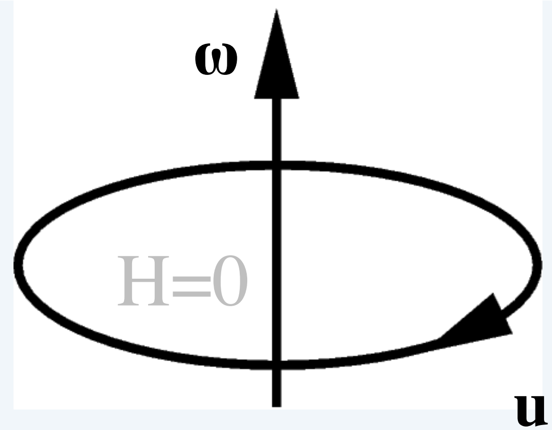
*Link to the **thermal winds***
(vertically sheared horizontal winds $\partial_z u_{perp}$)

H ≠ 0



Helicity

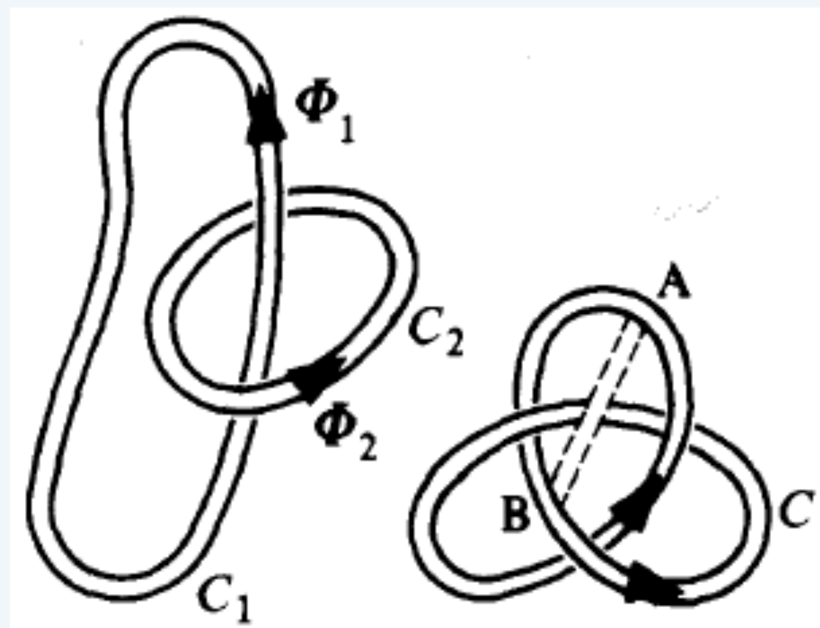
H is a pseudo (axial) scalar



$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} dV$$

$<u_i(\mathbf{k})u_j^*(-\mathbf{k})> = U_E(|k|)\, P_{ij}(|k|)$

$+ \quad \varepsilon_{ijl} k_l\, U_H(|k|)$



Helicity dynamics in HIT

- Evolution equation for the local helicity density in HIT
  (Matthaeus+ 2008):

$$\partial_t(\mathbf{v}.\,\omega) + \mathbf{v}.\,\mathbf{grad}(\mathbf{v}.\,\omega) = \omega.\mathbf{grad}(\mathbf{v}^2/2 - P) + \nu\Delta\,(\mathbf{v}.\,\omega)$$

→ ***v. ω (x)*** *can grow* ***locally*** *on a fast (nonlinear) time-scale even though it is conserved globally*

Taylor-Green vortex (non-helical) - Blow-up at peak of dissipation

Vorticity $\omega = \nabla \times \mathbf{v}$ & Relative helicity in $h = cos(\mathbf{v}, \omega)$

Local v-ω alignment (Beltramization) (*Tsinober & Levich, 1983; Moffatt, 1985, …).*

→ no mirror symmetry, together with weak nonlinearities in the small scales

Blue, h> 0.95 ; Red, h<-0.95

Taylor-Green vortex (non-helical) - Blow-up at peak of dissipation

Vorticity $\omega = \nabla \times \mathbf{v}$ & Relative helicity $h = \cos(\mathbf{v}, \omega)$

Local v-ω alignment (Beltramization) (*(Tsinober & Levich, 1983; Moffatt, 1985, …).*

→ no mirror symmetry, together with weak nonlinearities in the small scales

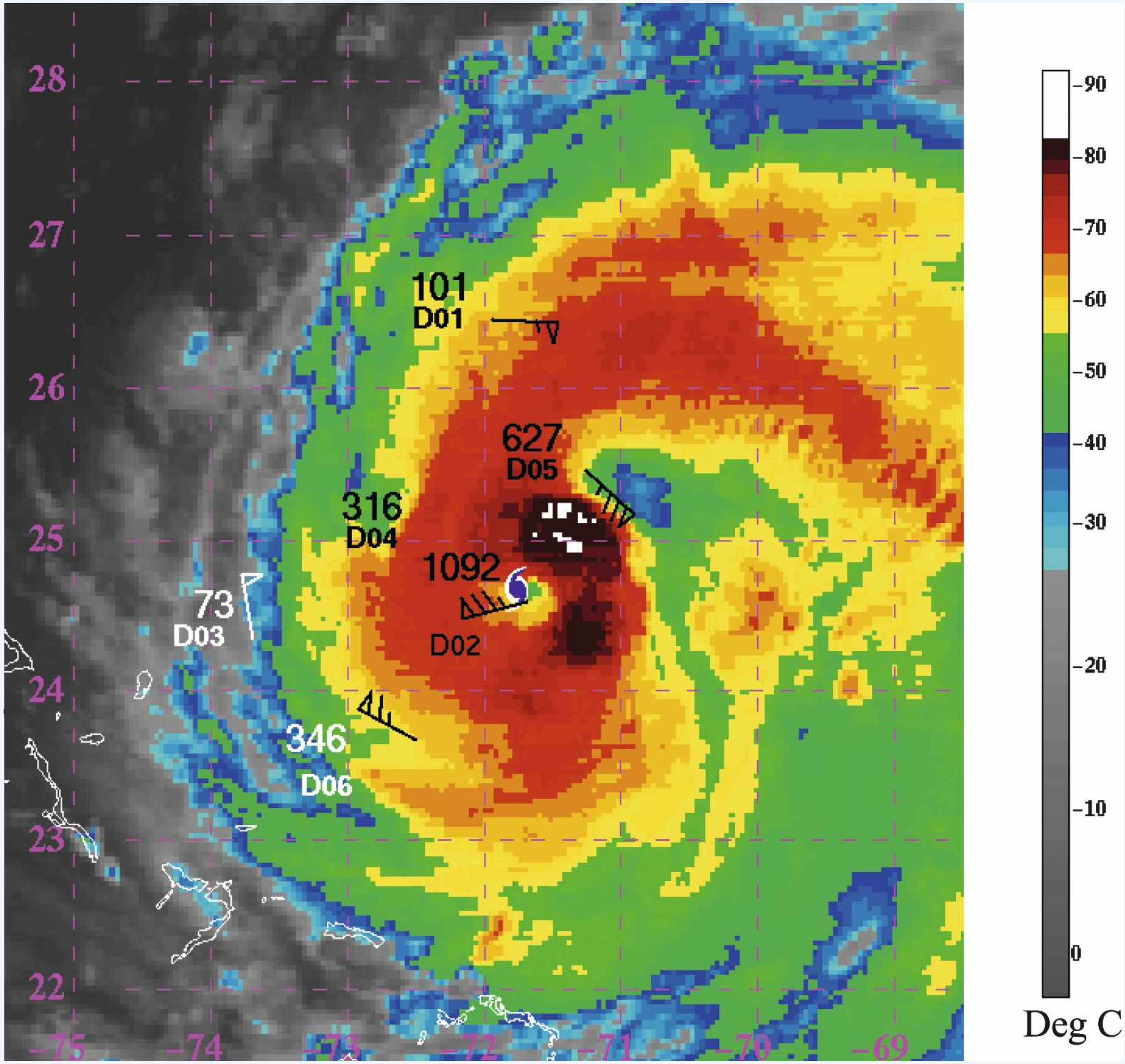
$E(k) \sim H(k) \sim k^{-5/3}$

$h(k) = H(k)/kE(k) \sim 1/k$

→ Link with intermittency

?

Blue, h> 0.95 ; Red, h<-0.95



$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} \, dV$$

Hurricane Bonnie (1998) (V~50m/s, shear 12m/s): helicity, winds and brightness temperature from tropospheric dropsondes

Molinari & Vollaro, 2008

Helicity and shear in tropical cyclones, *Molinari & Vollaro, '10*

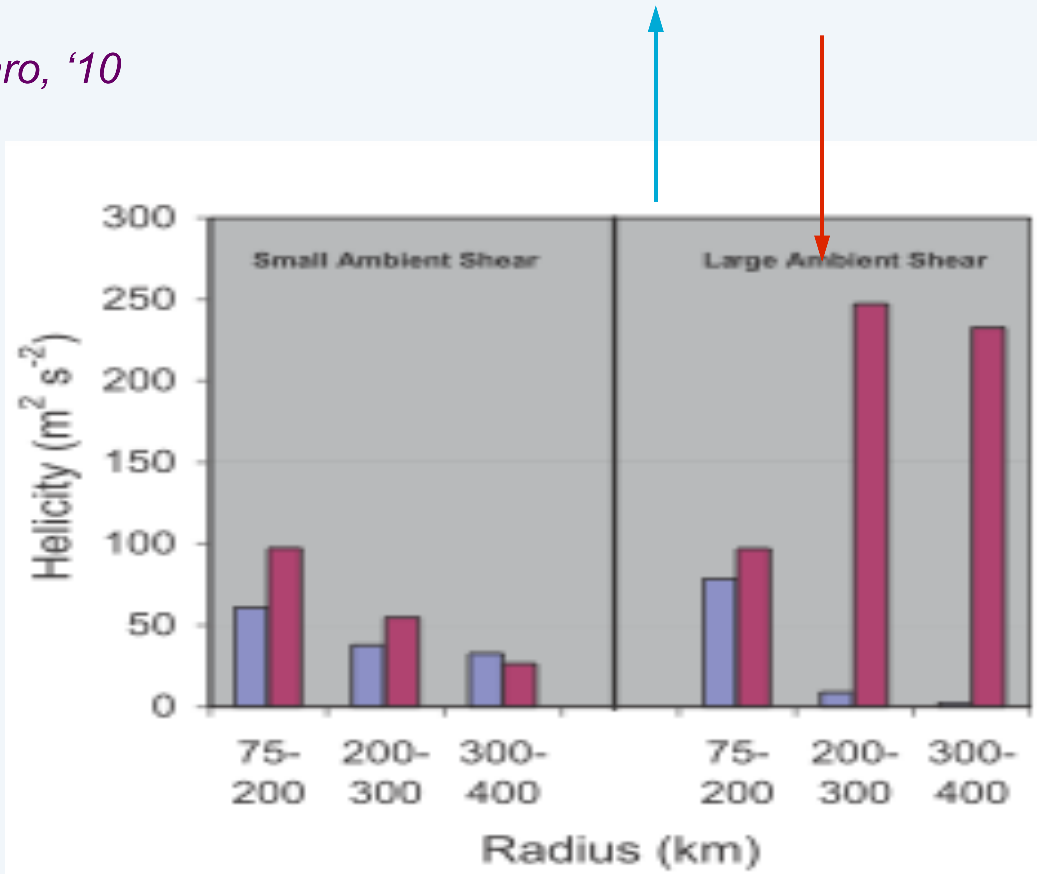


FIG. 4. Radial variation of 0–3-km helicity ($m^2 s^{-2}$) averaged over 75–200, 200–300, and 300–400 km for TCs experiencing (left) small and (right) large ambient shear. Upshear means are in blue and downshear in red.

$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} \, dV$$

Helicity spectrum in the Planetary Boundary Layer

Flat spectrum at night
(when more stable)

Koprov, 2005

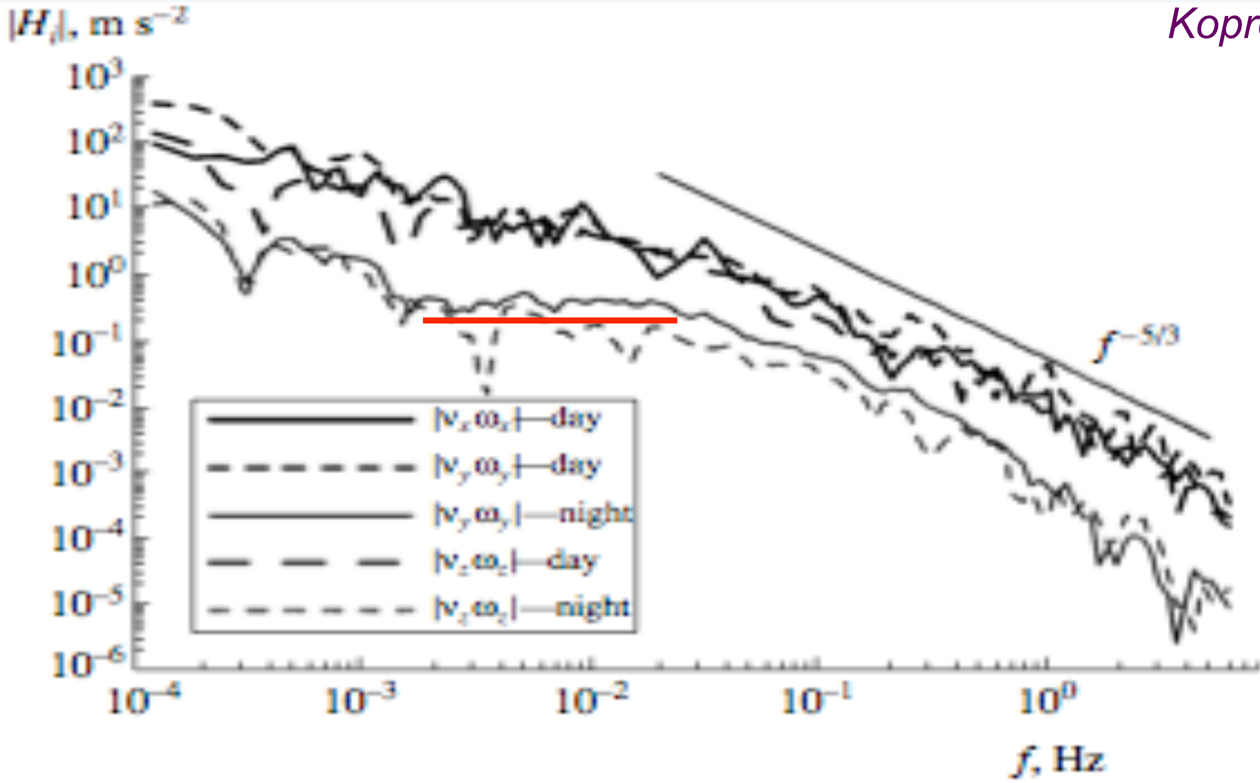


Fig. 4. Spectra of helicity components.

Helicity in other geophysical flows

→ Secondary currents in river bends, effect on salt distribution

→ Mixing in estuaries and interactions with tidal flows

→ Isopycnals are helical surfaces when eq. of state is nonlinear

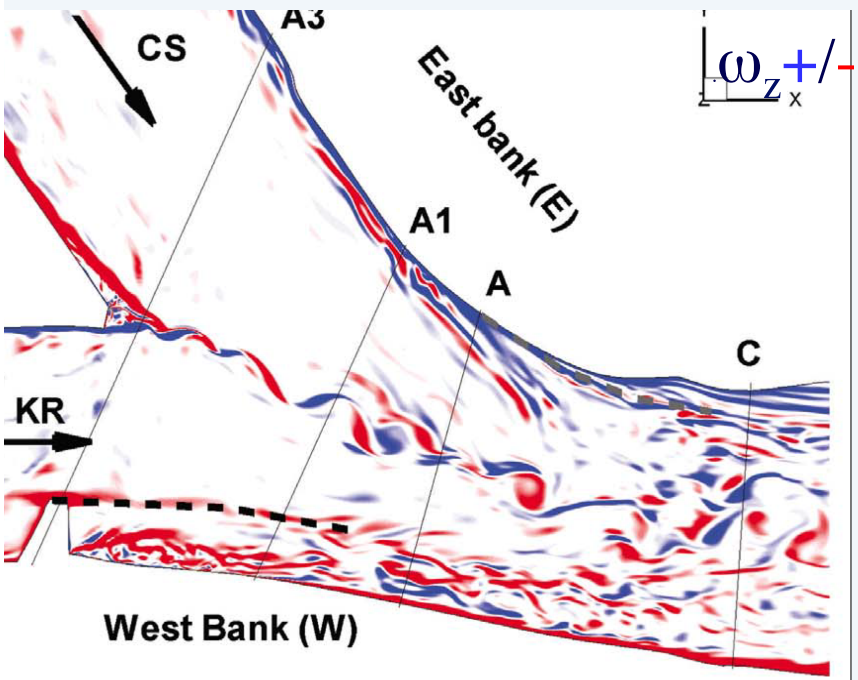
→ Helicity and large-scale instabilities, as in hurricanes

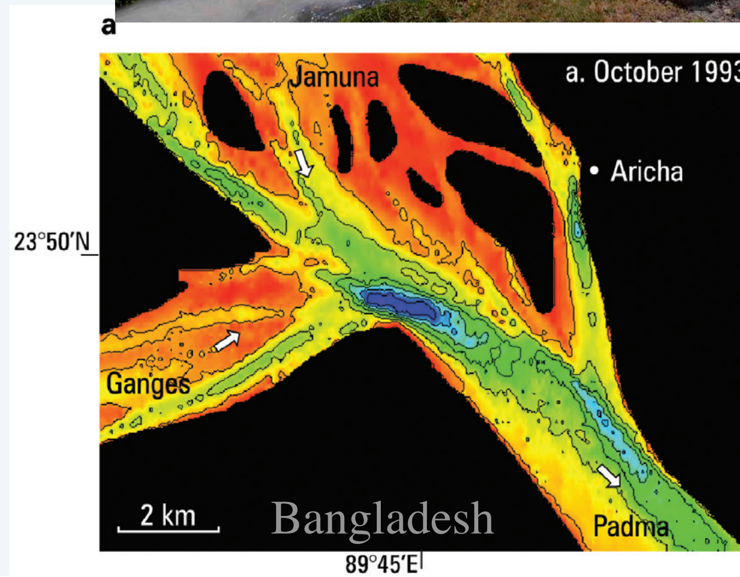
→ Production of large-scale helical magnetic fields *(& shear)*

River confluence, sedimentation, mixing, erosion, water quality control, morphology
...

Figure 5.6 Oblique aerial photograph of the junction of the Río Paraná and Río Paraguay. Note the contrast produced by the higher suspended sediment concentrations of the Río Paraguay and the vorticity present along the mixing interface.

From Rice et al. Eds., Wiley, 2008
And Karimpur & Smalley, 2011





Boussinesq equations

Geostrophic Balance

$$
\begin{aligned}
\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} &= -\nabla P - Nbe_z - 2\Omega e_z \times \mathbf{u} + \mathrm{F} \\
\partial_t b + \mathbf{u}\cdot\nabla b - \kappa\Delta b &= Nw\ , \\
\nabla\cdot\mathbf{u} &= 0\ .
\end{aligned}
$$

= 0

curl (GB) → thermal *vshw* winds: $f\,\partial_z u = N\partial_y b$, $f\,\partial_z v = -N\partial_x b$

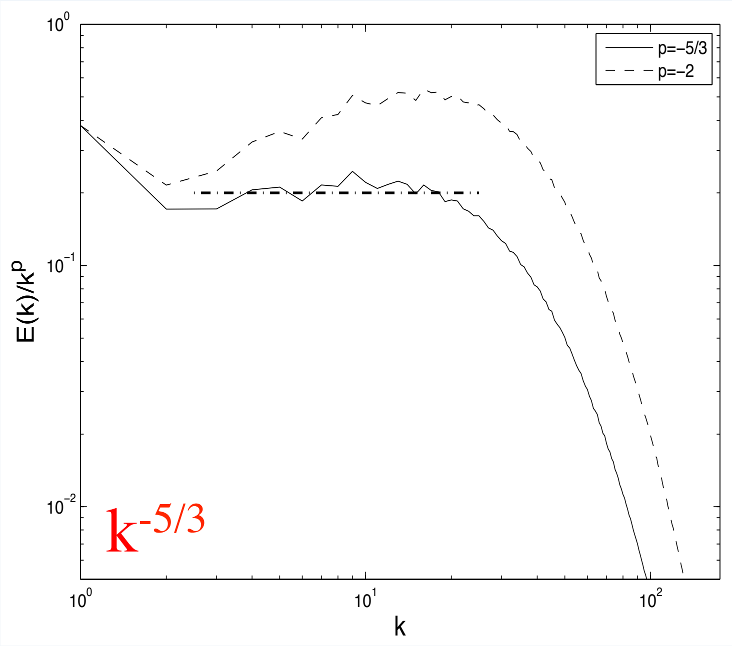
dot w. Coriolis force & spatial average →

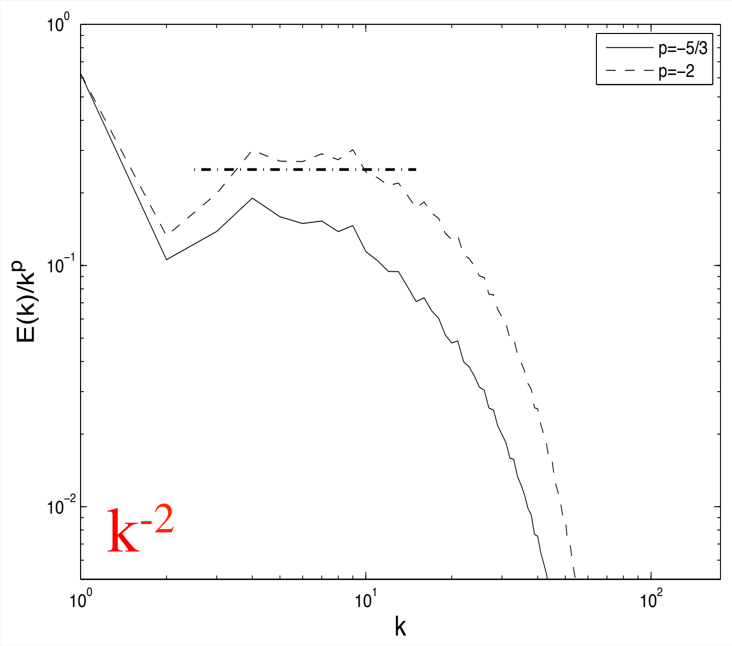
$$\langle H_\perp\rangle_\perp \equiv \langle u_\perp\cdot\nabla\times u_\perp\rangle_\perp = \frac{N}{f}\langle b\,\frac{\partial w}{\partial z}\rangle_\perp$$

f=2Ω

Hide, 1976

2 different compensations of total energy spectra
N/f ~ 3, Re~ 7000





Fr ~ 0.11, Ro ~ 0.39, R_B ~ 70

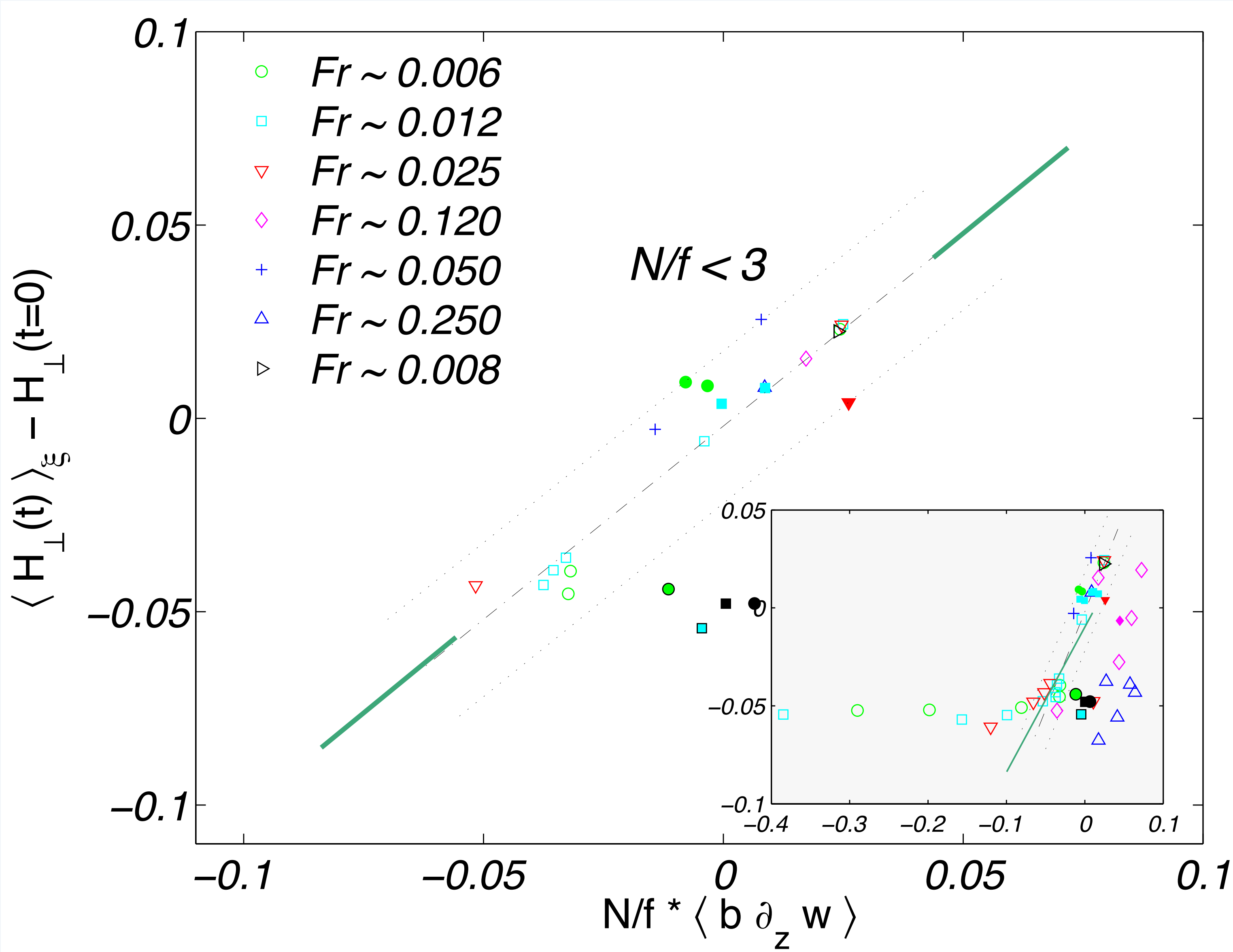
Fr ~ 0.006, Ro ~ 0.018, R_B ~ 0.3

E(k) ~ $k^{-5/3}$

E(k) ~ k^{-2}

Buoyancy scale L_B resolved in both cases

$L_{Ozmidov}$ resolved here only



Selection of data from 45 runs, 9 on 512^3 grids *(filled symbols)*

Criterion:

$ReFr^2 < 20$,
$ReRo^2 < 20$

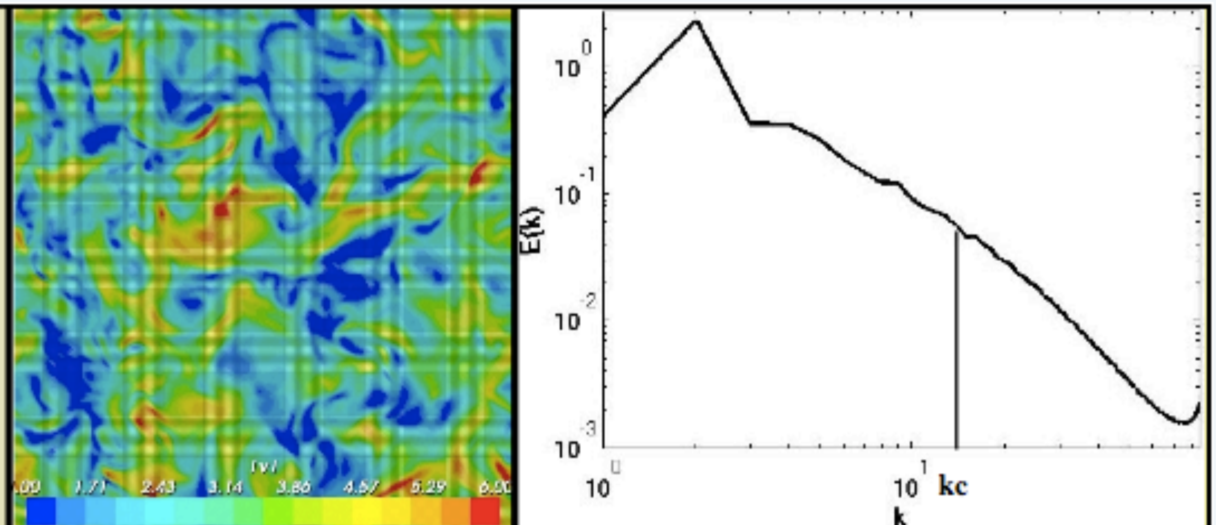
Shaded box:
all runs

$$\langle H_\perp \rangle_\perp \equiv \langle u_\perp \cdot \nabla \times u_\perp \rangle_\perp = \boxed{\frac{N}{f}} \langle b \, \frac{\partial w}{\partial z} \rangle_\perp$$

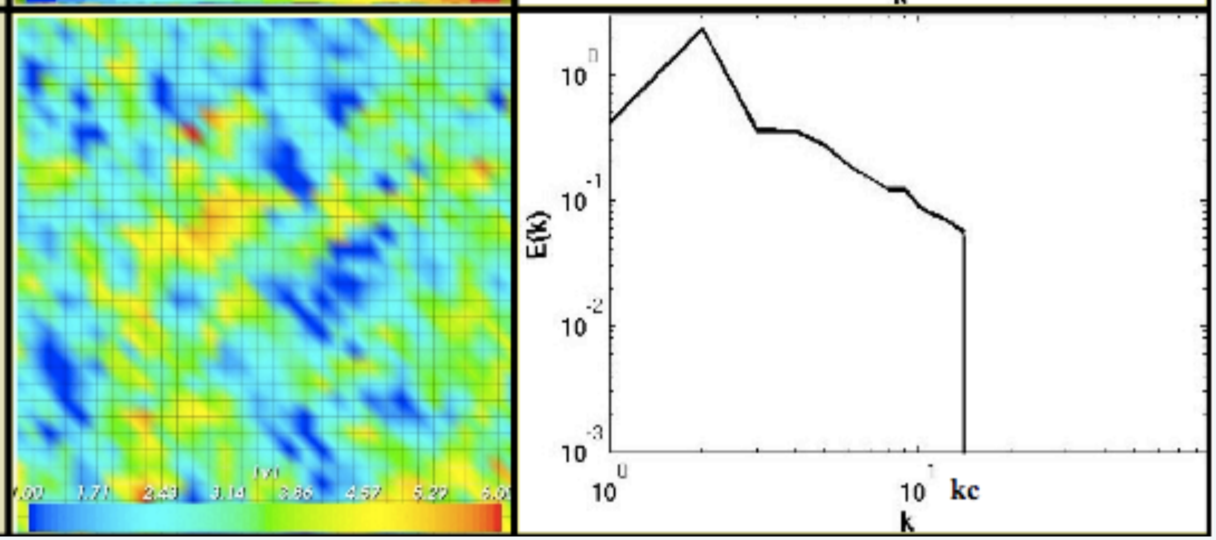
Marino et al., 2013

Numerical modeling

Direct numerical simulation with 256^3 grid-points



Filtered velocity field to 32^3 grid-points



Slide from Julien Baerenzung, 2008

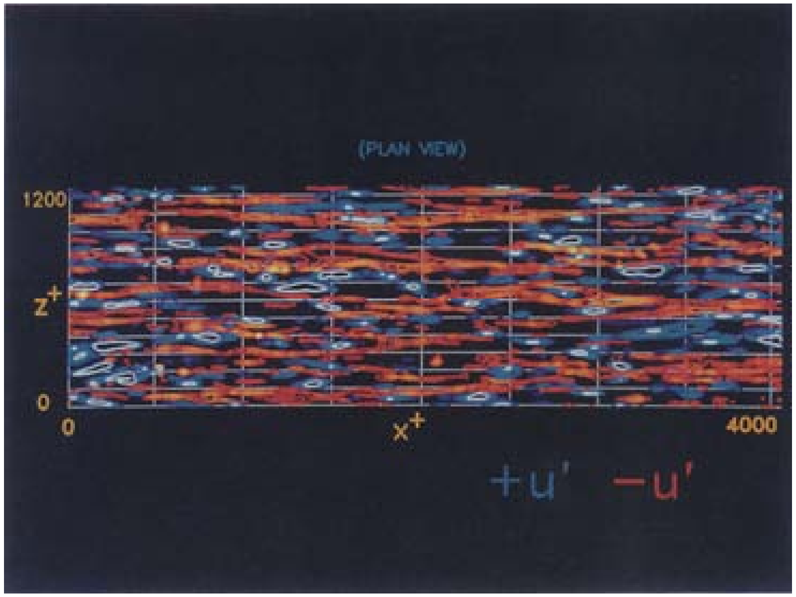
Modeling of helical flows

- Streaks in channel flow are strongly helical near the boundary, and in turn dissipation is weaker

- The Smagorinsky constant is adjusted to be half the value of the isotropic case*: helicity decreases nonlinearities and thus eddy everywhere,*

  except perhaps

  in shear layers



Contours of fluctuating streamwise velocity

Robinson et al. 1988 (after Yokoi, 2012)

Modeling of helical flows

$[\nu] \sim U^*L \rightarrow \nu^H_{turb} \sim L^3/U$ *(Yokoi, 2010)*

$\nu_{turb} k^2 v_k \rightarrow \nu_{turb} k^2 v_k + \nu^H_{turb} k^2 \omega_k$

à la Chollet-Lesieur (1981), *EDQNM-based closure, Baerenzung et al. 2008*

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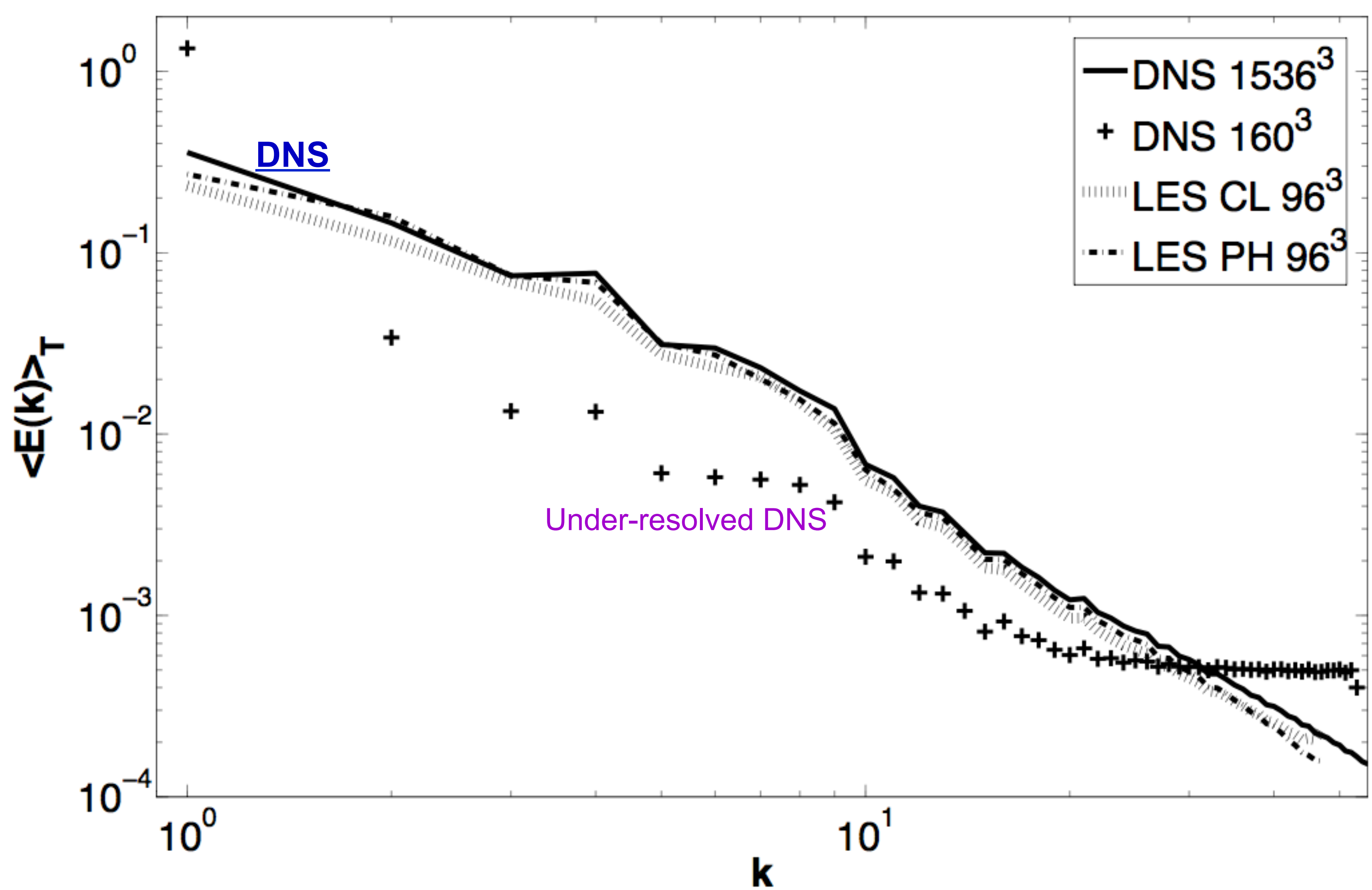
$\nu_{turb} k^2 v_k \rightarrow \boxed{\nu_{turb} k^2 v_k + \nu^H_{turb} k^2 \omega_k}$

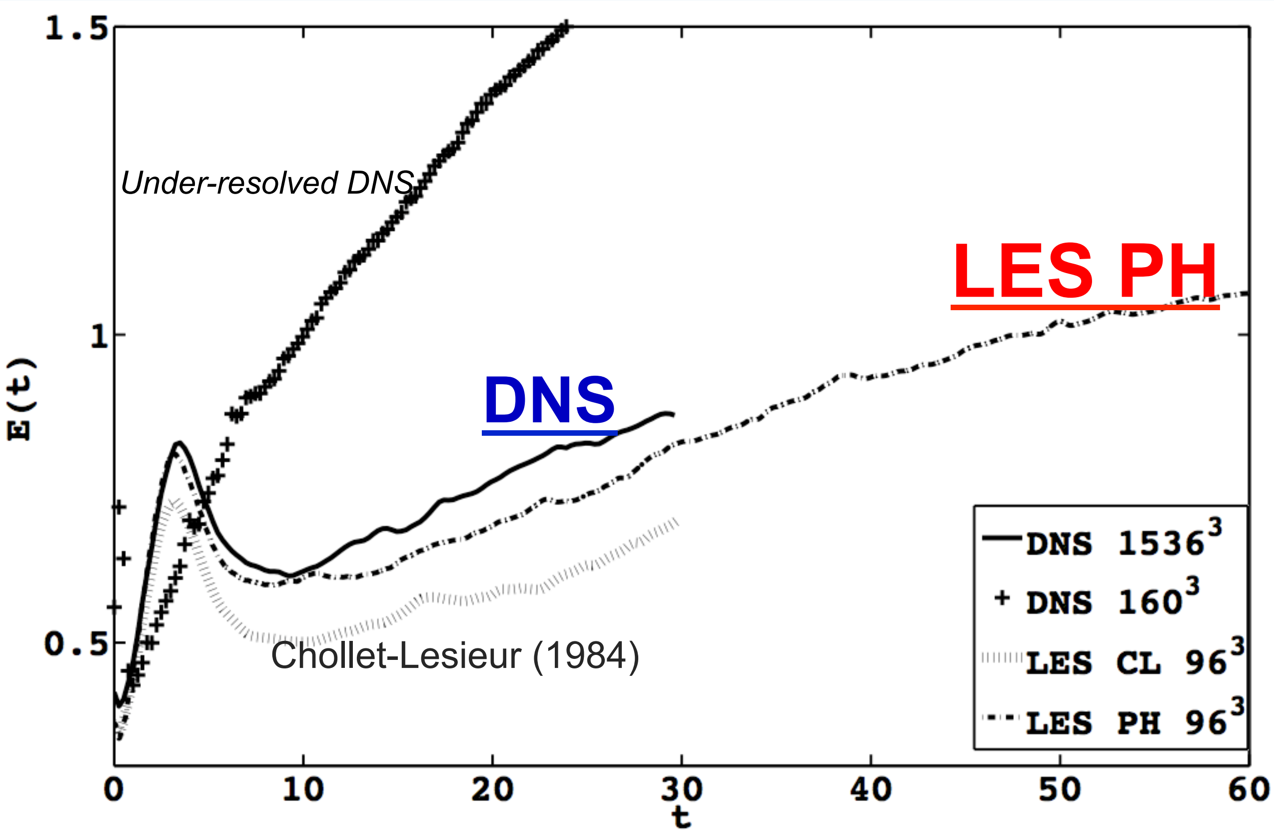
+ Eddy noise, or back-scatter
(Rose 1977, Mason & Thomson 1992, Sura 2011, Palmer 2012),
depending again on helicity

à la Chollet-Lesieur (1981), *EDQNM-based closure, Baerenzung et al. 2008*

Validation of LES in spectral space using Direct Numerical Simulations (DNS)

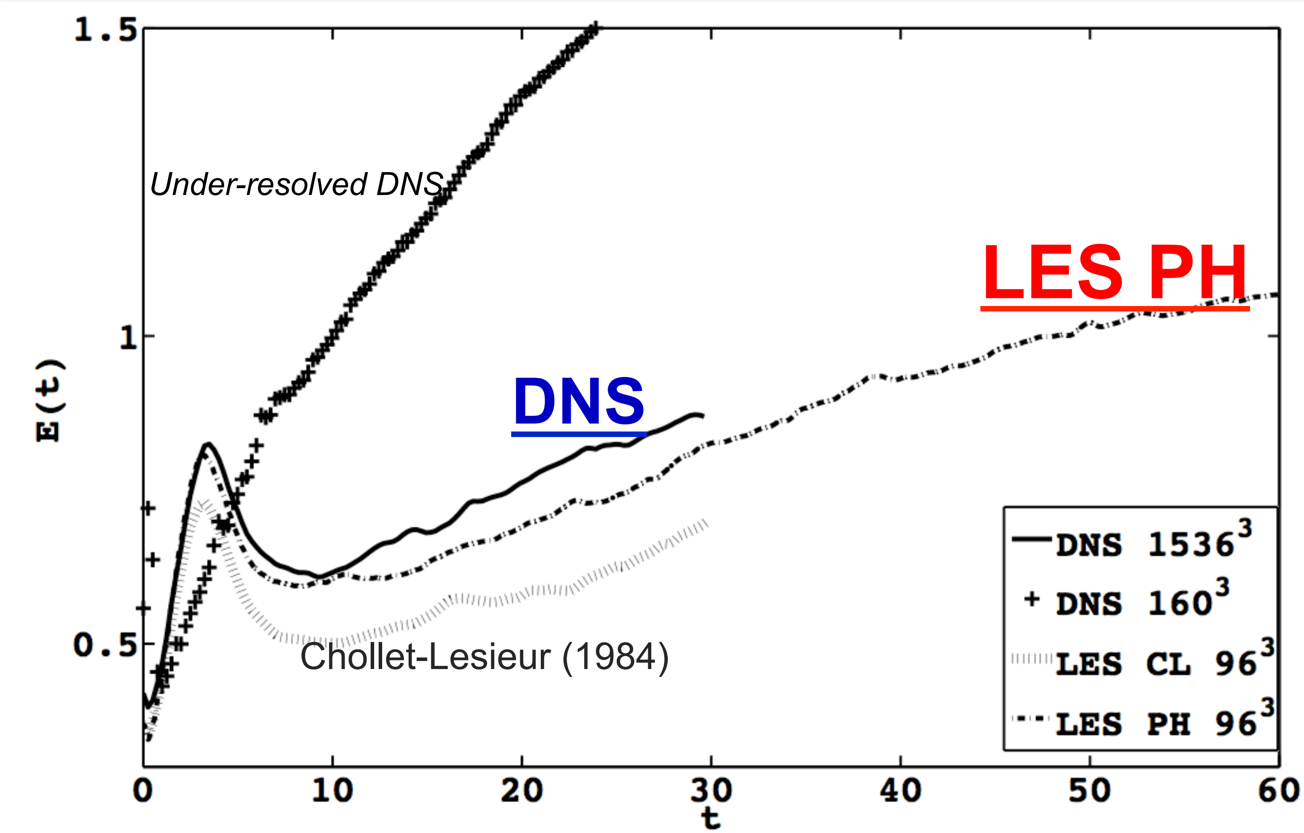


Validation of LES



Savings in CPU : 0.5*[1536/96]^4 ~ 30,000 *(also for memory)*

Validation of LES



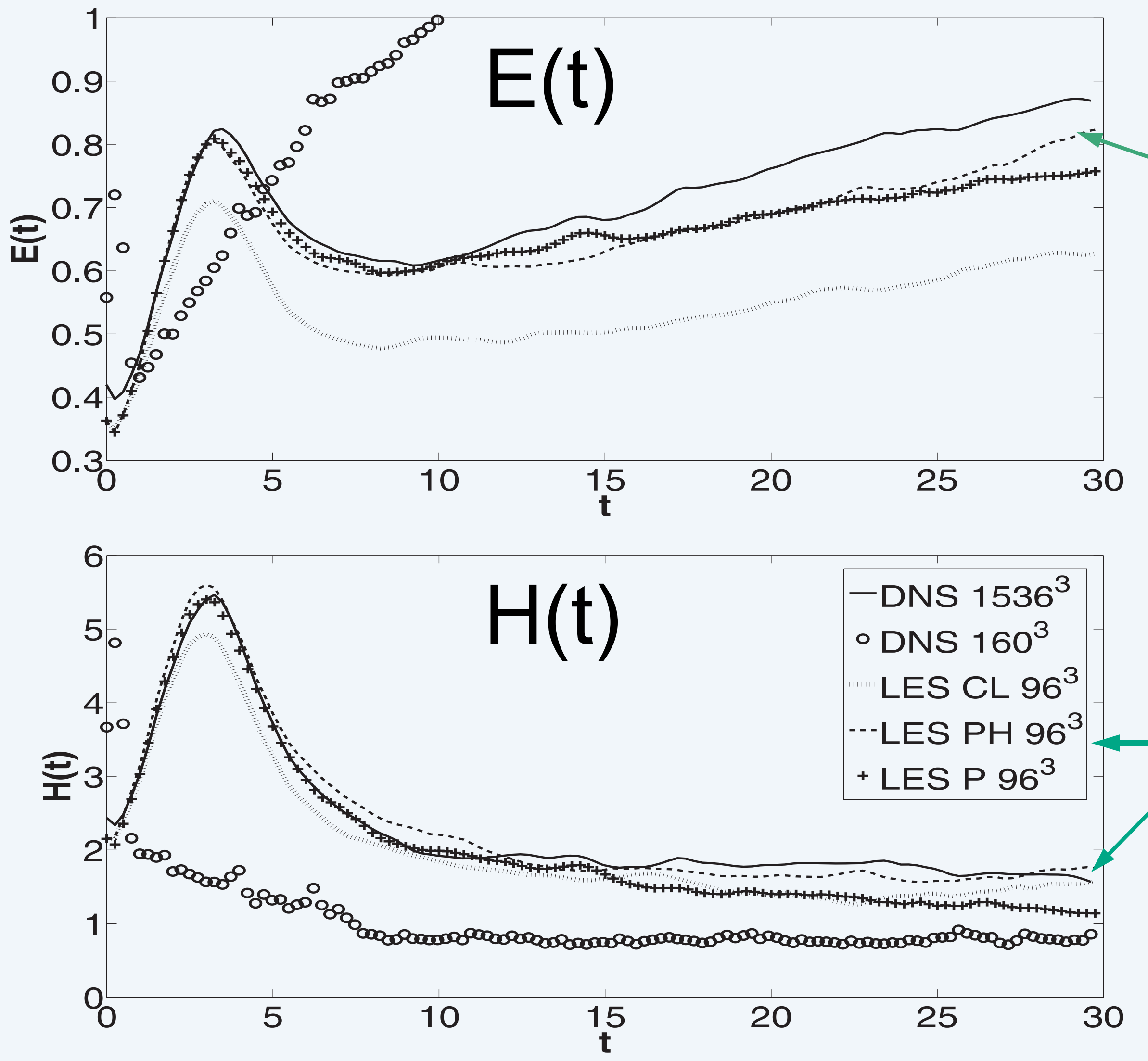
^ Include anisotropy

^ In the stratified case, include kinetic-potential energy exchanges

(Osborn-Cox, Mellor-Yamada; Zilitinkevich+, Galperin+, …)

Savings in CPU : $0.5*[1536/96]^4 \sim 30{,}000$ *(also for memory)*

Rotating turbulence



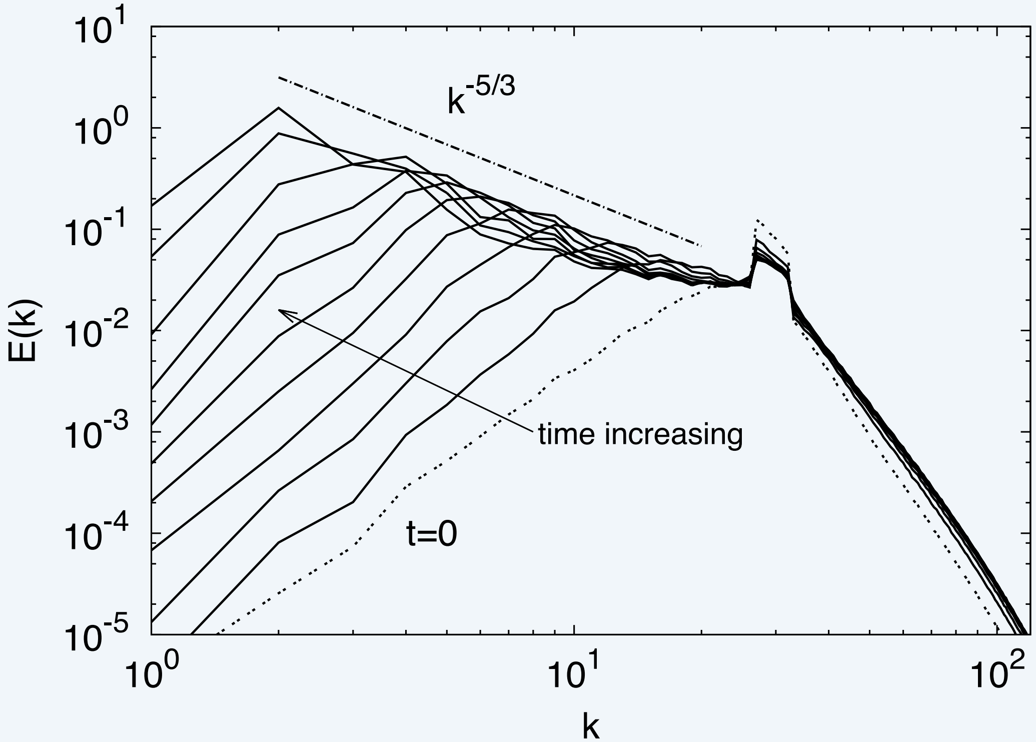
Helical model is closer to DNS

Baerenzung et al. 2011

Helicity

- Craya-Herring-Waleffe decomposition into ± circularly polarized waves
- Triad interactions (s,s',s'') where s,s',s''= ±

- Restrict Navier-Stokes dynamics to one-sign interactions:
  (+++) → inverse cascade of energy in 3D NS *(Biferale et al. 2012)*



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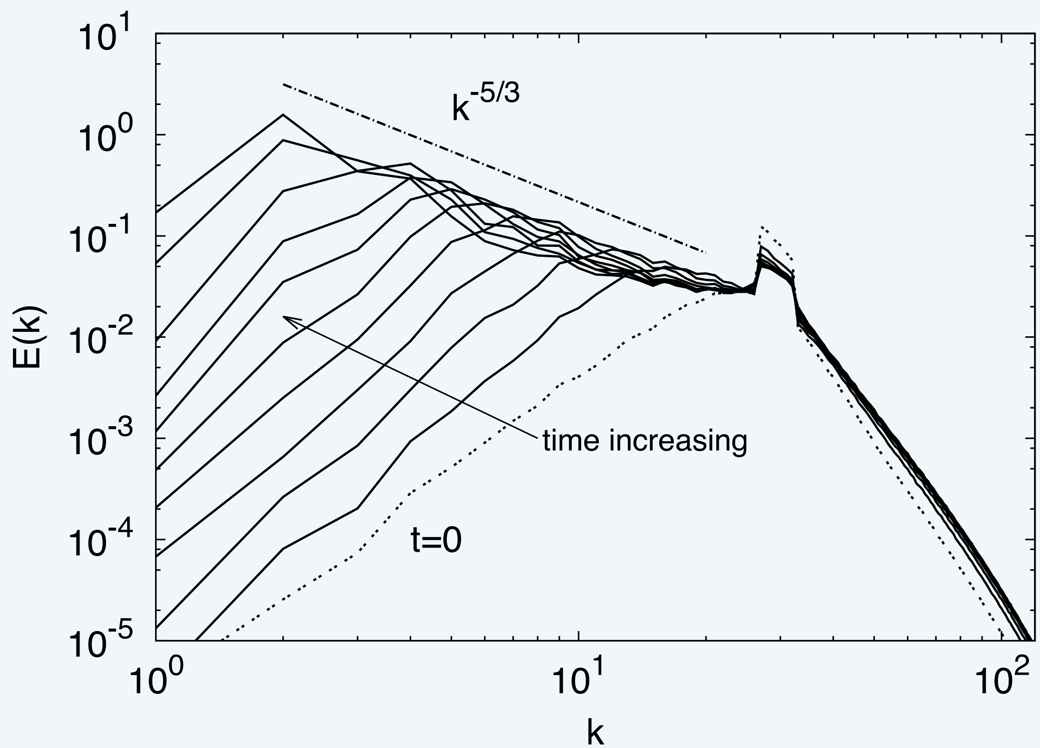

Also observed for rotating flows, forced with maximal helicity

(Mininni & Pouquet, 2010)

Helicity

- Craya-Herring-Waleffe decomposition into ± circularly polarized waves
- Triad interactions (s,s',s'') where s,s',s''= ±
- Restrict to one-sign interactions, say (+++) → inverse cascade of energy in 3D NS
- Kraichnan (1973): one-signed triad interactions are subdominant

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- Triad interactions (s,s',s'') where s,s',s''= ±
- Restrict to one-sign interactions, say (+++) → inverse cascade of energy in 3D NS
- Kraichnan (1973): one-signed triad interactions are subdominant

- Production of point-wise helicity *(Matthaeus et al. 2008)*
- Relative helicity $\sigma(k)=H(k)/[kE(k)] \sim 1/k$, but there are strongly helical vortex filaments in the dissipation range → local V determined by Biot-Savart (LIA)

- **Regularity** of the NS eqs. when restricted to 1-sign interactions *(Biferale &Titi, 2013)*

``*If the dynamics is restricted to the sub-set of modes with a well definite sign of helicity (i.e. positive), then the flow admits unique global weak solutions that depend continuously on the initial data*.''

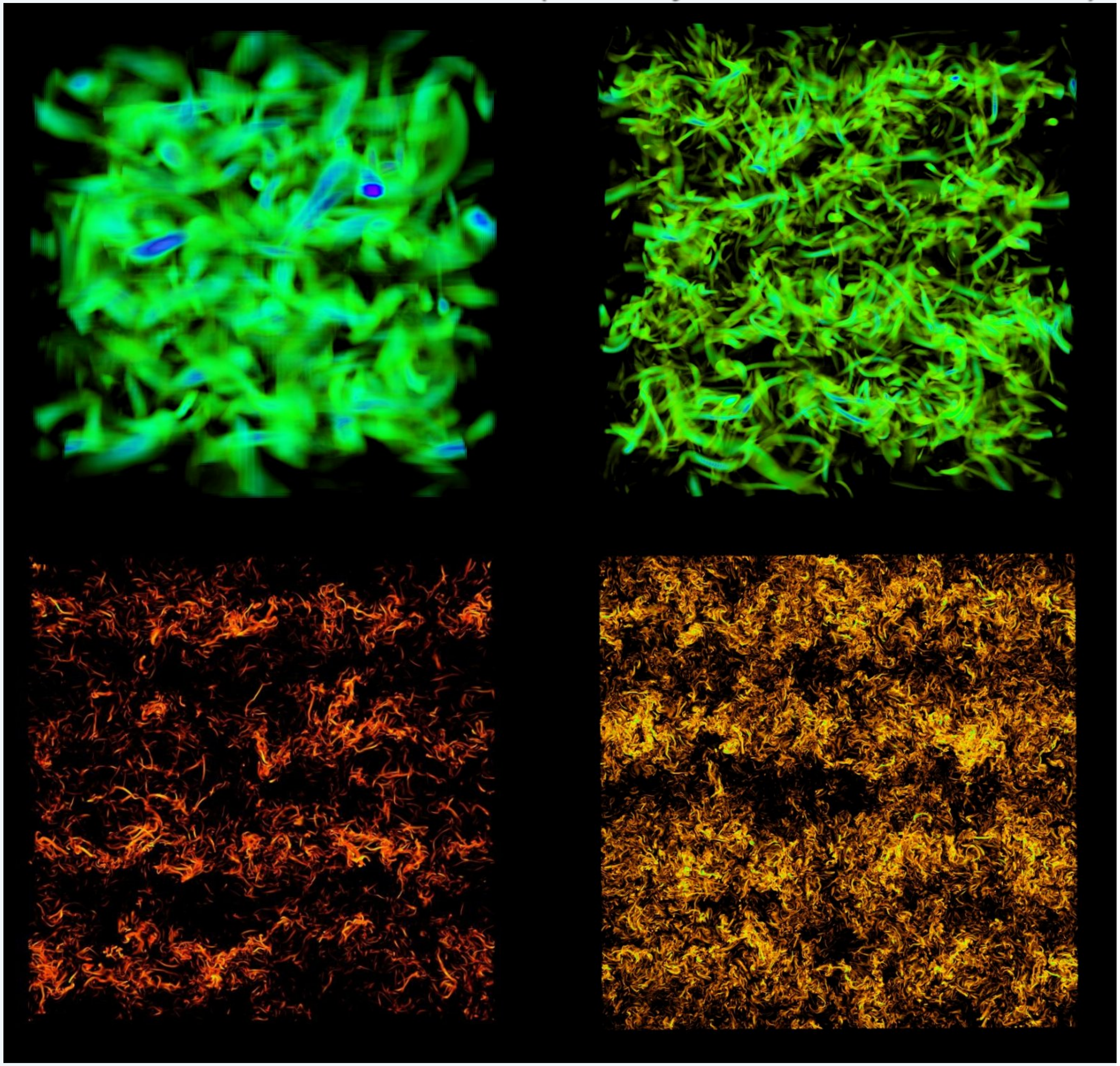
What does grid resolution buy you?

Navier-Stokes grids
with N^3 points

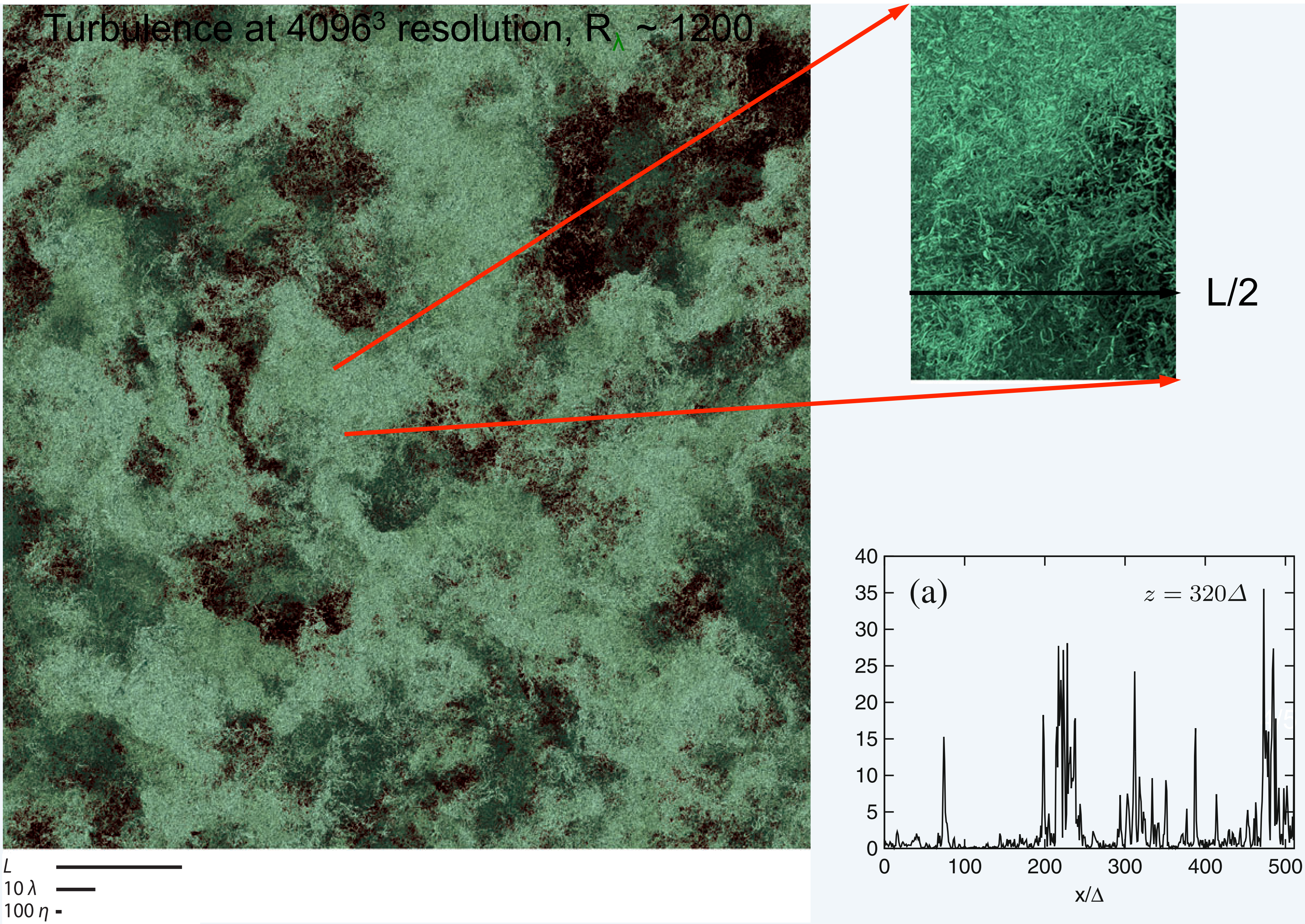
$Re = UL/\nu$

64^3 & 256^3
1024^3 & 2048^3

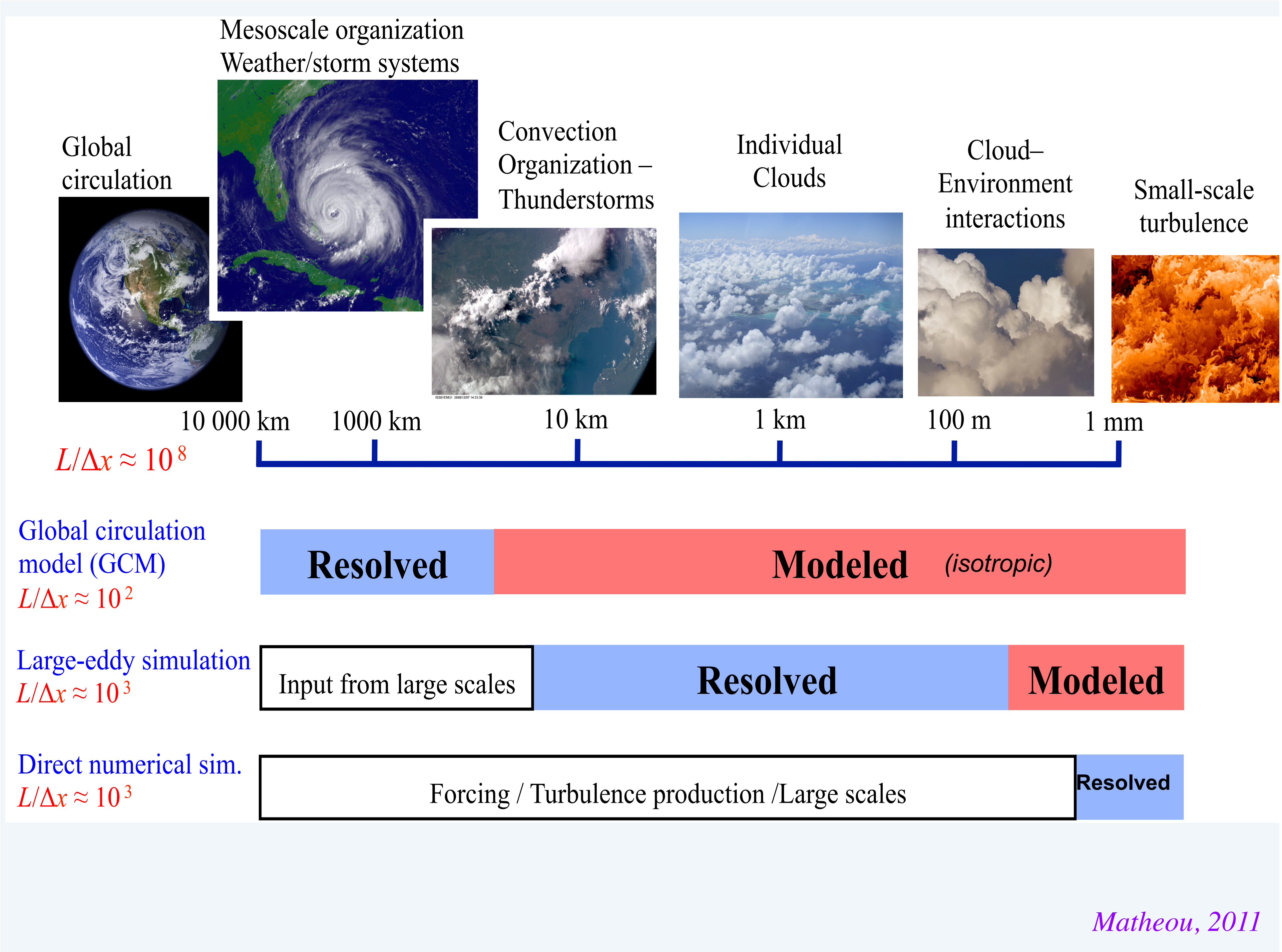
VAPOR freeware (John Clyne & Alan Norton, NCAR)



Multi-scale interactions & persistence



Kaneda et al. 2003,
Ishihara et al. 2009


Mesoscale organization
Weather/storm systems
Global
circulation
Convection
Organization –
Thunderstorms
Individual
Clouds
Cloud–
Environment
interactions
Small-scale
turbulence
10 000 km
1000 km
10 km
1 km
100 m
1 mm
$L/\Delta x \approx 10^8$
Global circulation
model (GCM)
$L/\Delta x \approx 10^2$
Resolved
Modeled
(isotropic)
Large-eddy simulation
$L/\Delta x \approx 10^3$
Input from large scales
Resolved
Modeled
Direct numerical sim.
$L/\Delta x \approx 10^3$
Forcing / Turbulence production /Large scales
Resolved
Matheou, 2011

What is different in Rotating &/or Stratified Turbulence? (RST)

- *Bi-directional constant-flux energy cascade (Marino's talk)*
- *Anisotropy*

- IA- Non-conservation of helicity *(velocity-vorticity correlations)*
- IB- Intermittency of the vertical velocity & temperature fields *[f=0]*
- IIA- Bolgiano-Obukhov scaling and the role of potential energy
- IIB- Anomalous mixing, dissipation & the role of potential energy

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Cushman-Roisin; Ghil; MacWilliams; Pedlovsky; Vallis, …

Homogeneous and isotropic case

Incompressible Navier-Stokes equations

$$\begin{aligned} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} &= -\nabla P \qquad + \mathrm{F} \\ \partial_t b + \mathbf{u}\cdot\nabla b - \kappa\Delta b &= 0 \ , \\ \nabla\cdot\mathbf{u} &= 0 \ . \end{aligned}$$

e.g., chemical tracer

$\dfrac{T_{\text{dissipation}}}{T_{\text{nonlinear}}}$

$\mathrm{Re} = U_0 L_0/\nu \gg 1$ *Reynolds number*

Non-linear term → convolution in Fourier space

→ coupling between scales

Modeling through both eddy viscosity & eddy noise

Rotating stratified flows

Incompressible Boussinesq equations

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} = -\nabla P - Nbe_z - 2\Omega e_z \times \mathbf{u} + \mathrm{F}$$

$$\partial_t b + \mathbf{u}\cdot\nabla b - \kappa\Delta b = Nw\,,$$

$$\nabla\cdot\mathbf{u} = 0\,.$$

Effective buoyancy

$$\frac{T_{wave}}{T_{nonlinear}}$$

Fr = $U_0/NL_0 < 1$ *Froude number*

Ro = $U_0/fL_0 < 1$ *Rossby number*

→ Inertia-gravity waves

→ Interplay between fast inertia-gravity waves and nonlinear eddies

Rotating stratified flows

Incompressible Boussinesq equations

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} - \nu \Delta \mathbf{u} = -\nabla P - Nb e_z - 2\Omega e_z \times \mathbf{u} + \mathrm{F}$$

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$$\nabla \cdot \mathbf{u} = 0 \ .$$

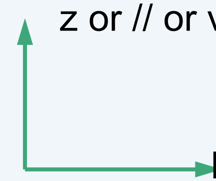
$$\frac{T_{wave}}{T_{nonlinear}}$$

$Fr = U_0/NL_0 < 1$ *Froude number*

$Ro = U_0/fL_0 < 1$ *Rossby number*

Frequency of inertia-gravity waves:

ω_k = [1/k]\sqrt{N^2 k_{perp}^2 + f^2 $k_{//}^2$}



Stratified flows (f=0)

$Fr = U_0/[NL_0] < 1$ *Froude number*

Scale at which Fr =1?

→ $L_B = U_0/N$ Buoyancy scale

And for a Kolmogorov spectrum, $u(l) \sim \varepsilon^{1/3} l^{1/3}$

→ $L_{Ozmidov} = [\varepsilon/N^3]^{1/2}$

→Buoyancy Reynolds number: $R_B = Re\, Fr^2$

$R_B = 1$ for $L_{Oz} = \eta = [\varepsilon/\nu^3]^{-1/4}$ *(Kolmogorov dissipation scale)*

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→Buoyancy Reynolds number: $R_B = Re\, Fr^2$ **$= [L_{oz}/\eta]^{4/3}$**

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$R_B = 1$ for $L_{Oz} = \eta = [\varepsilon/\nu^3]^{-1/4}$ *(Kolmogorov dissipation scale)*

Rotating flows (N=0): *N* → *f*

Ro = $U_0/[fL_0]$ < 1 *Rossby number*

Scale at which Ro =1?

→ $L_?$ = U_0/f what scale?

And for a Kolmogorov spectrum, $u(l) \sim \varepsilon^{1/3}\, l^{\,1/3}$

→ $L_{Zeman} = [\varepsilon/f^3]^{1/2}$

→Micro Rossby number: $R_\omega = [\mathrm{Re}\ \mathrm{Ro}^2]^{1/2}$

$R_\omega = 1$ for $L_{Zeman} = \eta = [\varepsilon/\nu^3]^{-1/4}$

Rotating flows (N=0): $N \rightarrow f$

Ro = $U_0/[fL_0]$ < 1 *Rossby number*

Scale at which Ro =1?

→ $L_?$ = U_0/f what scale?

And for a Kolmogorov spectrum, $u(l) \sim \varepsilon^{1/3}\, l^{\,1/3}$

→ $L_{Zeman} = [\varepsilon/f^3]^{½}$

→Micro Rossby number: $R_\omega = [\mathrm{Re}\ \mathrm{Ro}^2]^{1/2}$ **$= \omega_{rms}/f$**

R_ω = 1 for $L_{Zeman} = \eta = [\varepsilon/\nu^3]^{-1/4}$

Rotating flows (N=0): $N \rightarrow f$

$Ro = U_0/[fL_0] < 1$ *Rossby number*

Scale at which Ro =1?

$\rightarrow L_? = U_0/f$ what scale?

And for a Kolmogorov spectrum, $u(l) \sim \varepsilon^{1/3} l^{1/3}$

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$\rightarrow$ Micro Rossby number: $R_\omega = [Re\ Ro^2]^{1/2} = \boldsymbol{\omega_{rms}/f}$

$R_\omega = 1$ for $L_{Zeman} = \eta = [\varepsilon/\nu^3]^{-1/4}$

$[1/\text{Micro-buoyancy}]^2$:
Richardson number
$Ri = N^2/\text{Shear}^2$
$= N^2/\langle du_{perp}/dz\rangle^2$
And so:

$Fr_z = Ri^{-1/2}$

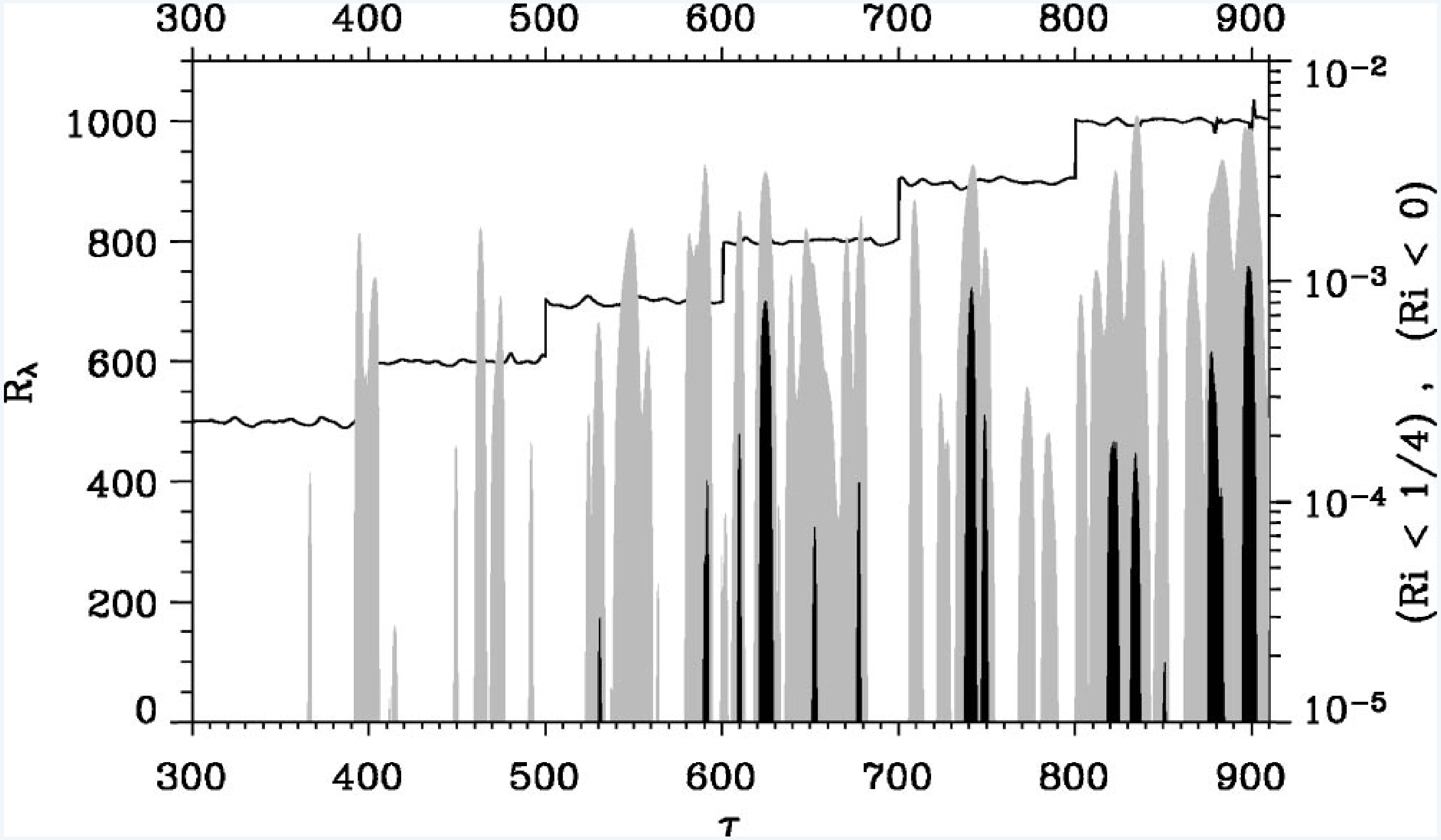


FIG. 4. Time evolution of R_λ (solid line) and the volume fraction of the domain with local Ri$<$0.25 (filled gray area) and with local Ri$<$0 (filled black area). There was no occurrence of Ri $<$0.25 for $0<\tau<300$.

Laval et al. 2003

Rotating and stratified flows

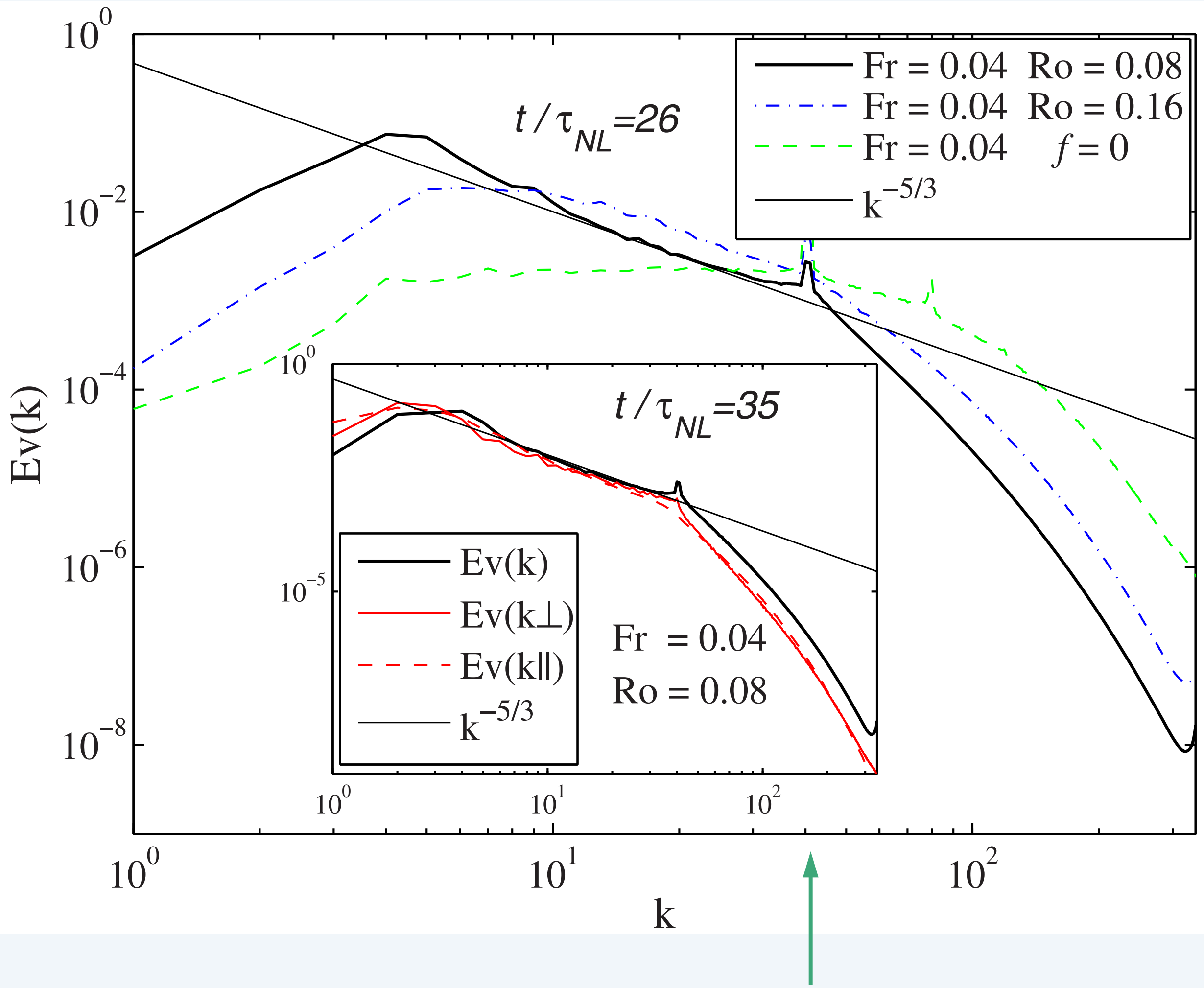
$L_D = [N/f]\, L_0$ *Deformation radius* *(Charney, '60s, …)*

$L_z \sim 1/N$ *or* $L_z \sim [f/N]\, L_{perp}$ *(Billant Chomaz, 2001)*

Also:

$\omega_k = [1/k]\sqrt{ N^2 k_{perp}^2 + f^2 k_{//}^2 }$

→ $N/L_{perp} \sim f/L_z$ *or* $N/f \sim L_{perp} / L_z$ *(cf. e.g. MacWilliams 2006)*



L_{Def} = [N/f] L_0

K_F=40

N/f=2

K_{Def}=20

Marino+ 2013

Rotating and stratified flows

$L_D = [N/f]\, L_0$ *Deformation radius* *(Charney, '60s)*

$L_z \sim 1/N$ *or* $L_z \sim [f/N]\, L_{perp}$ *(Billant Chomaz, 2001)*

Also:

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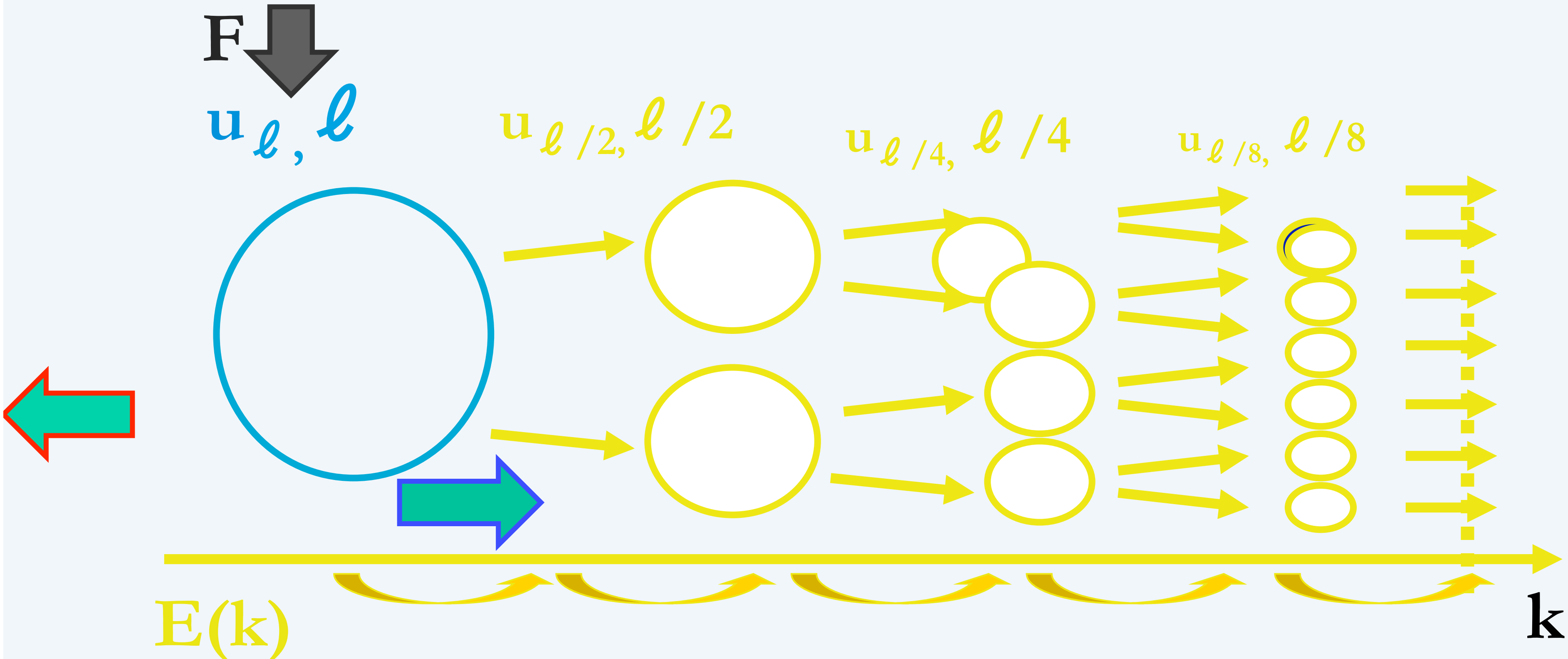
→ $N/L_{perp} \sim f/L_z$ *or* $N/f \sim L_{perp} / L_z$

Which of these scales (L_B, $L_{Ozmidov}$, L_{Zeman}, L_D, η) are resolved in a given simulation? Does it matter?
And if so, why and how?

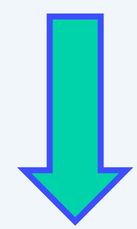
Inverse cascade, small-scale modeling, hyper-viscosity

*Less classical picture of **quasi-2D** turbulence*



$\epsilon = dE/dt$: energy dissipation rate

$E \sim kE(k)$ (locality) and $\tau \sim \ell/u_\ell$ *(eddy turn-over time),*



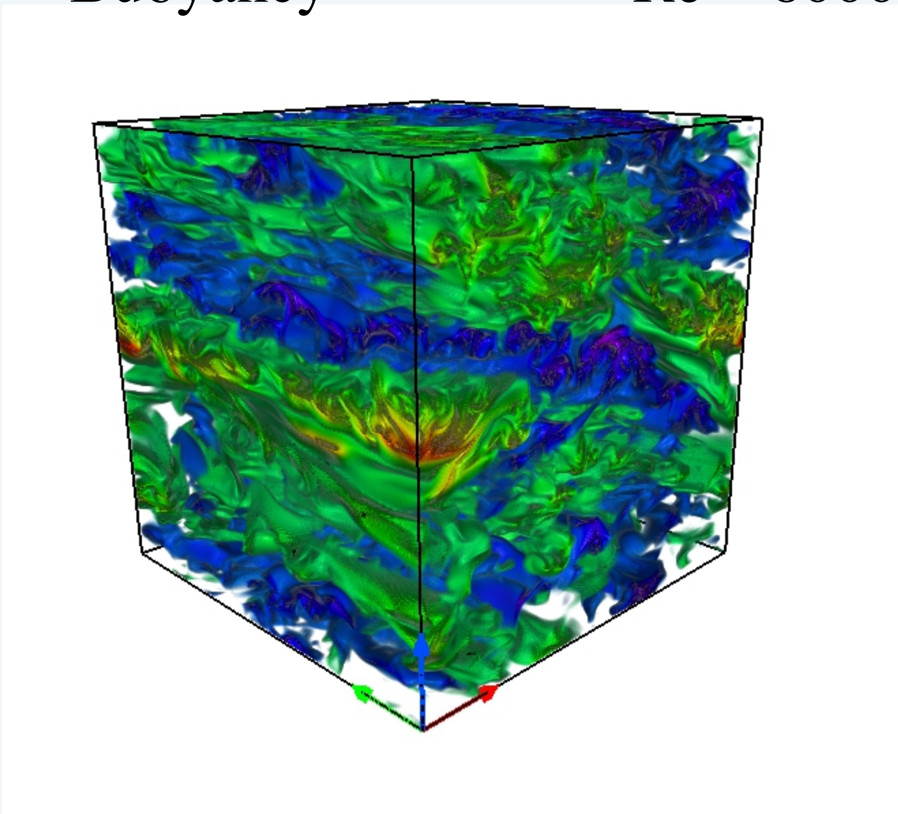
So: $\epsilon \sim u_\ell^{3}/\ell$

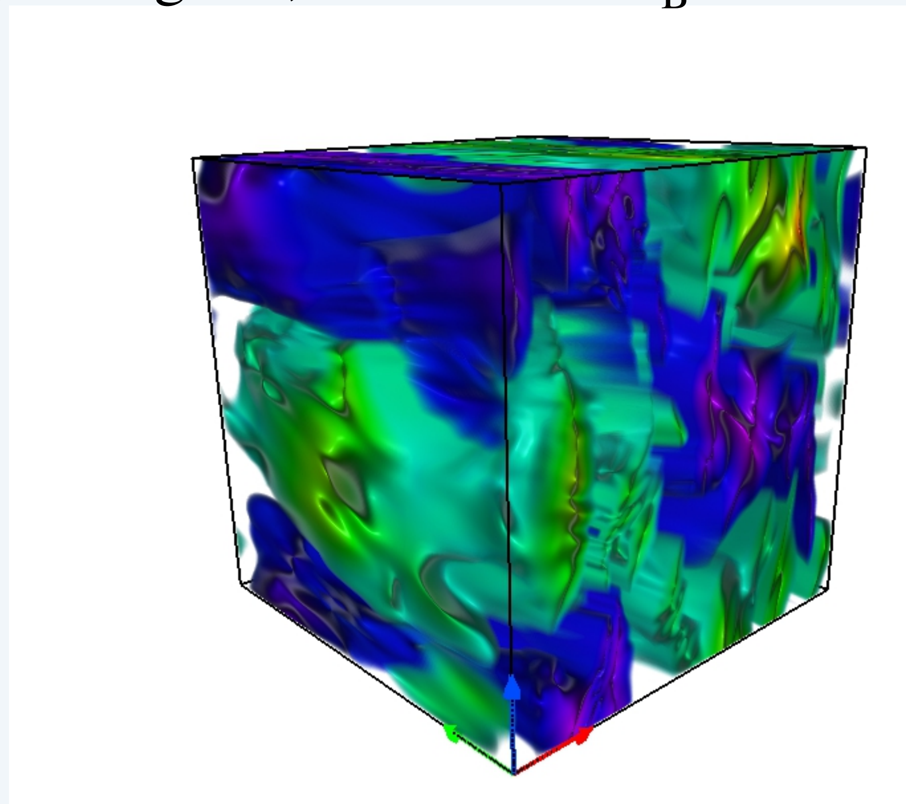
and $E(k) = C_K\, \epsilon^{2/3}\, k^{-5/3}$

Marino's talk: dual constant-flux energy cascade

ν

Buoyancy Re ~ 8000, 512^3 grids, $R_B = ReFr^2$





Fr ~ 0.11, Ro ~ 0.4,
R_B ~ 100, N/f ~ 3.6

Fr ~ 0.025, Ro ~ 0.05,
R_B ~ 5, N/f = 2

Marino et al., 2013

Recovered classical single-scale models: *Rot. Strat. Turb.*

$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\frac{t}{\varepsilon}, \boldsymbol{x}, \frac{z}{\varepsilon})$	Linear small scale internal gravity waves
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(t, \boldsymbol{x}, z)$	Anelastic & pseudo-incompressible models
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\varepsilon t, \varepsilon^2 \boldsymbol{x}, z)$	Linear large scale internal gravity waves
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\varepsilon^2 t, \varepsilon^2 \boldsymbol{x}, z)$	Mid-latitude Quasi-Geostrophic Flow
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\varepsilon^2 t, \varepsilon^2 \boldsymbol{x}, z)$	Equatorial Weak Temperature Gradients
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\varepsilon^2 t, \varepsilon^{-1} \xi(\varepsilon^2 \boldsymbol{x}), z)$	Semi-geostrophic flow
$\mathbf{U}^{(i)} = \mathbf{U}^{(i)}(\underline{\varepsilon^{3/2} t}, \underline{\varepsilon^{5/2} x}, \underline{\varepsilon^{5/2} y}, z)$	Kelvin, Yanai, Rossby, and gravity Waves

Klein, 2010

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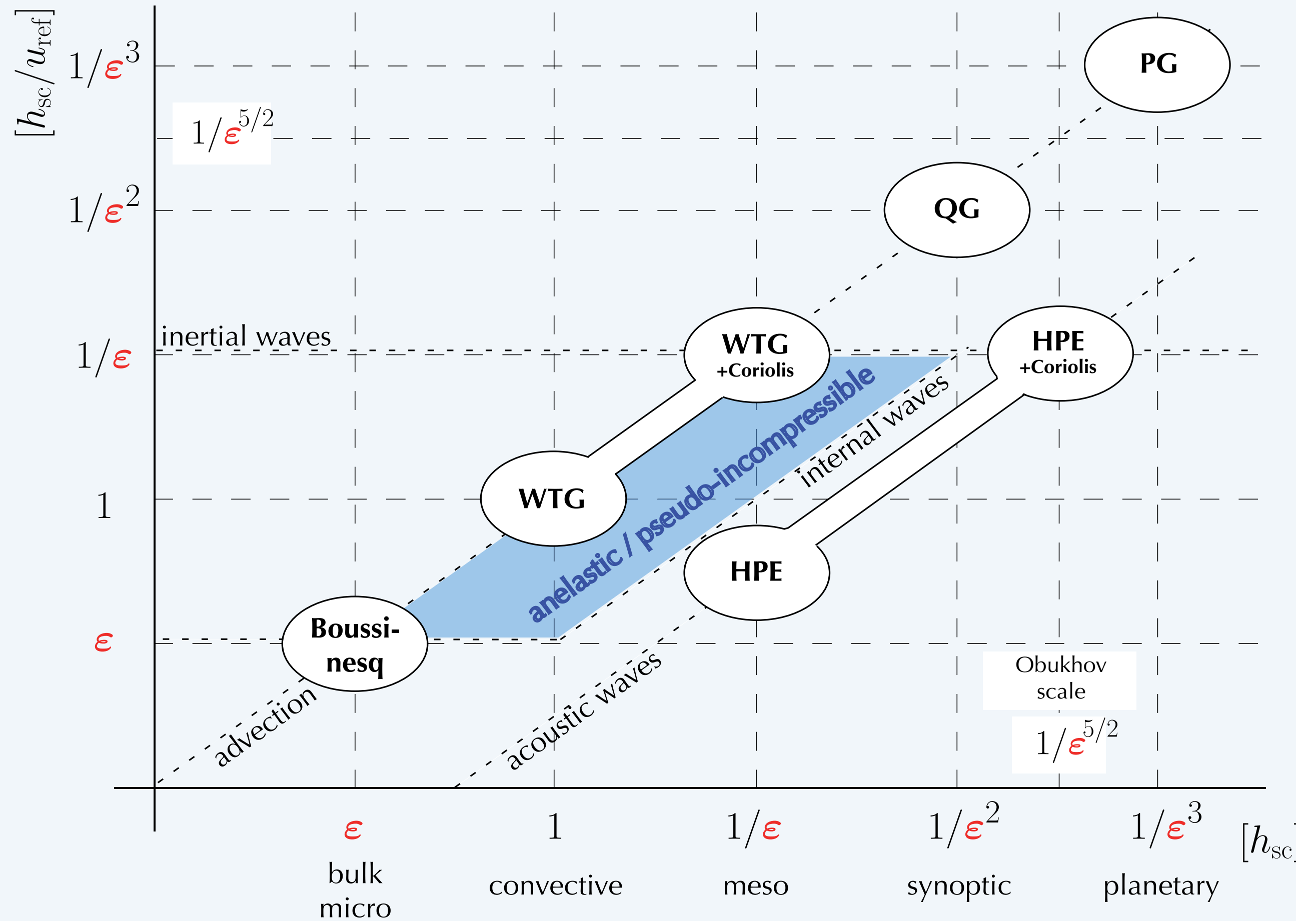
Klein, 2010

Atmospheric Flow Regimes



R.K., Ann. Rev. Fluid

Scaling regimes and model equations for atmospheric flows. The weak-temperature-gradient (WTG) and hydrostatic primitive equation (HPE) models cover a wide range of spatial scales assuming the associated advective and acoustic timescales, respectively. The anelastic and pseudoincompressible models for realistic flow regimes cover multiple spatiotemporal scales (Section 4.3). For similar graphs for near-equatorial flows, see Majda 2007b, Majda & Klein 2003. PG, planetary geostrophic; QG, quasi-geostrophic.

h_{sc}: density scale height; ε: *Froude number*

Klein, Ann. Rev. Fluid Mech. 2010

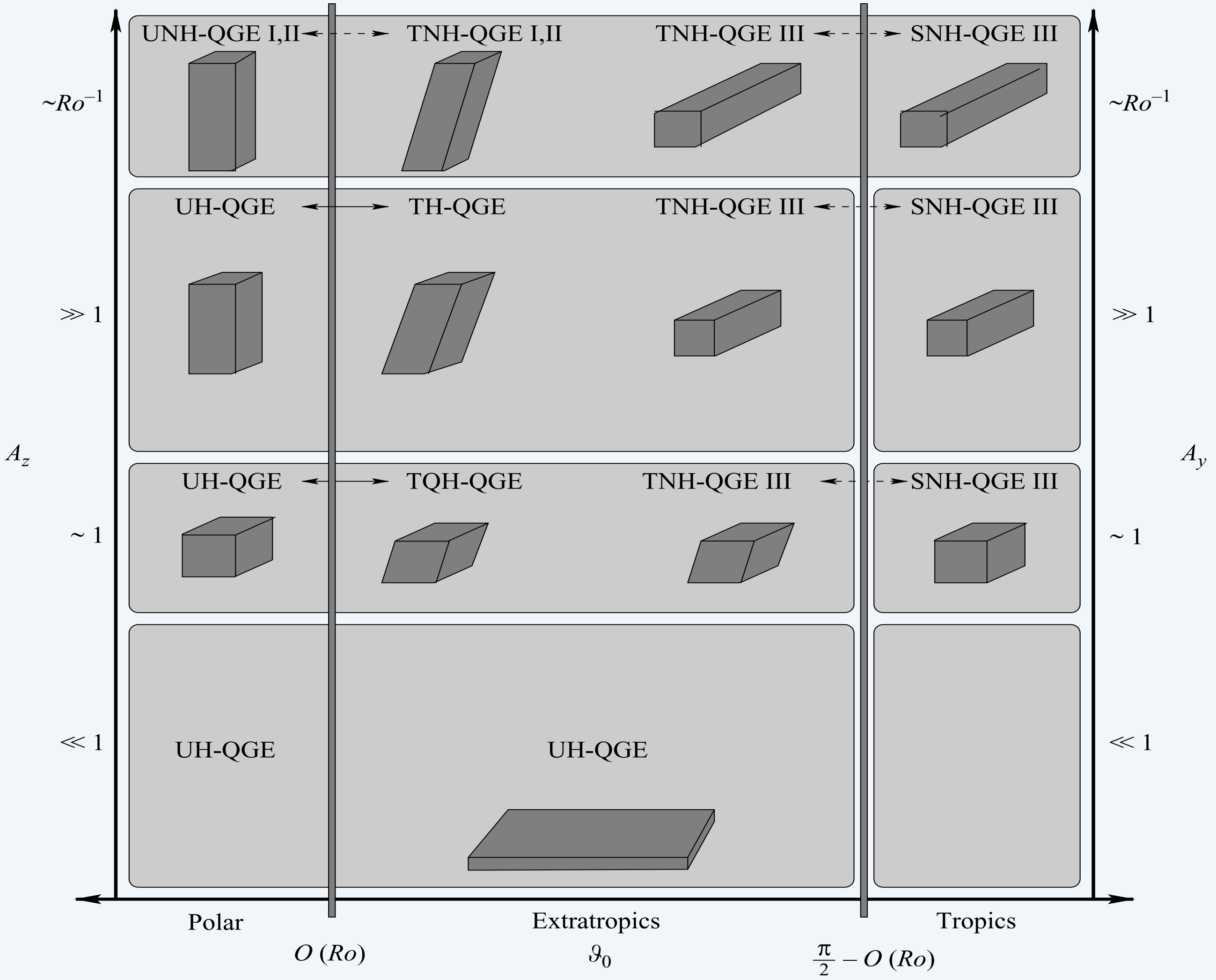


FIGURE 4. Classification of the reduced U–Upright, T–Tilted, S–Sideways QG models (see table 4) as a function of the colatitude ϑ_0, and the spatial aspect ratios A_z or A_y. H–hydrostatic, QH–quasi-hydrostatic, NH–non-hydrostatic. With the exception of TNH-QGE III A_z distinguishes between all models in the polar and extratropical regions where $A_y = O(1)$, while A_y distinguishes between the tropical QGE and TNH-QGE III for which $A_z = O(1)$. The symbol $\longleftrightarrow$ indicates a continuous transition between different models while ⇠⇢ indicates extension of a model to the polar or equatorial regions.

1) $\nabla \cdot \mathbf{u} = 0 \to k_\perp u_\perp \sim k_Z W \to \tau_{NL}^{\perp} \sim \tau_{NL}^{Z}$

2) $\mathcal{H} : u_\perp >> W \leftarrow \to k_Z >> k_\perp$

3a) Defs: $Fr \equiv \frac{u_\perp}{L_\perp N}$; $Re \equiv \frac{u_\perp L_\perp}{\nu}$

3b) $\mathcal{H} : F_Z \equiv \frac{u_\perp}{L_Z N} = 1$ *(Billant Chomaz 2001)*

$\to L_B = \frac{u_\perp}{N} = L_Z$ *(Buoyancy scale)*

and $\to \frac{L_Z}{L_\perp} = \frac{k_\perp}{k_Z} = \frac{W}{u_\perp} = Fr$

4) Saturation spectrum: $W \partial_Z W \sim N\theta$

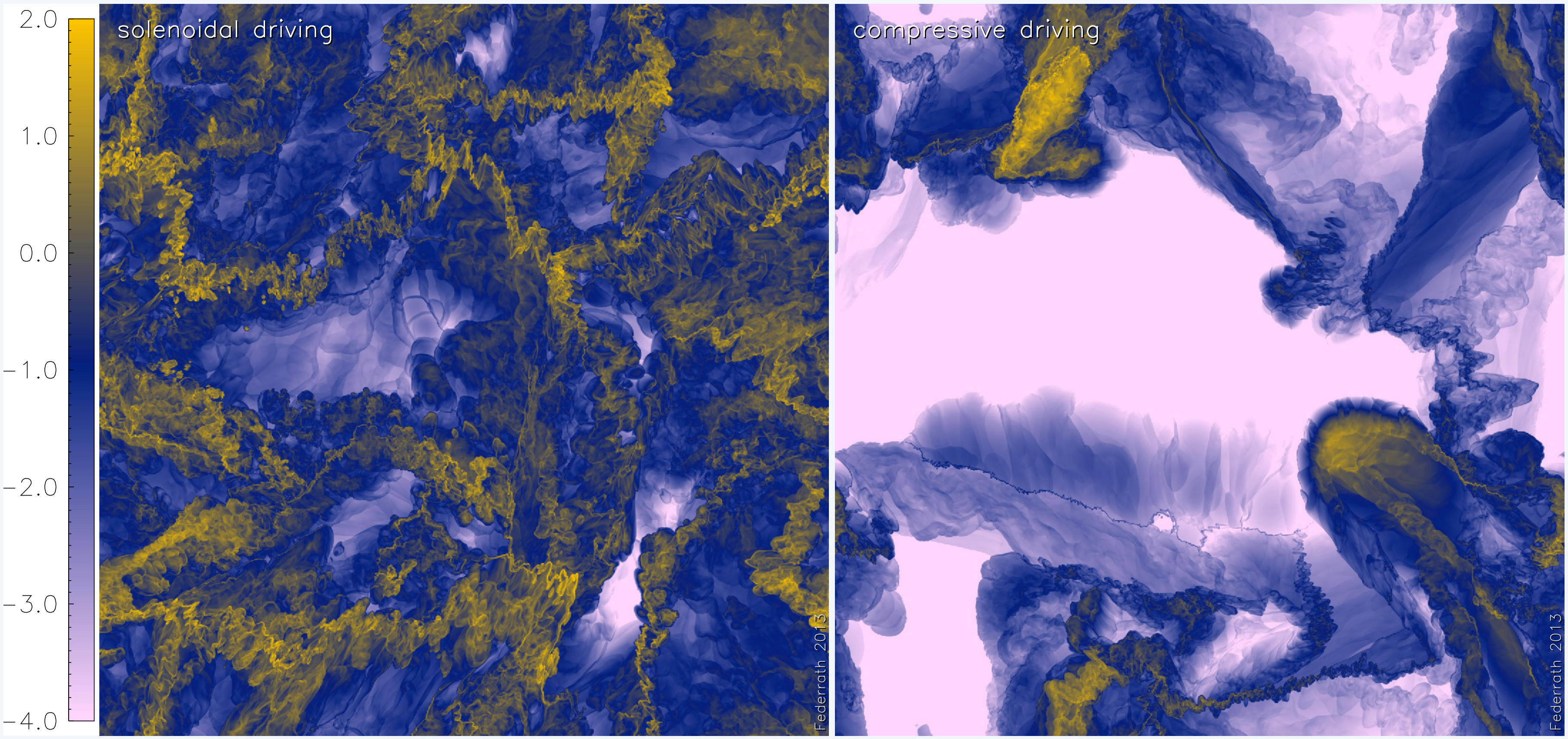
$\to E_W(k_Z) = E_P(k_Z) \sim N^2 k_Z^{-3}$ *(observed)*

5) $\epsilon \equiv \frac{DE}{DT} \sim \frac{u_\perp^2}{\tau_{tr}}$, $\tau_{tr} \in [\tau_W = 1/N,\ \tau_{NL},\ \tau_{NL}/Fr,\ \tau_{sw}, ...]$

Dimensionless parameters - Universality?

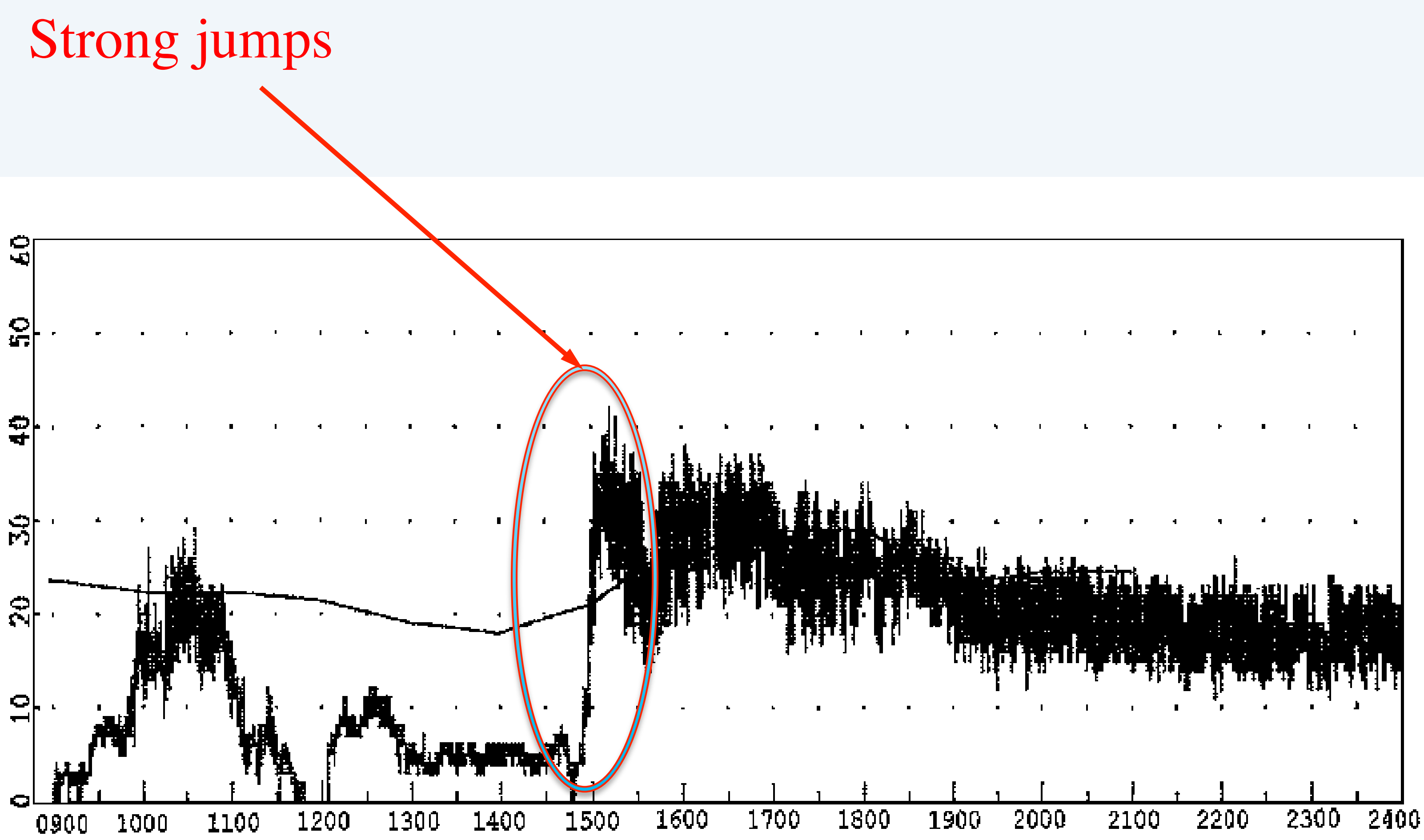
- Re, Ro, Fr and Pr (=1)
- Scale of initial field and/or of forcing
- Isotropy or not of initial conditions and/or forcing
- Presence or not of temperature fluctuations, and if so, balanced or not
- Role of:
  - inviscid invariants such as $P_{ot}V_{ort}$, linear or not
  - resolving characteristic scales for a given parameter set
  - local/nonlocal scale interactions
  - large-scale friction

Density slice, supersonic-Mach ~17, 4096^3 grid



Different (solenoidal/compressible) driving

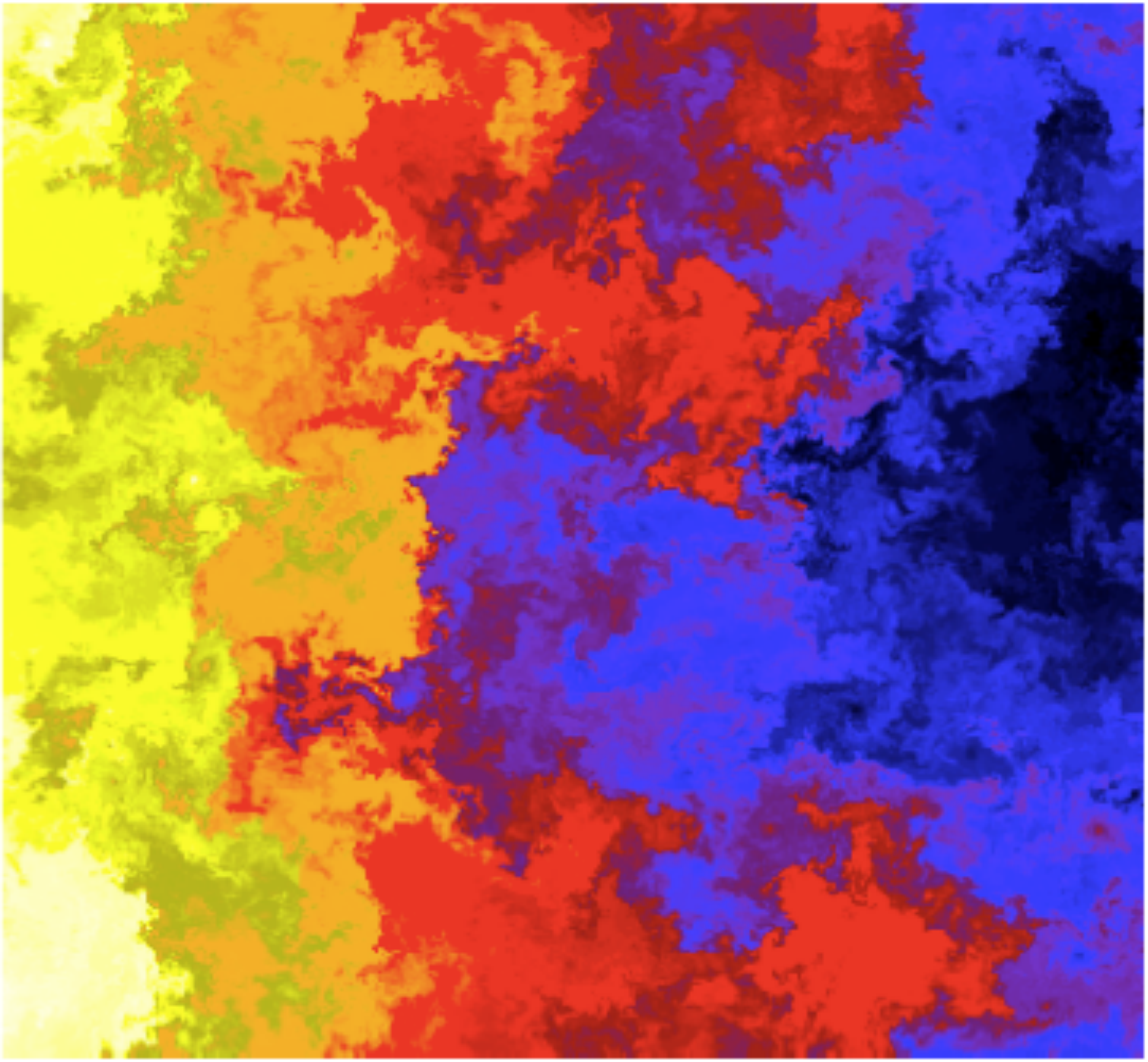
Federrath, 2013
Flash code



igure 6. Anemograph trace for Bellambi Point on 26 December 1996 (wind speed in knots), taken from Batt and Leslie (1998), Fig. 7.

Intermittency which manifests itself as long tails in PdF

Problem for e.g. wind farms



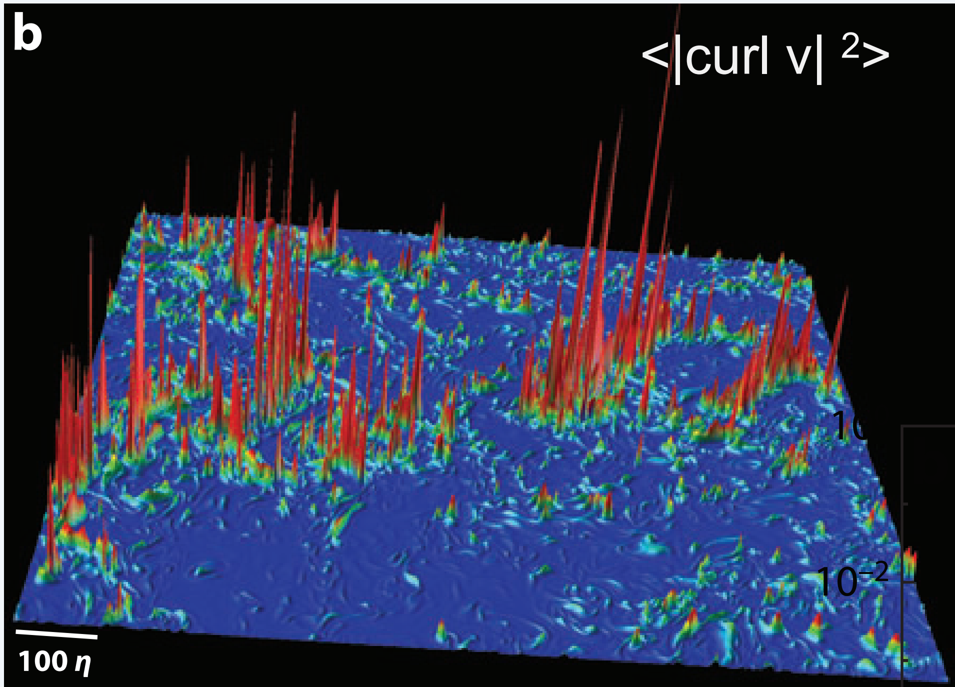
Celani & Vergassola, Phys. Rev. Lett. **86**, 424 (2001)

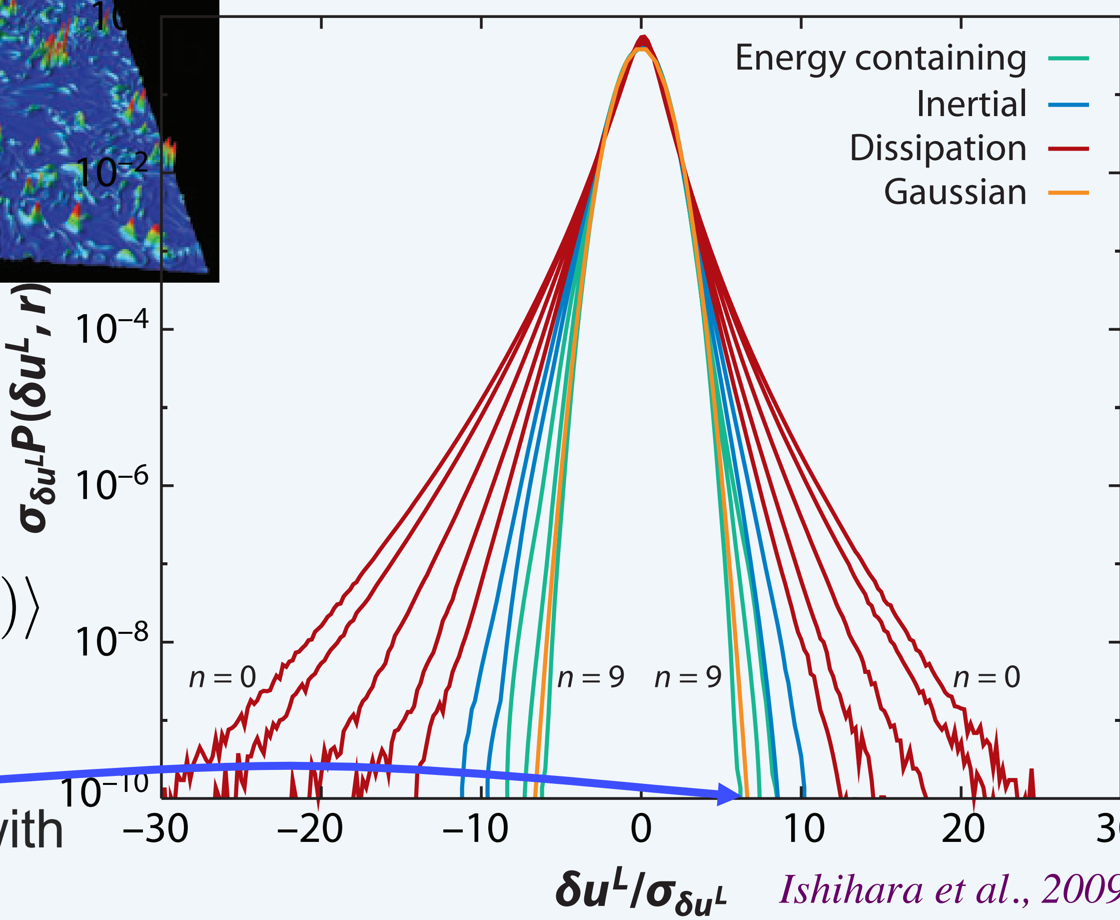
Two-dimensional passive scalar:

Sharp fronts

4096^2 points

Fluid turbulence at 4096^3 resolution, $R_\lambda \sim 1200$





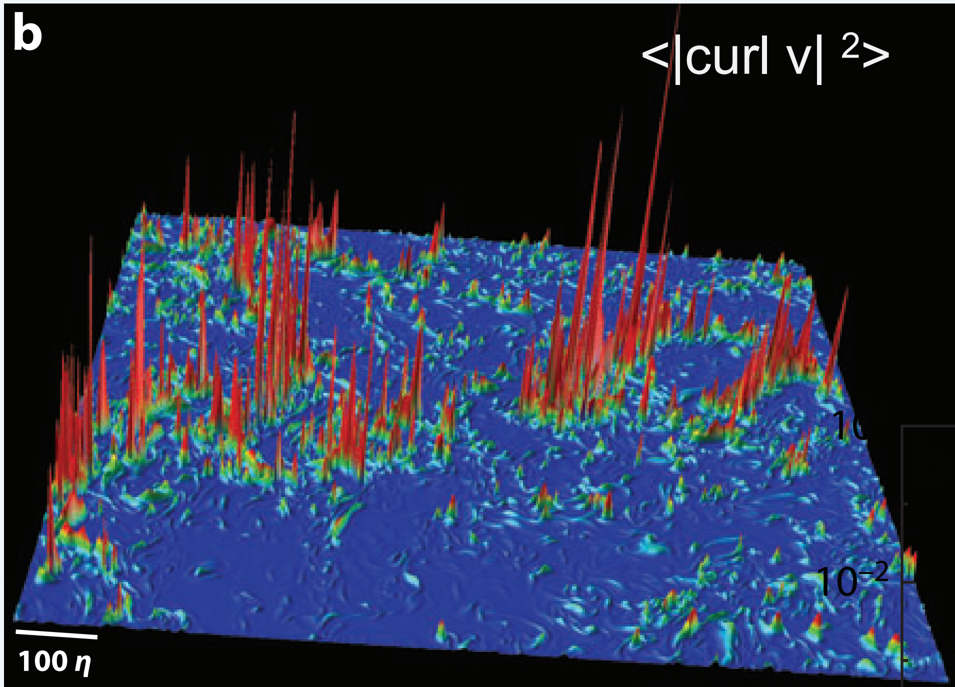
Ishihara et al., 2009

Velocity differences $\delta u(l)$ on distances $l \sim 2^n \Delta x$

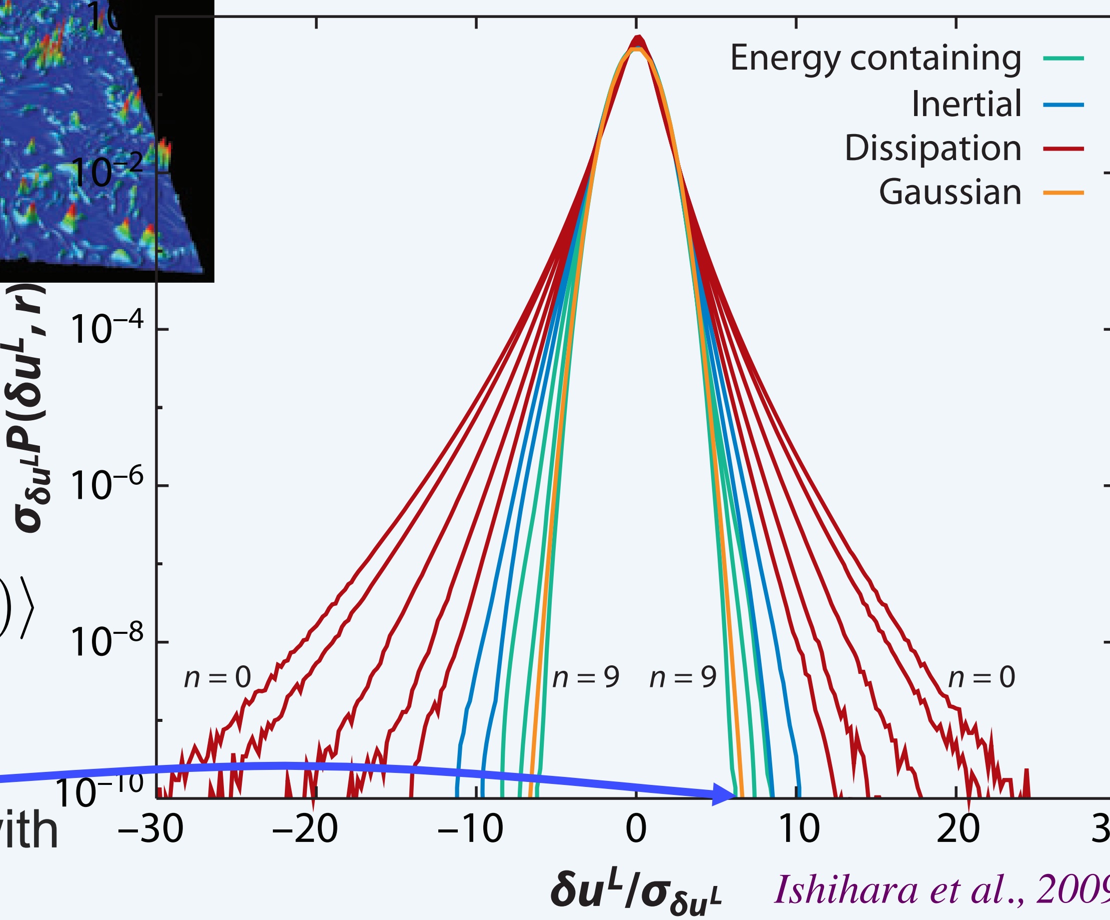
$$\delta u_x(\ell) = \langle u_x(x+\ell) - u_x(x) \rangle$$

Gaussian at large scale, and with heavy tails at small scales

Fluid turbulence at 4096^3 resolution, $R_\lambda \sim 1200$



Velocities are Gaussian
Gradients are intermittent



Ishihara et al., 2009

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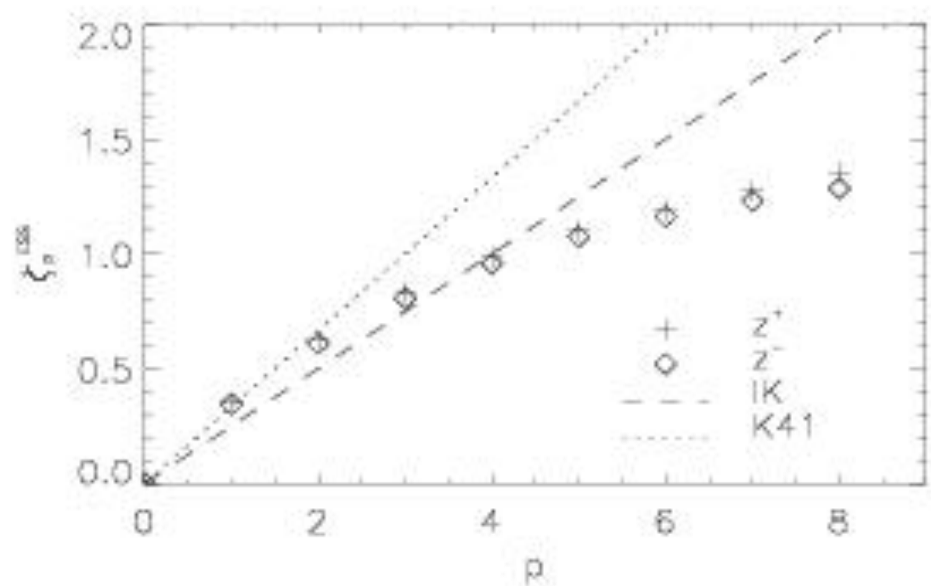
Gaussian at large scale, and with heavy tails at small scales

Isotropic MHD scaling at peak of dissipation

- Anomalous exponents of structure functions for Elsässer variables, with isotropy assumed *(similar results for v and B)*
- K41: $u(l) \sim l^{1/3}$
- IK: $b(l) \sim l^{1/4}$
- Curvature: intermittency



Extreme event

- Scaling exponents

Abramenko, review (2007)

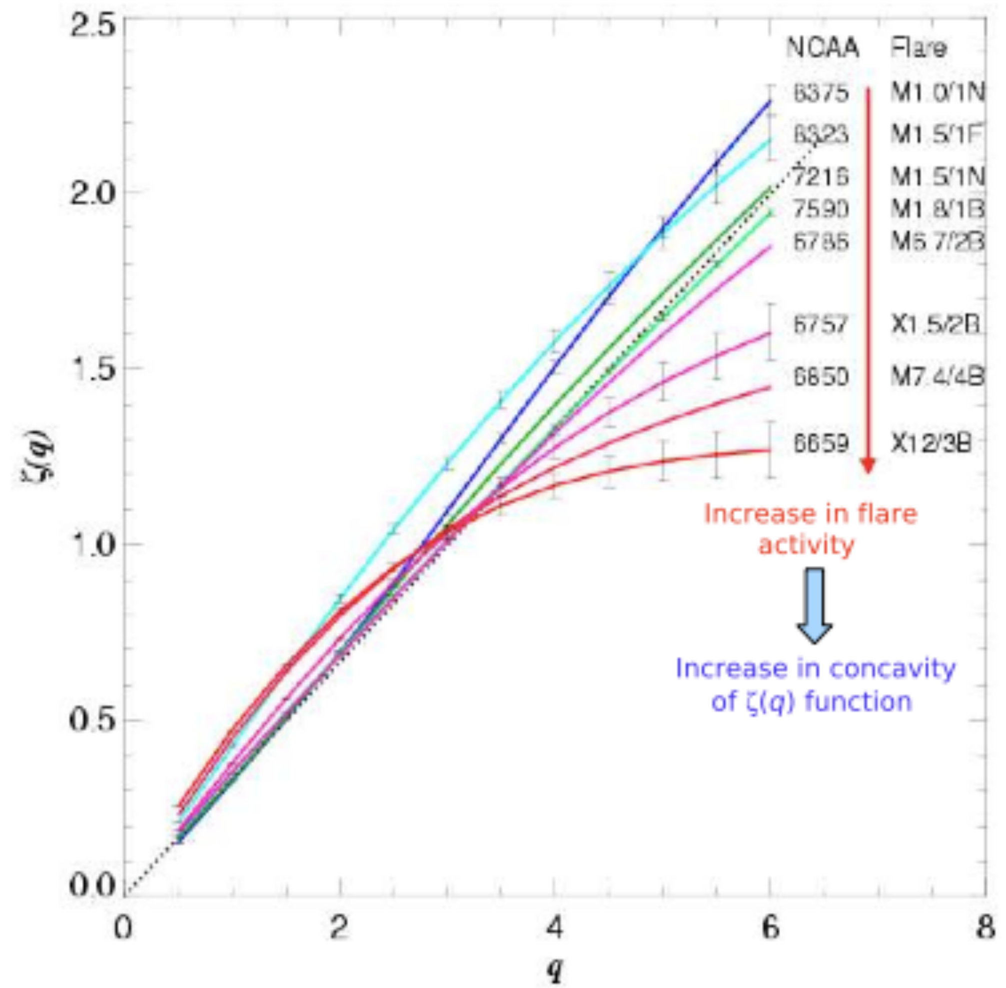


Figure 16: Scaling exponents $\zeta(q)$ of structure functions of order q calculated for eight active regions by Abramenko et al. (2002). The straight dotted line has a slope of $1/3$ and refers to the state of Kolmogorov turbulence. The NOAA number and the strongest flare (X-ray class/optical class) of each active region is shown. Increase of the flaring activity of active regions (from the top down to the bottom) is accompanied by general increase in concavity of $\zeta(q)$ functions.

How do **waves** alter the dynamics?

Stable Boussinesq stratification → gravity waves

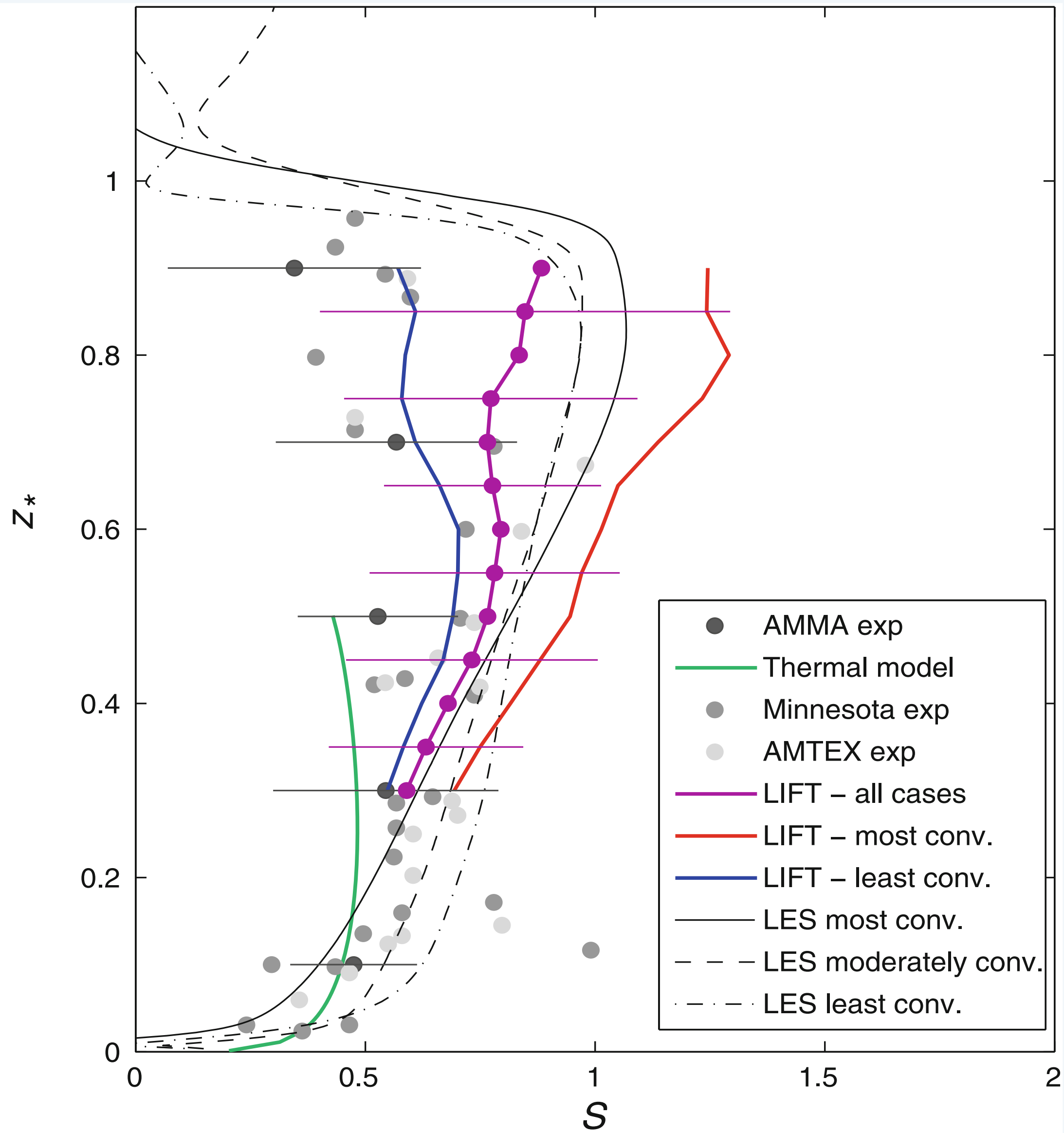
$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} = -\nabla P - Nbe_z \quad + \mathrm{F}$$
$$\partial_t b + \mathbf{u}\cdot\nabla b - \kappa\Delta b = Nw \ ,$$
$$\nabla\cdot\mathbf{u} = 0 \ .$$

$\frac{T_{dissipation}}{T_{nonlinear}}$ Re = U_0L_0/ν *Reynolds number*

$\frac{T_{wave}}{T_{nonlinear}}$ Fr = $U_0/[L_0N]$ *Froude number*

$\nu = \varkappa$ *Unit Prandtl nb.*

Skewness of vertical velocity in the convective planetary boundary layer



LIFT: LIdar in Flat Terrain

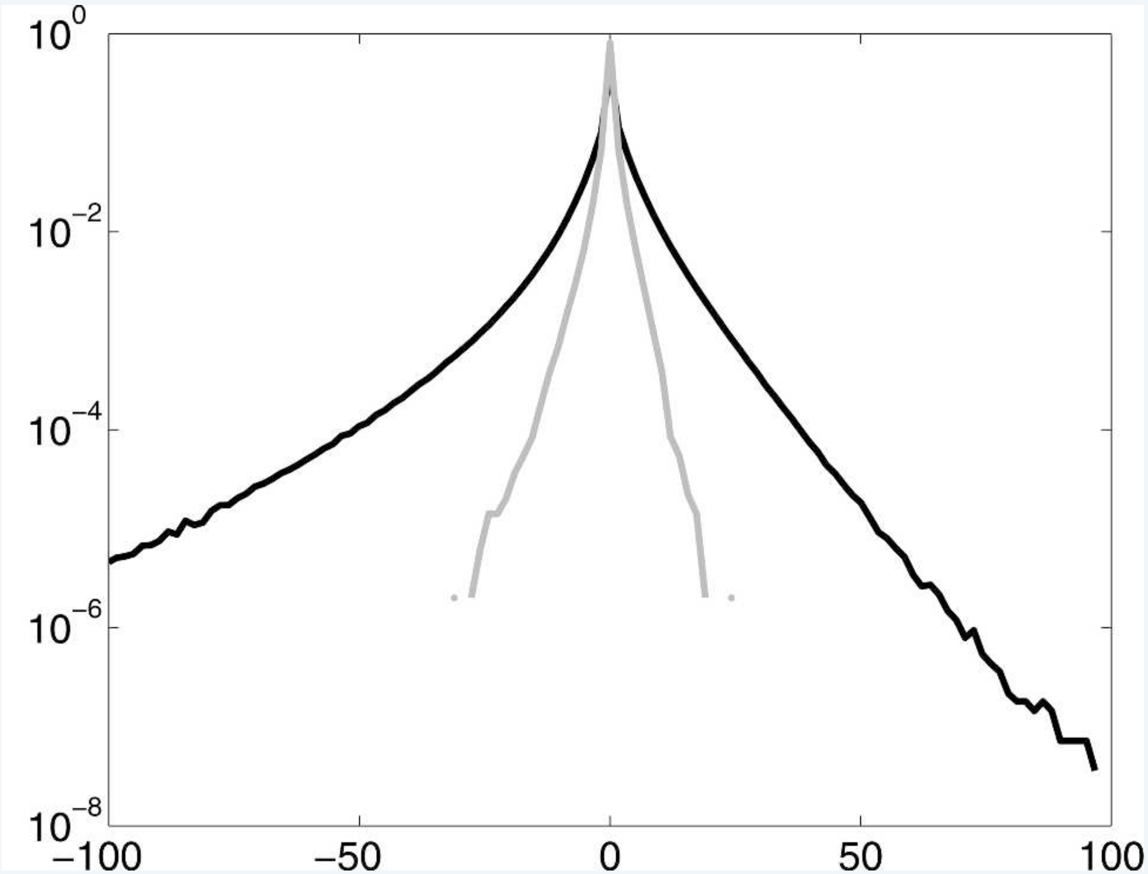
30m res.; 1s res. over 110 hrs

z_I : Conv. Bound. Layer depth

$z_* = z/z_I$

Lenschow et al., 2012

PdF of vertical velocity in an oceanic model

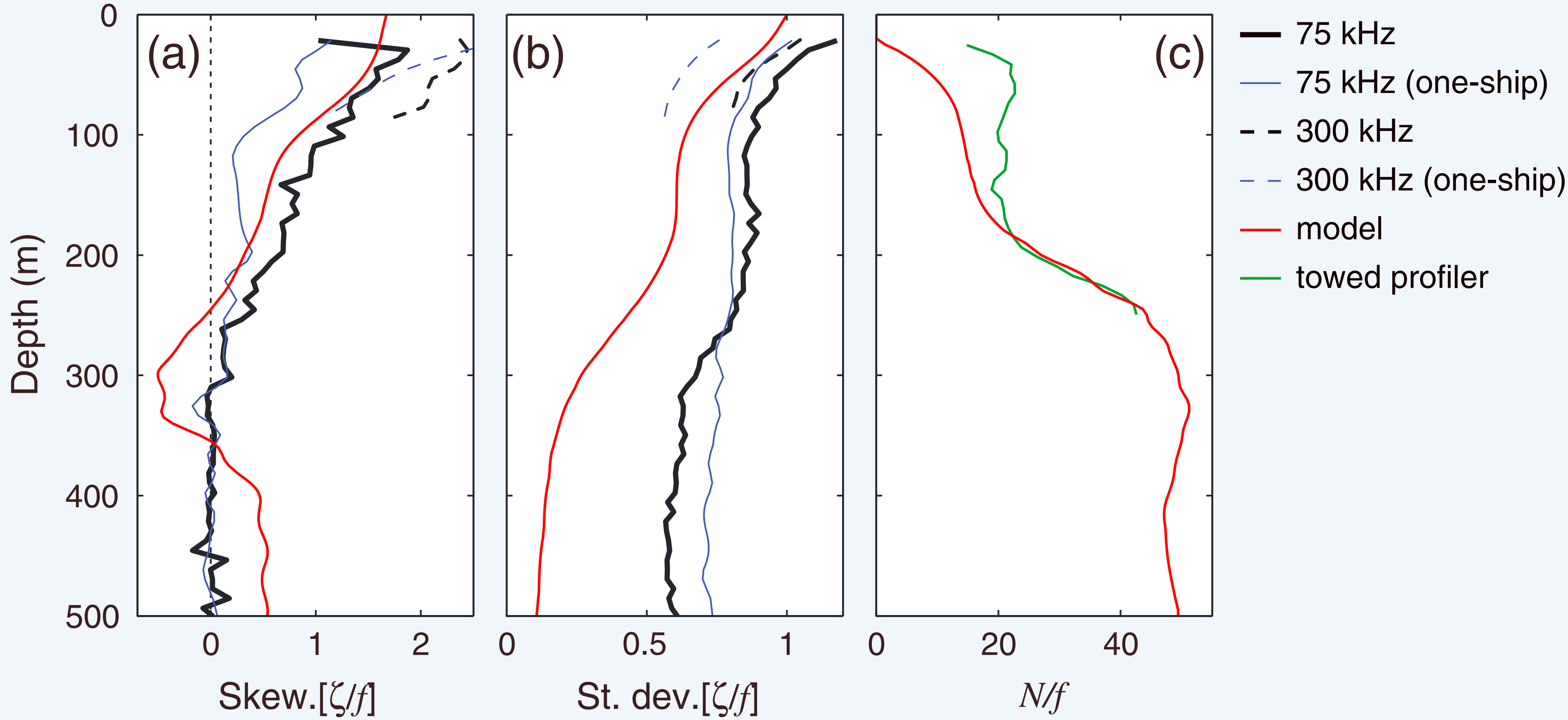


Two depths
ROMS with atmospheric forcing
~ 1000^2 X 40 res.
California current

Wings associated with frontogenesis

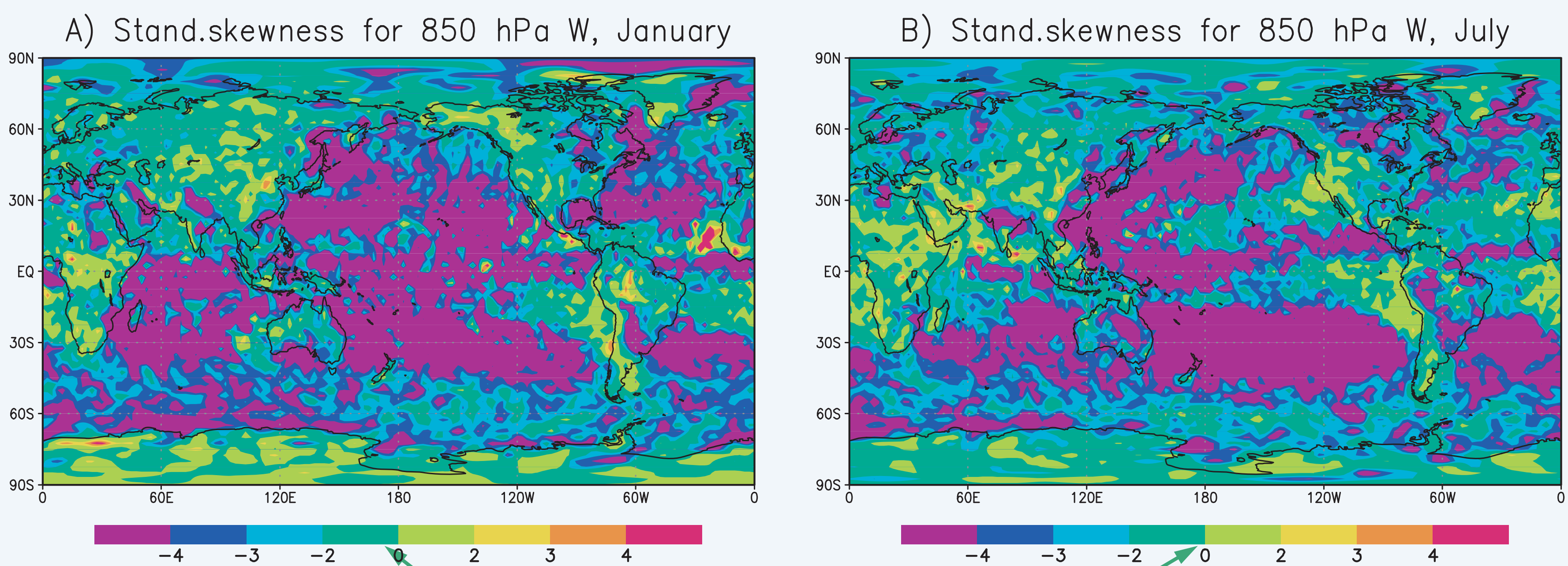
Capet et al., 2008a

Skewness of vorticity in upper ocean



Shcherbina et al. 2013

Intermittency in the free troposphere
ERA40: ECMWF 40km res., 1945- 1976-2002, daily data
Skewness of vertical velocity

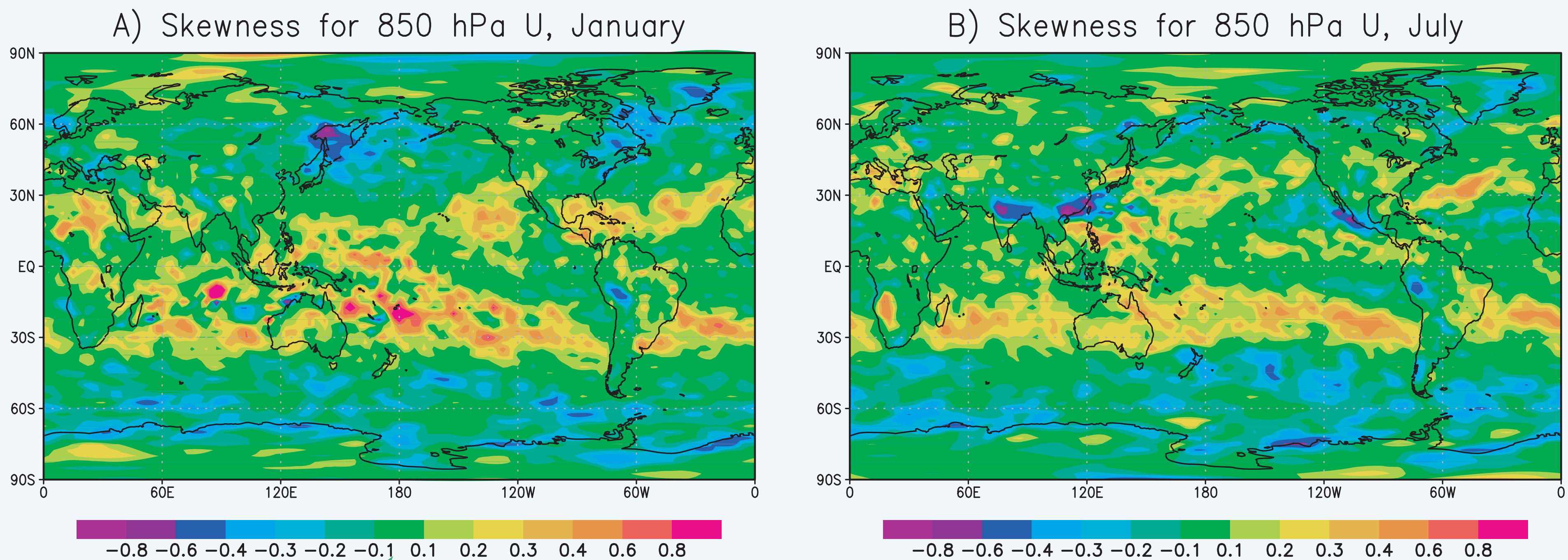


Gaussian

Petoukhov et al., 2008

Skewness of horizontal velocity

Associated with cyclones

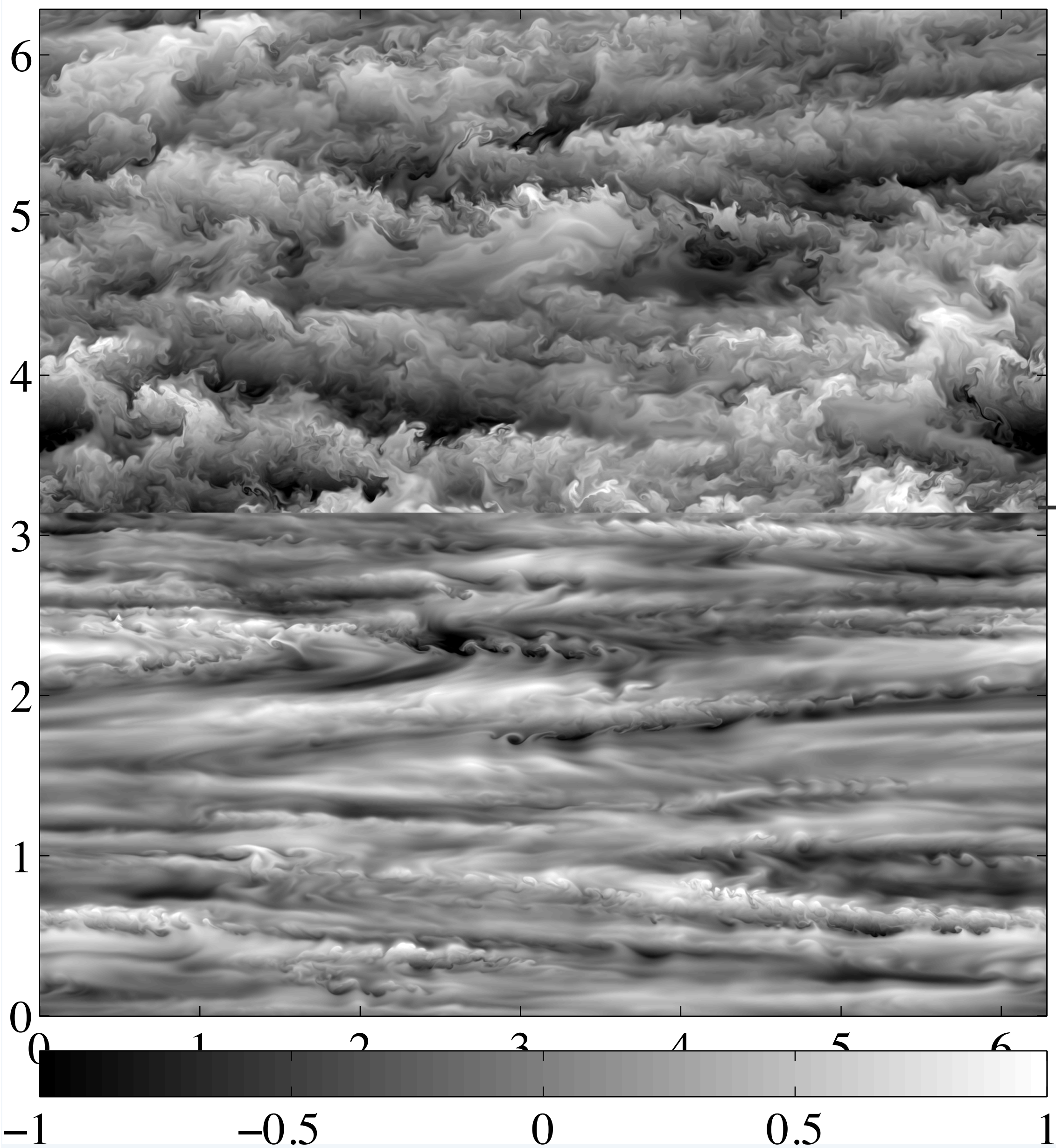


Gaussian

Petoukhov et al., 2008

Stratification, no rotation: Temperature fluctuations, xz slice,
Re ~ 24000, 2048^3 grids, K_F ~2-3

$R_B = ReFr^2$: buoyancy

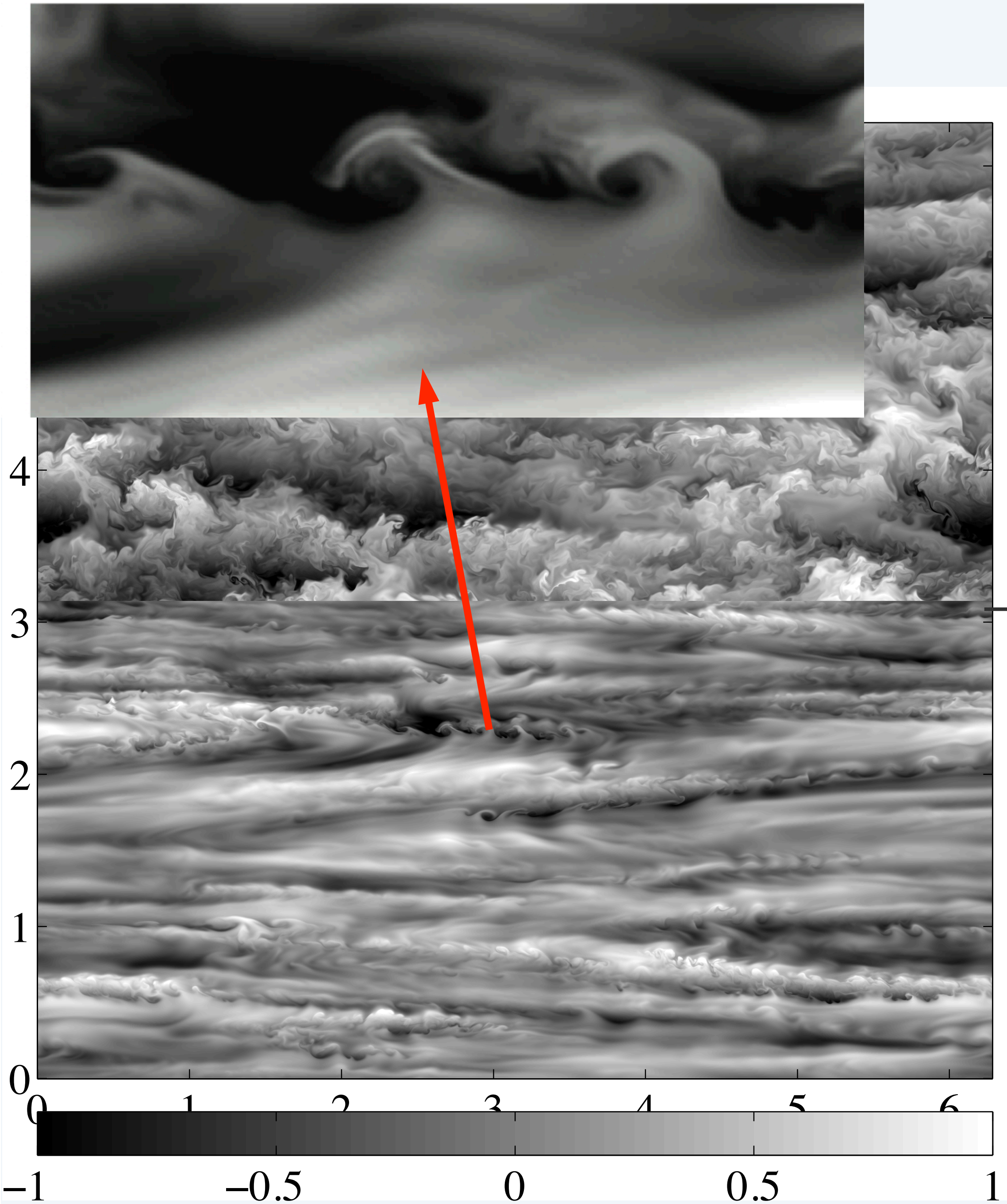


Fr ~ 0.11 *(N=4)*
R_B ~ 300

z

Fr ~ 0.03 *(N=12)*
R_B ~ 22

Rorai et al., 2014

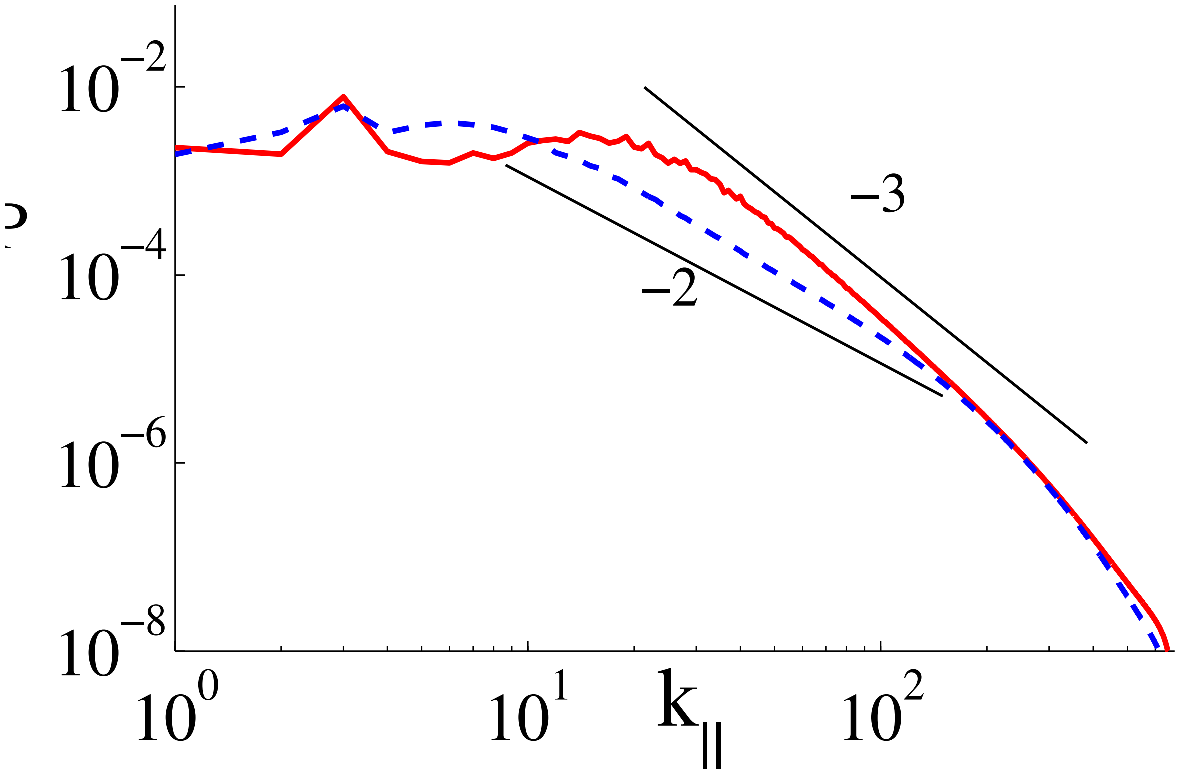


Pure stratification

$Fr \sim 0.11, R_B \sim 300$

$Fr \sim 0.03, R_B \sim 22$

Rorai et al., 2014, 2015



Re~2.4x10^4

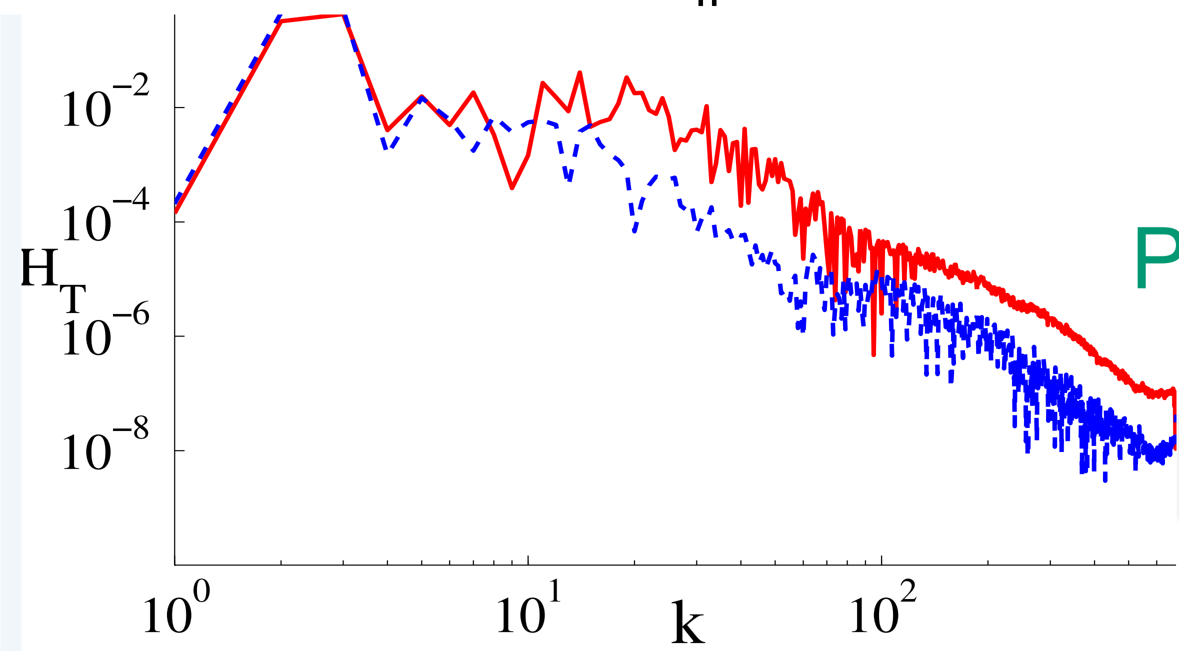
N=4 *(Fr~0.1)*

N=12 *(Fr~0.03)*

Potential energy

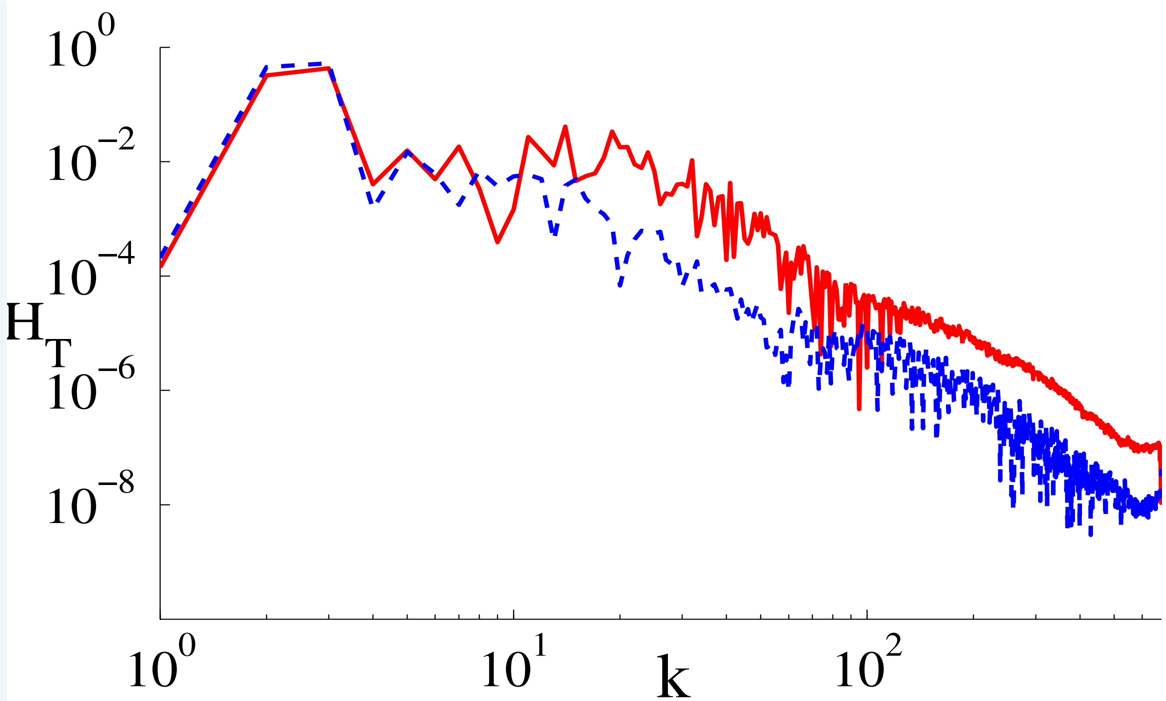
&

Helicity



Plateau until k_B~N/U

The buoyancy scale



Re~2x10^4

N=4 *(Fr~0.1)*

N=12 *(Fr~0.03)*

Helicity

Energy spectrum break at L_B

Break also in oceanic data,

D'Asaro & Lien 2000

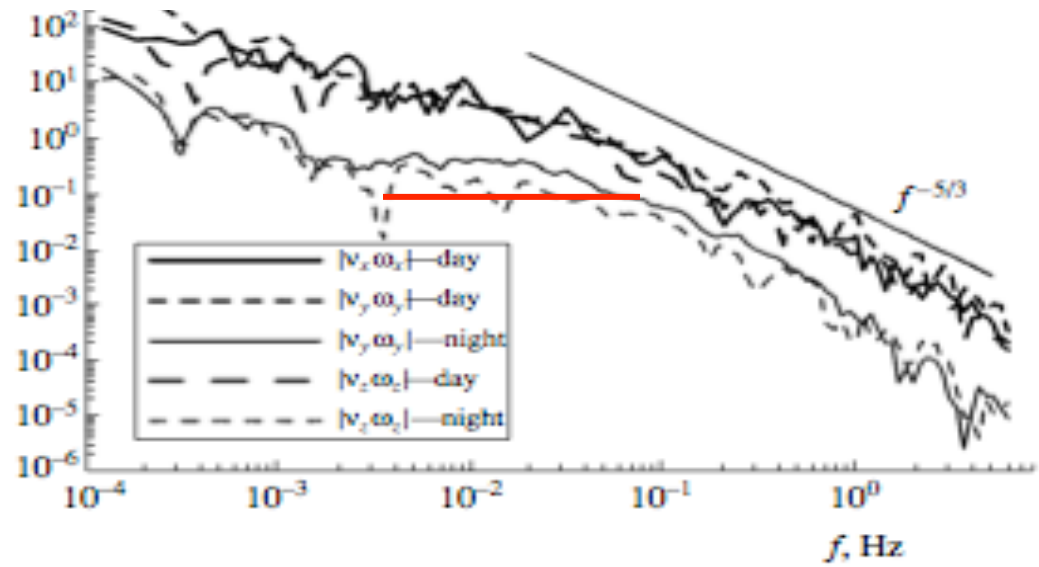
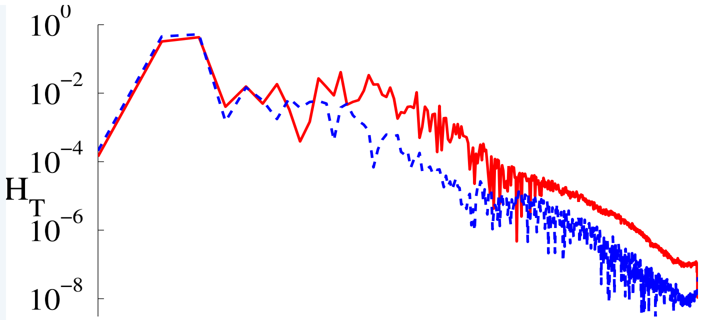
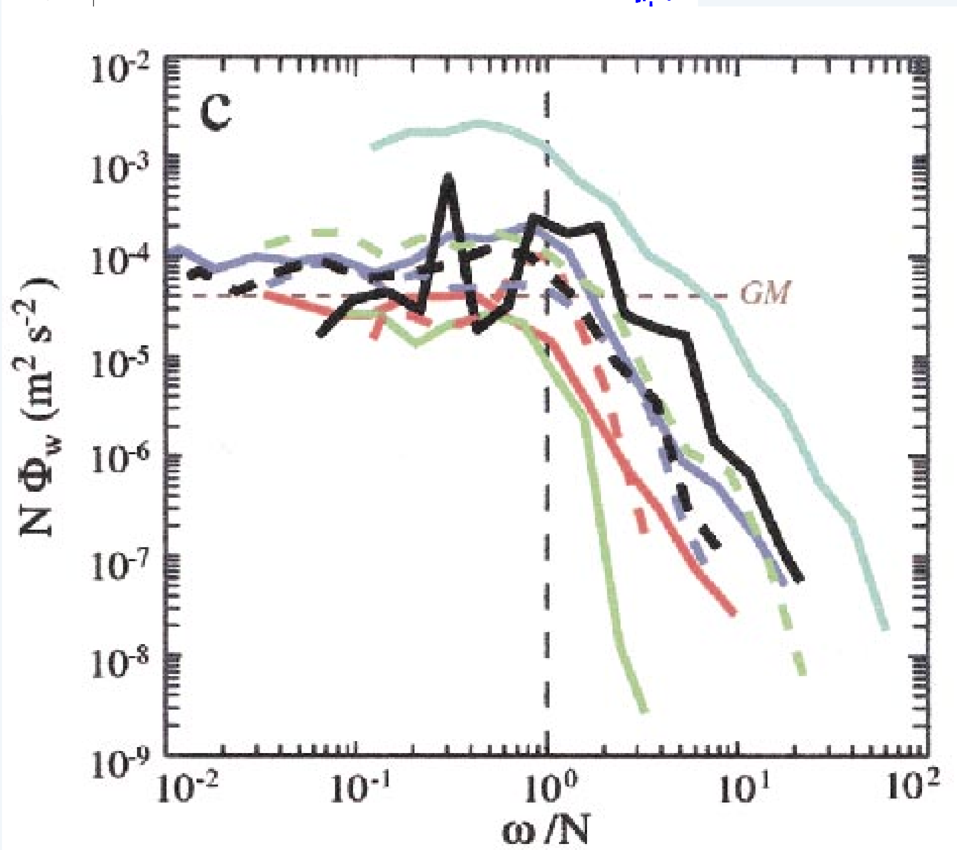


Fig. 4. Spectra of helicity components.

Koprov 2005





Re~2x10^4

N=4 *(Fr~0.1)*

N=12 *(Fr~0.03)*

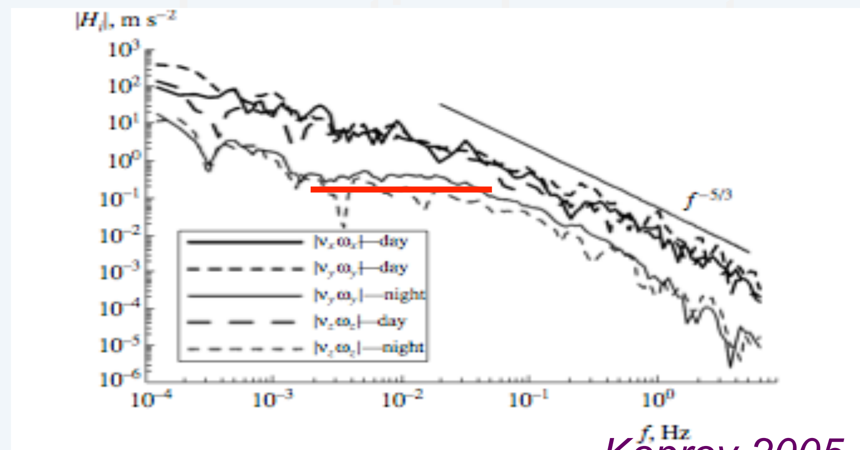


Fig. 4. Spectra of helicity components.

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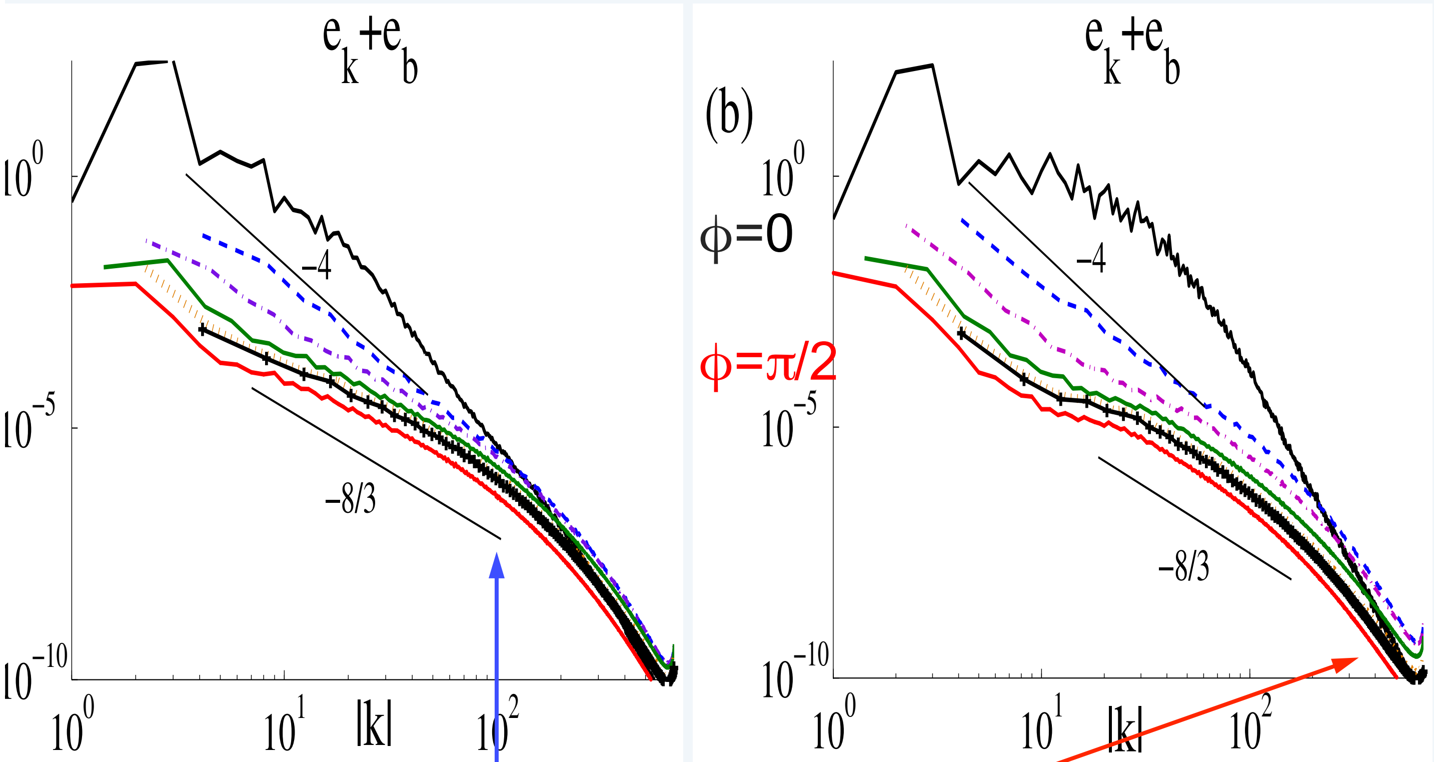
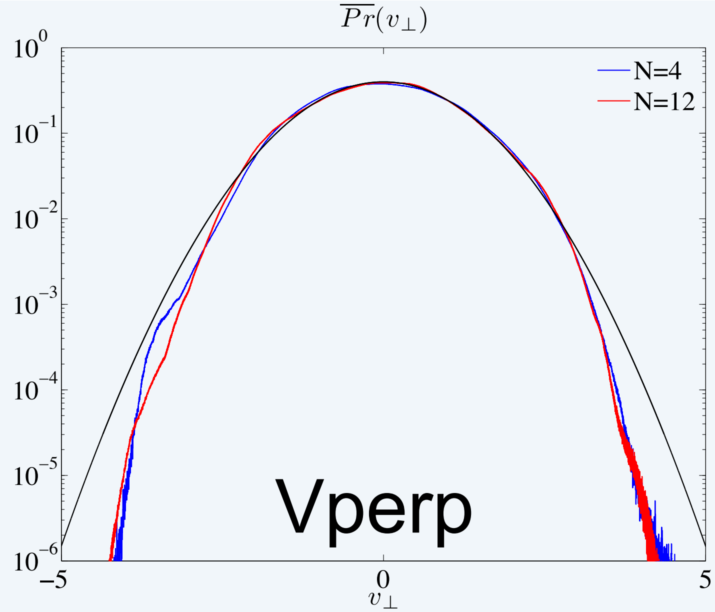
N=4 *(Fr~0.1, R_B~300)* *and* N=12 *(Fr~0.03, R_B~22)*



2D energy spectra for co-latitudes φ

Isotropy at $k_{Oz} \sim [N^3/\varepsilon]^{1/2}$: -5/3 beyond?

The Ozmidov scale



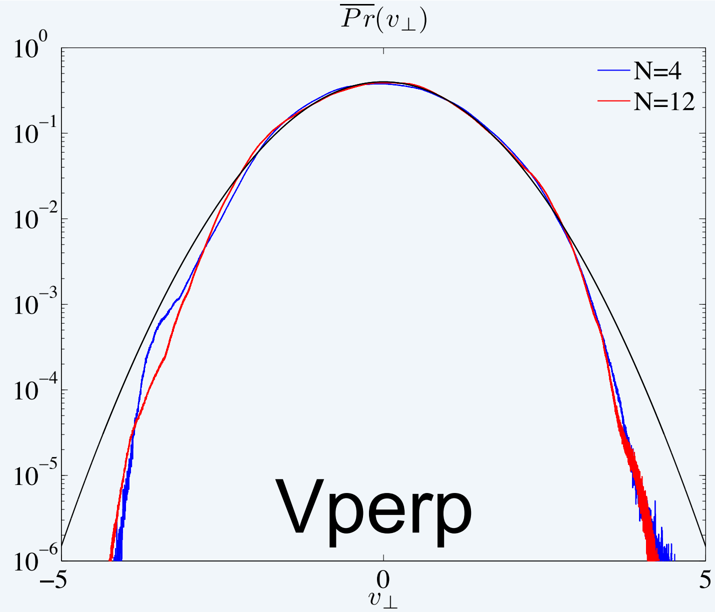
N=4

Gaussian in black

N=12

DNS 2048^3, Re=24000

Time Averaged PDFs after the peak of dissipation

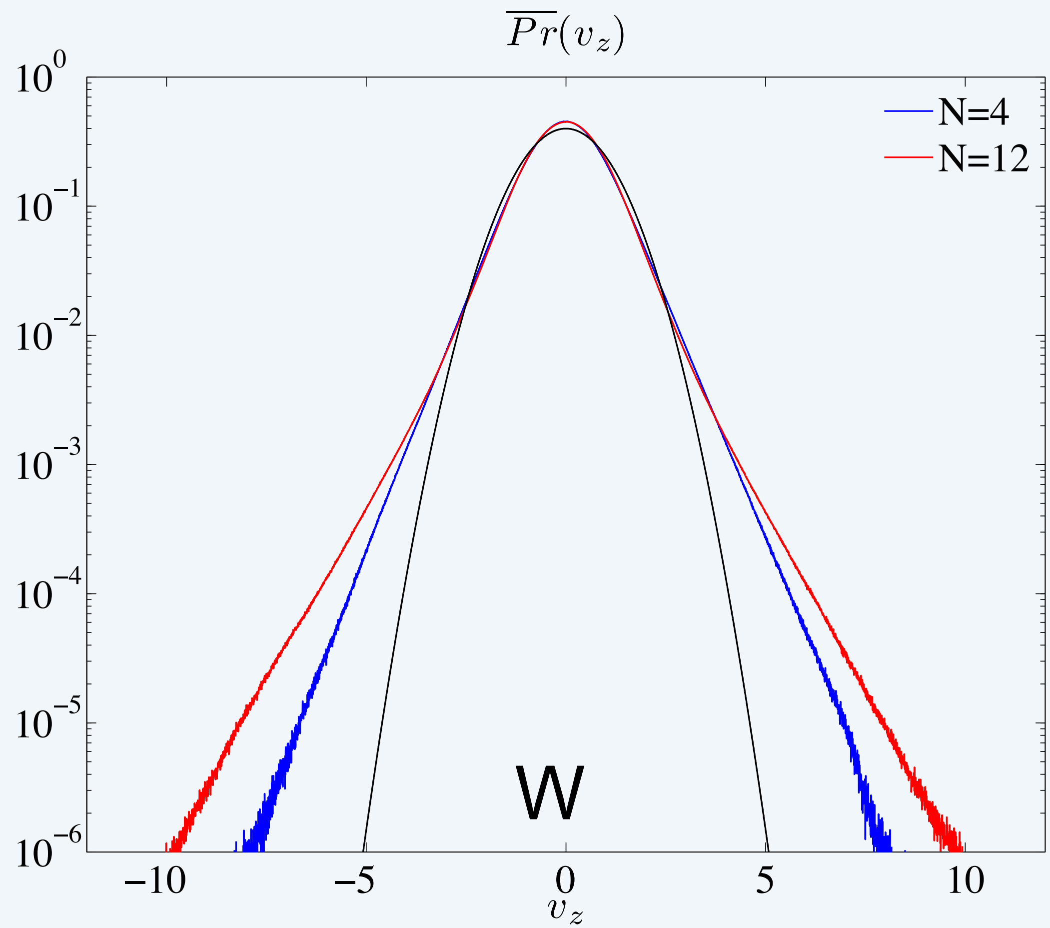


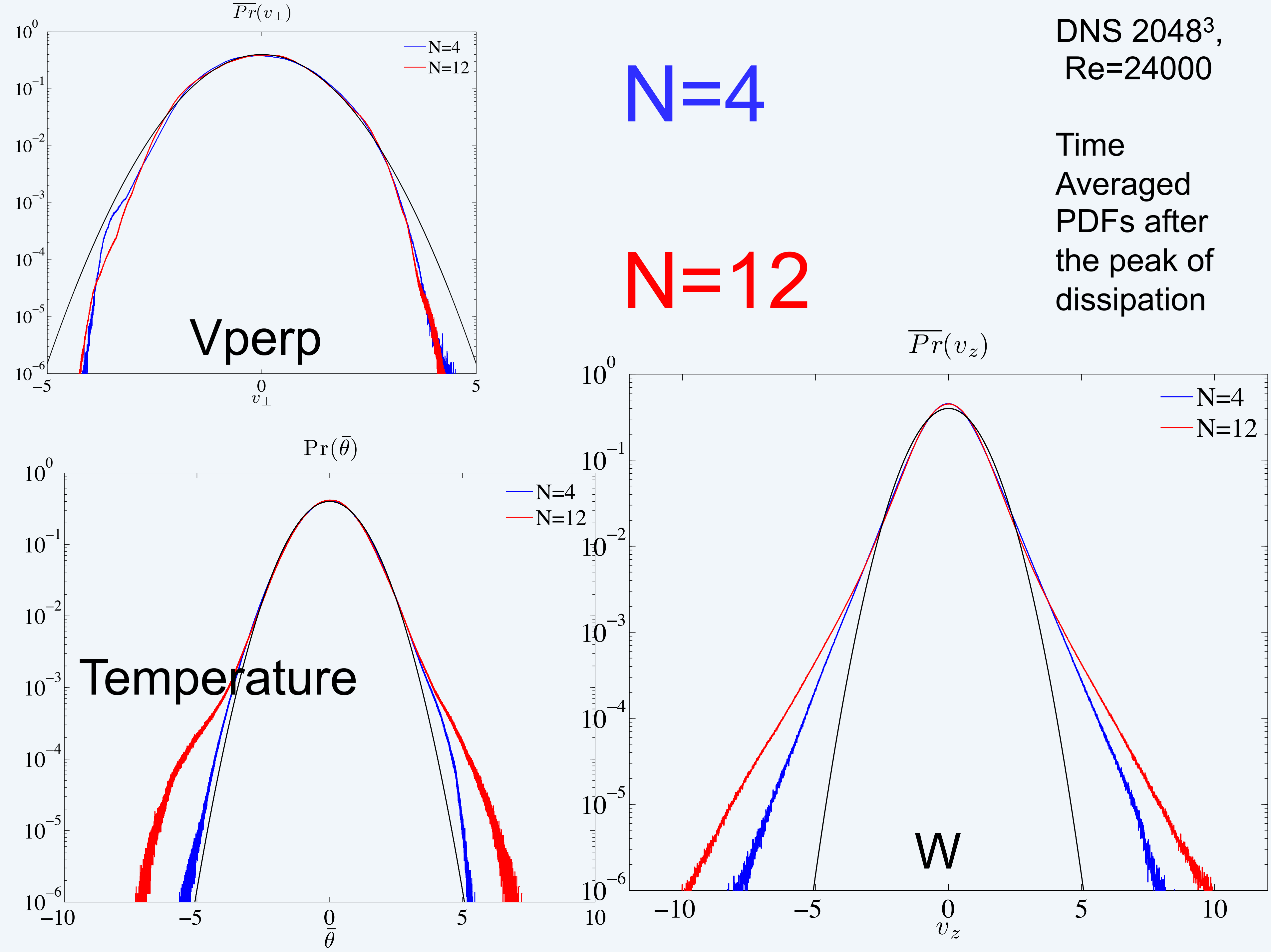
N=4

N=12

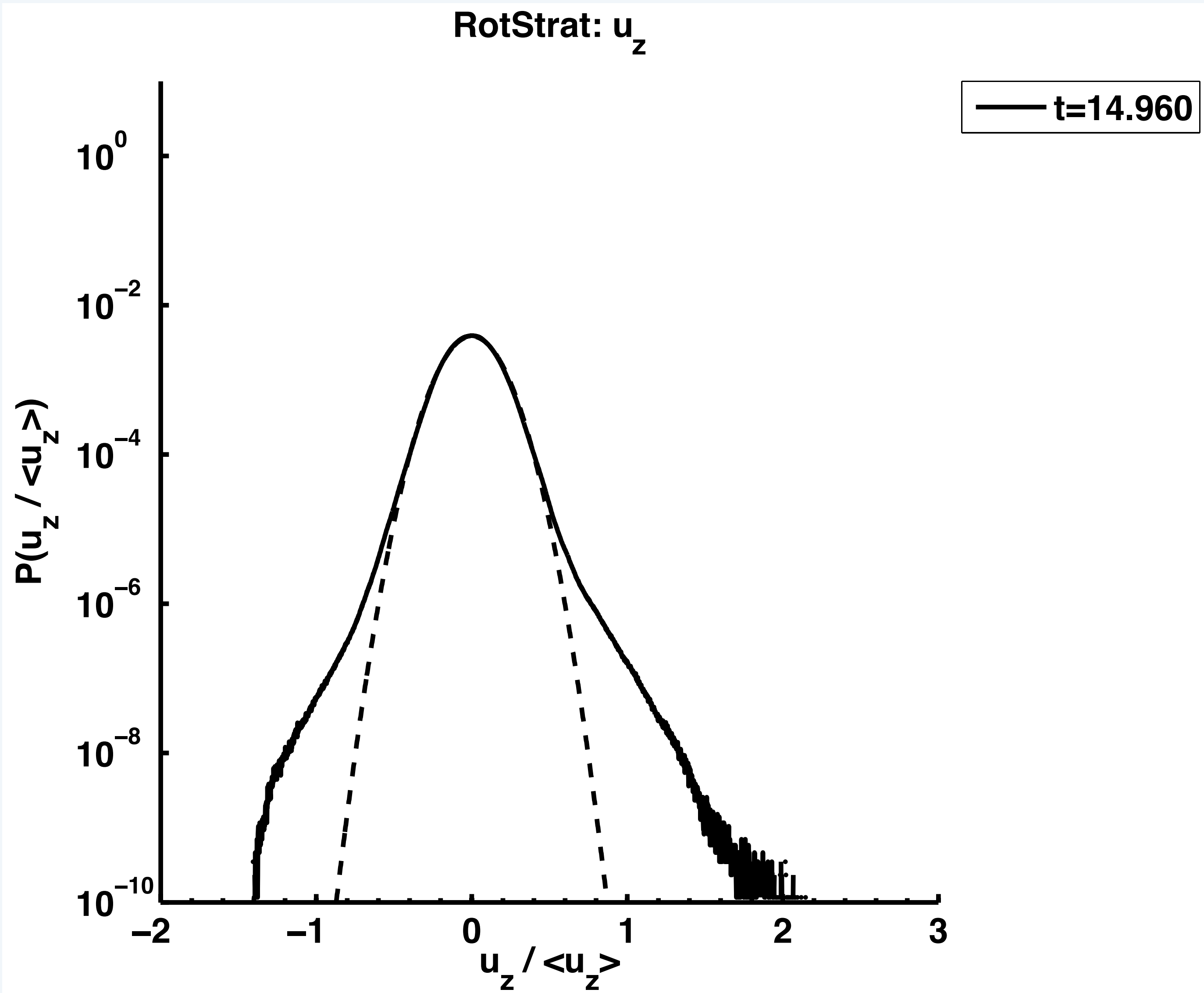
DNS 2048^3, Re=24000

Time Averaged PDFs after the peak of dissipation





DNS 2048^3,
Re=24000
Time Averaged PDFs after the peak of dissipation
N=4
N=12
$\overline{Pr}(v_\perp)$
Vperp
$v_\perp$
$\overline{Pr}(v_z)$
W
v_z
$\Pr(\bar{\theta})$
Temperature
$\bar{\theta}$
N=4
N=12



Marino et al., 2017

Leorat, PhD; Vieillefosse 1982, 1984

$$\frac{d\delta w}{dt} = -\frac{\delta w^2}{\ell}$$

Stratified turbulence model (N is the Brunt-Vaissala frequency):
vertical differences of fluctuations of vertical velocity w
and temperature θ over a vertical distance $l = l_z$

$$\frac{d\delta w}{dt} = -\frac{\delta w^2}{\ell} - N\delta\theta,$$

$$\frac{d\delta\theta}{dt} = -\frac{\delta w\delta\theta}{\ell} + N\delta w.$$

→ 3 regimes

* N small: hydrodynamic of intermittent strong turbulence

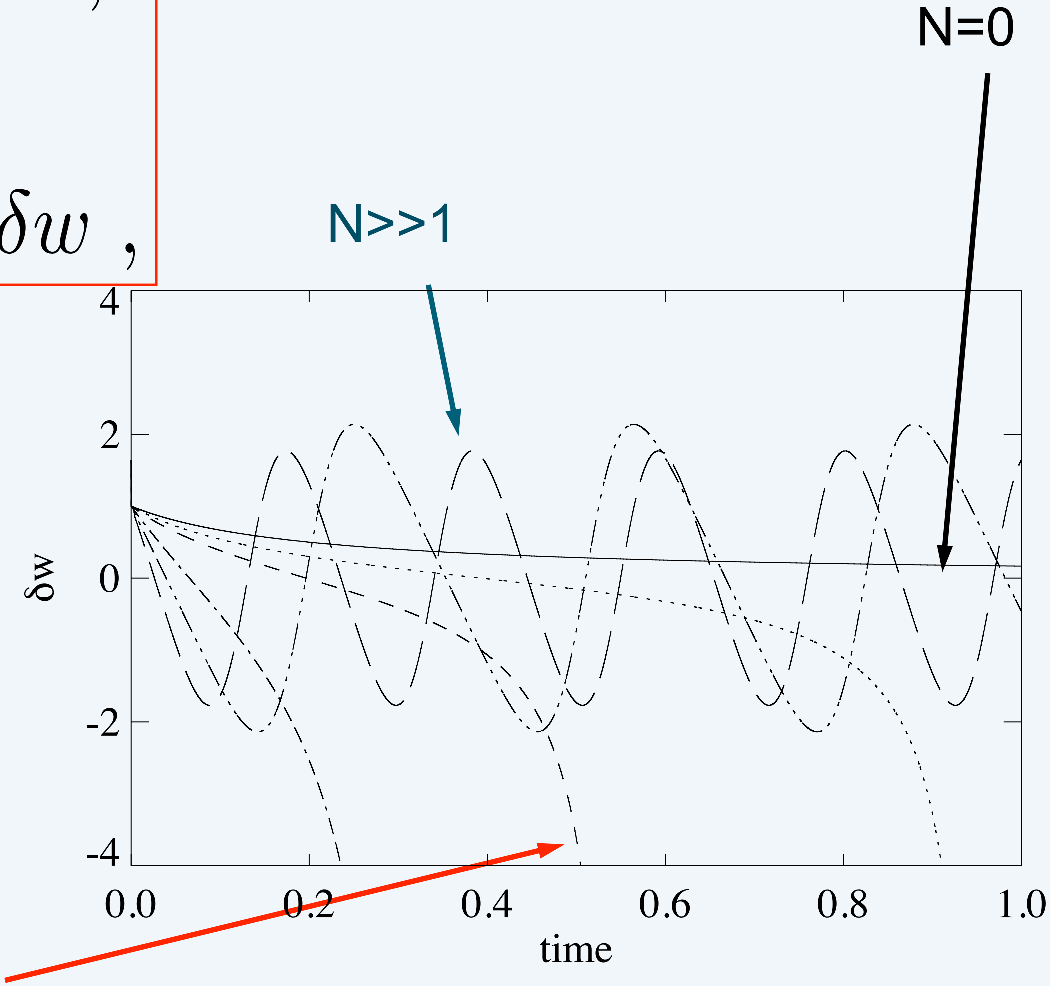
* N large: harmonic oscillator of frequency N

- $N\theta\, l_z \sim w^2$, balance compatible with "*saturated*" spectrum
$E_w(k_z) \sim E_P(k_z) \sim N^2 k_z^{-3}$

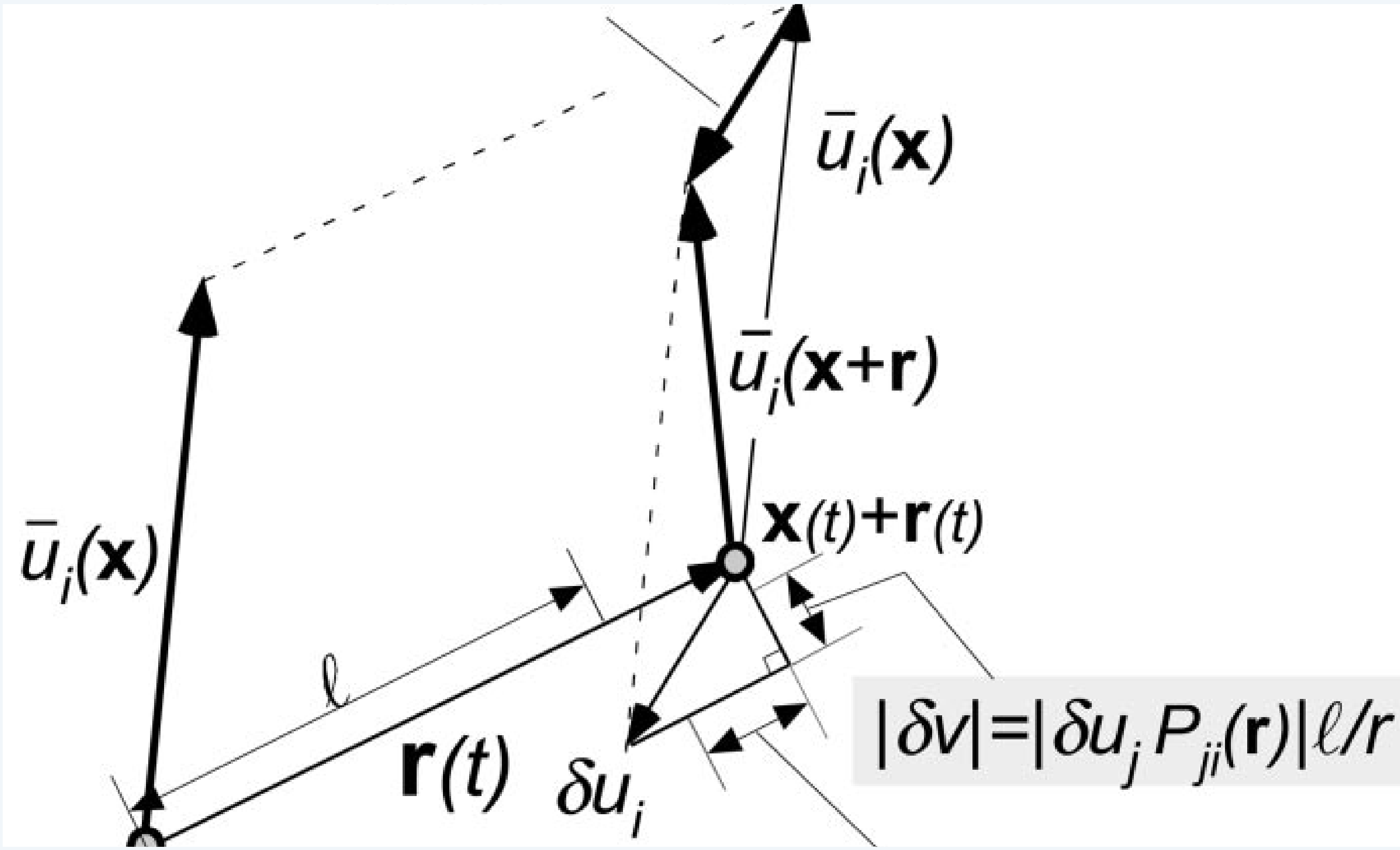
The model for vertical differences of fluctuations of vertical velocity w, over distance *l*, N is the BV frequency

$$d_t \delta w = -\delta w^2/\ell - N\delta\theta \quad ,$$

$$d_t \delta\theta = -\delta w \delta\theta/\ell + N\delta w \ ,$$



Intermediate N: faster growth
of negative gradients
in the *saturated* regime



Meneveau, Ann. Rev. Fluid Mech. 2011

- *Gradient matrix:* $A_{ij} = \partial u_i / \partial x_j$
- *Decomposition:* $A_{ij} = S_{ij} + \boldsymbol{\Omega}_{ij}$,

where $S_{ij} = (A_{ij} + A_{ji})/2$

$\boldsymbol{\Omega}_{ij} = (A_{ij} - A_{ji})/2$

Define:

$Q_2 = -[A_{im} A_{mi}]/2$, $R_3 = -[A_{im} A_{mn} A_{ni}]/3$,

- $Q_S = -[S_{im} S_{mi}]/2$,
- $R_S = -[S_{im} S_{mn} S_{ni}]/3$,
- $V^2 = S_{in} S_{im} \omega_m \omega_n$

Meneveau, Ann. Rev. Fluid Mech. 2011

$$d_t A_{ij} = -A_{ik} A_{kj} + 1/3[A_{mk} A_{km} \delta_{ij}] + H_{ij}^{\,p} + H_{ij}^{\nu}$$

with $H_{ij}^{\,p} = -(\partial^2 p/[\partial x_i \partial x_j] - 1/3 \nabla^2 p \delta_{ij})$

$$H_{ij}^{\nu} = \nu \partial^2 A_{ij} / [\partial x_k \partial x_k]$$

- Model pressure Hessian *(Chevillard et al. 2011, Meneveau 2011 …)*
- Isotropic? Local?

- Add transverse velocities
- Add rotation *(Li 2010)*, passive scalar, …

Other models of intermittency for the nocturnal planetary boundary layer: some degree of nonlinearity over an otherwise linear system:

- ^ *parametric instability*
- ^ *on-off intermittency*
- ^ *sub-critical transitions*

BUT:

Vertical Froude nber of order unity *(Billant Chomaz 2001)*
→ intrinsic nonlinear role in the vertical

Conclusions

1) Helicity is created in rotating stratified flows:

^ What are the emerging helical structures?

^ How much helicity when the flow is more turbulent?

2) Internal gravity waves can enhance in substantial ways the negative gradients of a turbulent flow leading to strong intermittency in strongly stable flows, as is well known from observations, but not necessarily well modeled

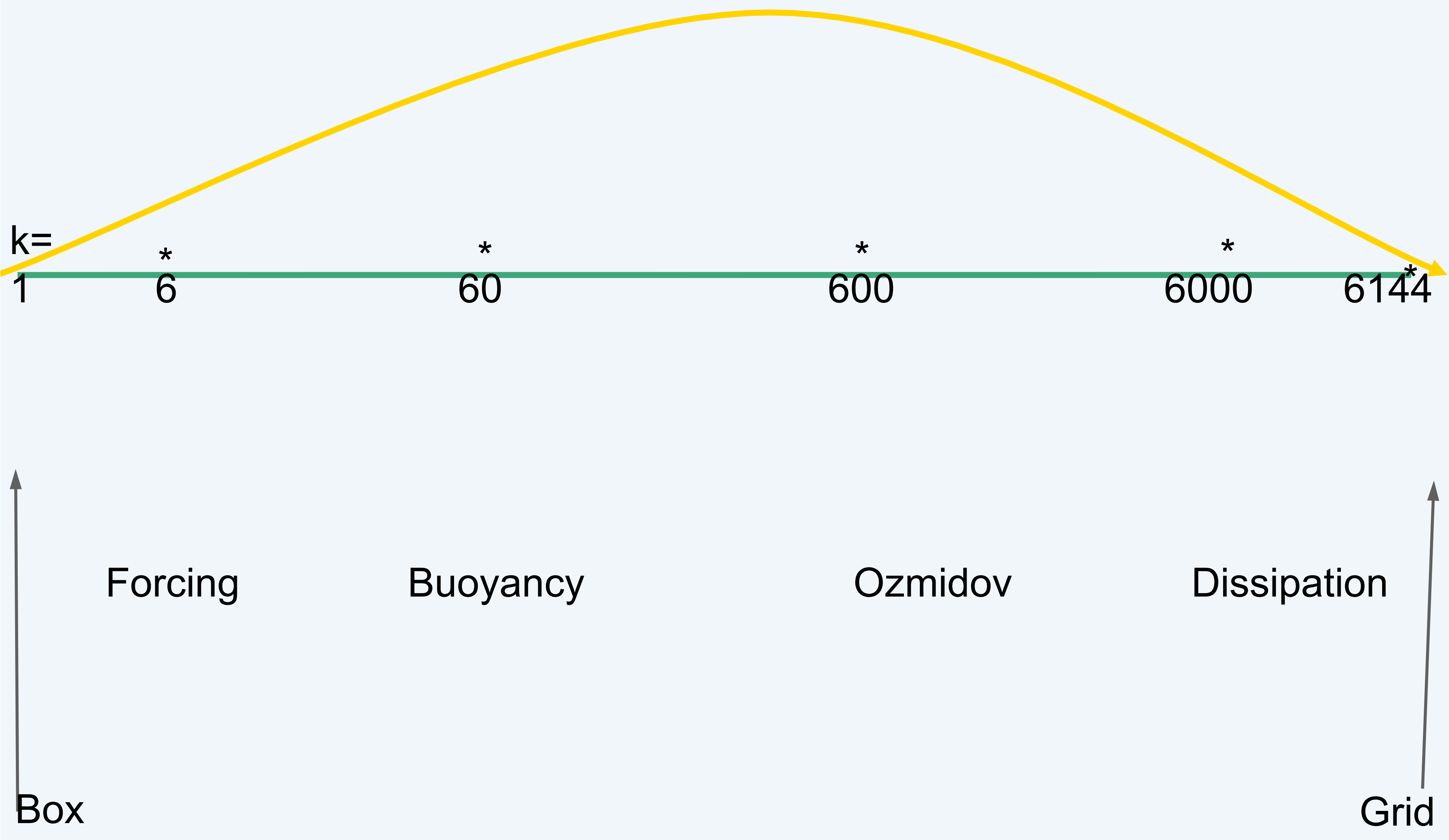
- *Link with structures (Kelvin-Helmoltz billows and secondary instabilities, fronts, …) and with mixing?*
- *Lifetime and spatial extent of transients?*
- *Expand the model*
- *Role of forcing scale?*
- *Role of parameters (Re, Fr, R_B)?*
- *Role of rotation (and of inverse cascade)?*

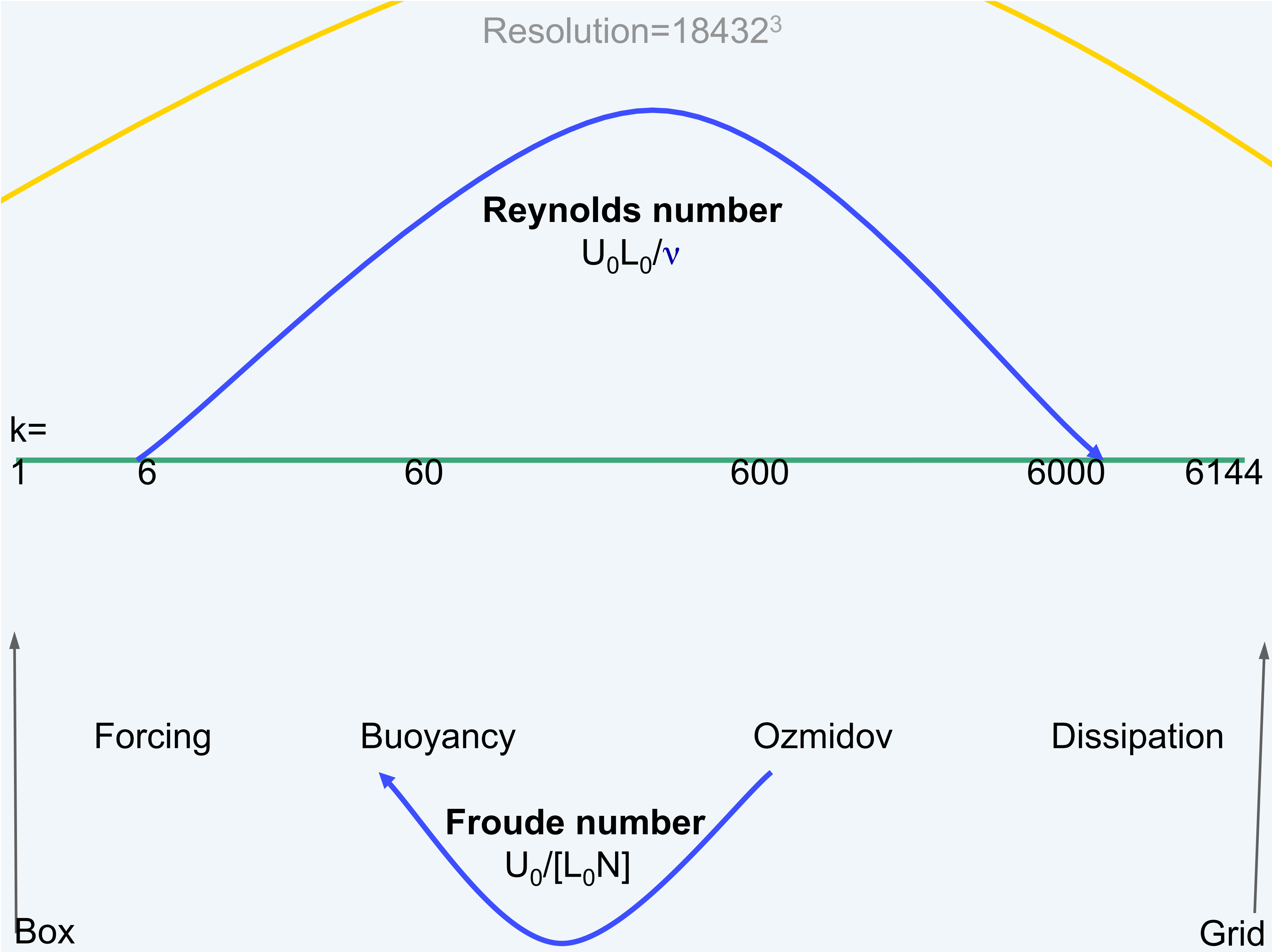
- As a matter of conclusion:

  – The lack of resolution when there is more than one inertial range: the emergence of two characteristic scales (buoyancy and Ozmidov)

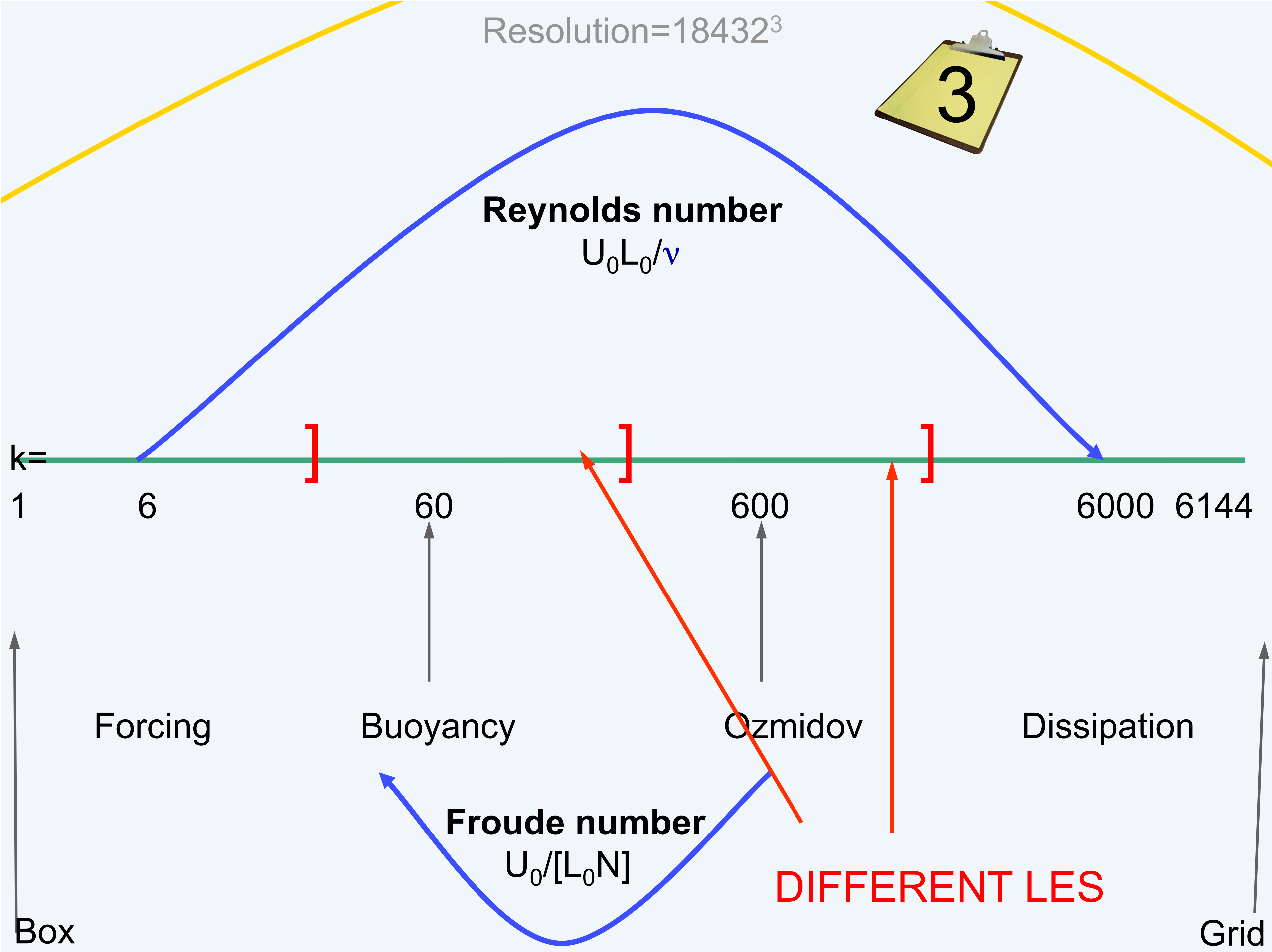
→ *A proposition for what would be a really big run of stratified (and rotating?) turbulence*

18432^3 points, stratified turbulence





Resolution=18432^3
Reynolds number
U_0L_0/ν
k=
1
6
60
600
6000
6144
Forcing
Buoyancy
Ozmidov
Dissipation
Froude number
$U_0/[L_0N]$
Box
Grid



Resolution=18432³
3
Reynolds number
U_0L_0/ν
k=
1
6
60
600
6000
6144
Forcing
Buoyancy
Ozmidov
Dissipation
Froude number
$U_0/[L_0N]$
DIFFERENT LES
Box
Grid