

Rotating stably stratified turbulence: Bolgiano-Obukhov scaling, wave-vortex partition and mixing

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INCITE/DOE DE-AC05-00OR22725 ; NSF/TG-PHY100029 ; NCAR/Yellowstone

Cargese, August 2016

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Kolmogorov:

“I soon understood that there was little hope of developing a pure, closed theory, and because of absence of such a theory the investigation must be based on hypotheses obtained on processing experimental data.”

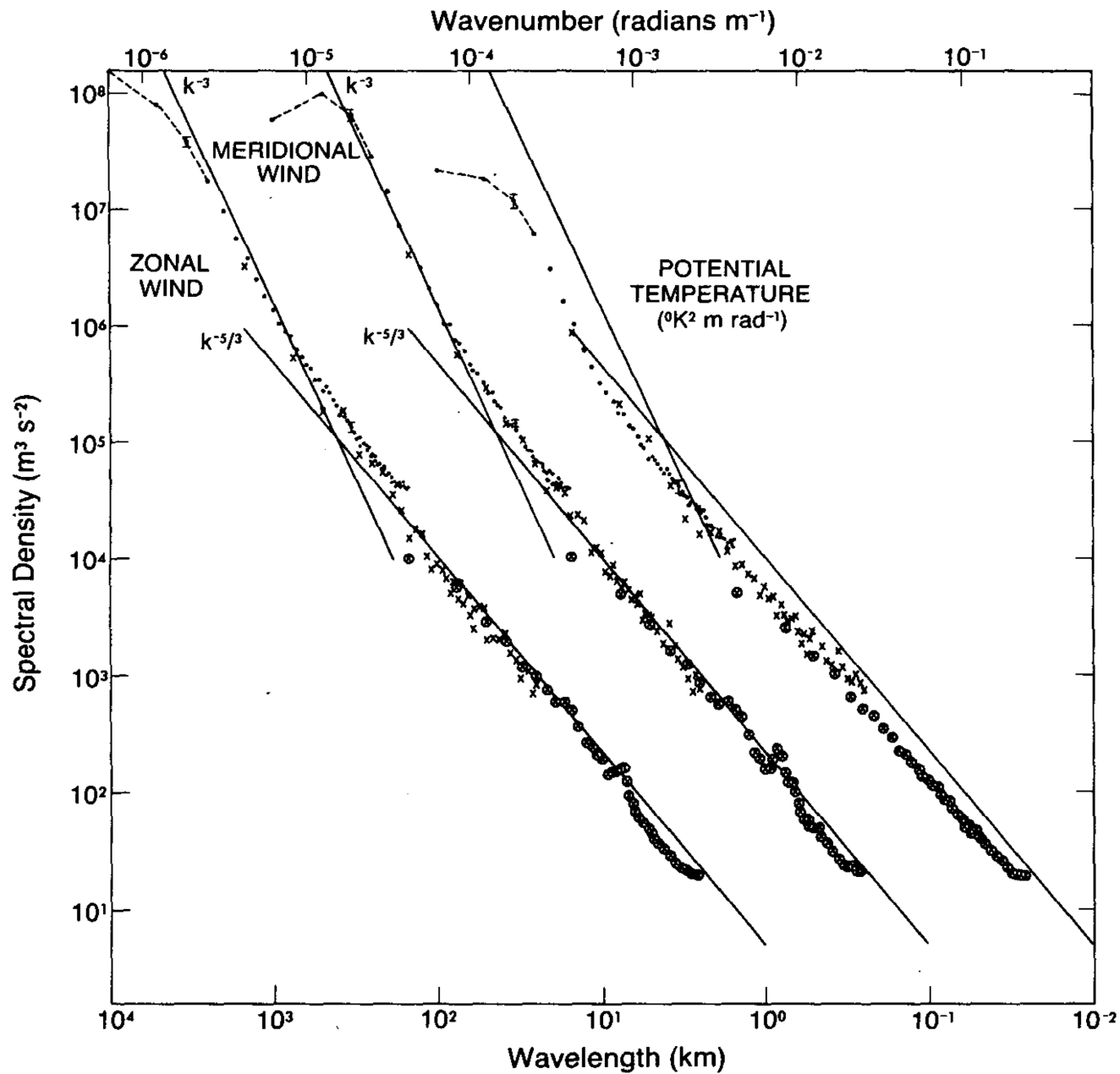


FIG. 3. Variance power spectra of wind and potential temperature near the tropopause from GASP aircraft data. The spectra for meridional wind and temperature are shifted one and two decades to the right, respectively; lines with slopes -3 and $-5/3$ are entered at the same relative coordinates for each variable for comparison.

$U \sim 20 \text{ m/s}$
 $N \sim 0.01$
 $L_{Oz} \sim 100 \text{ m}$

$\eta < 1 \text{ mm} \rightarrow$

Nastrom + 1985,87
Lindborg 2007³

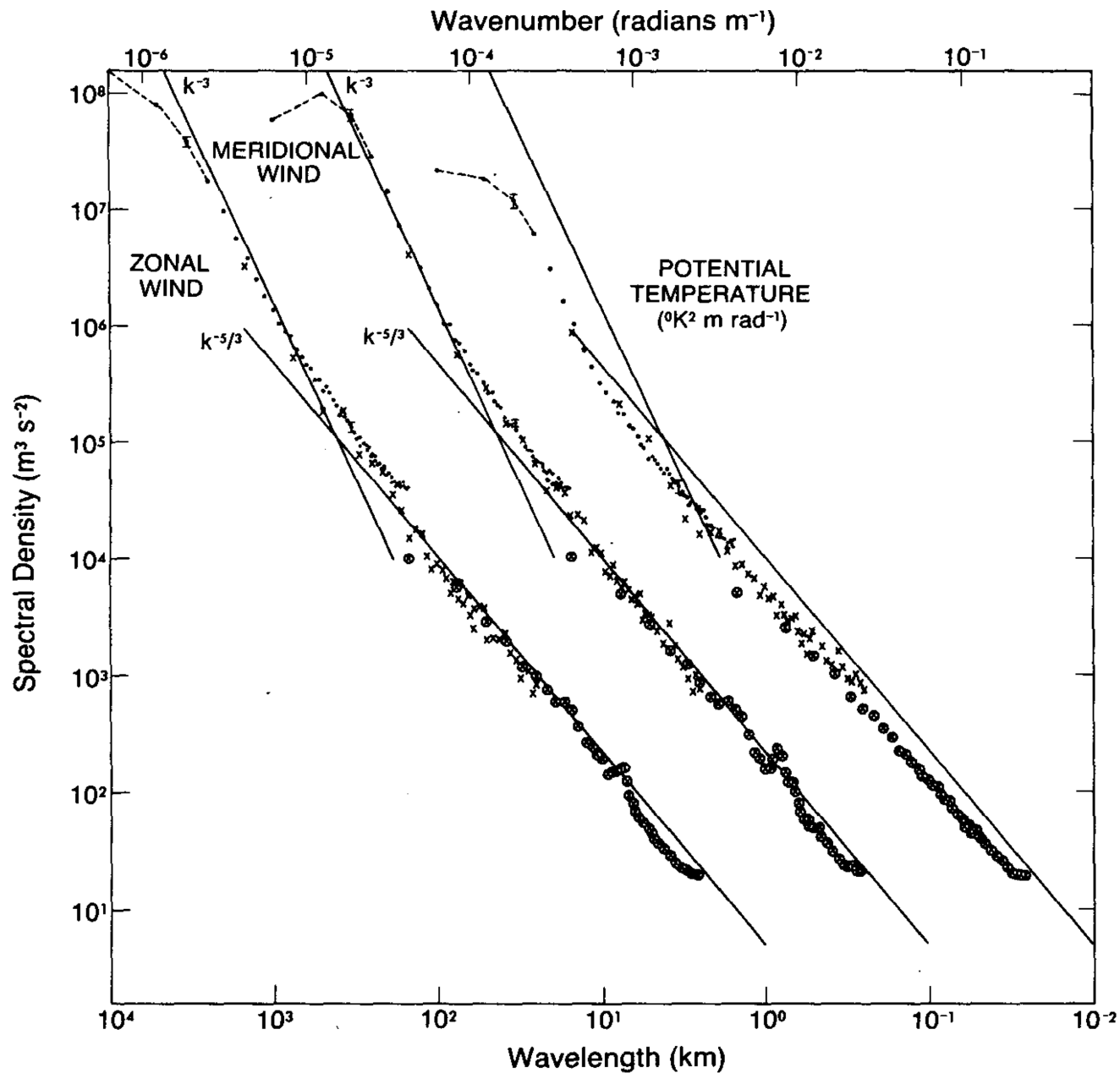


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Direction of
energy flux?

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Lindborg 2007⁴

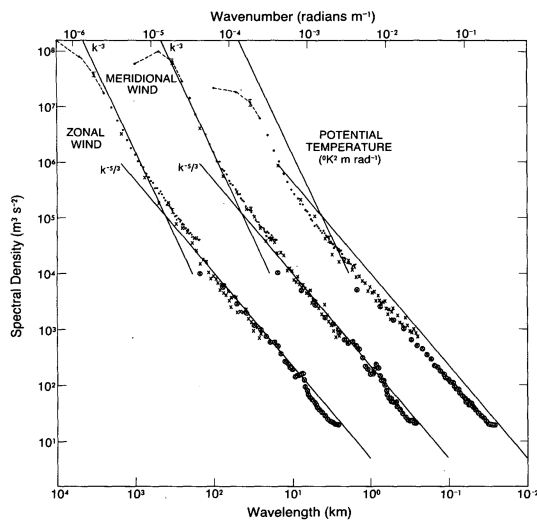
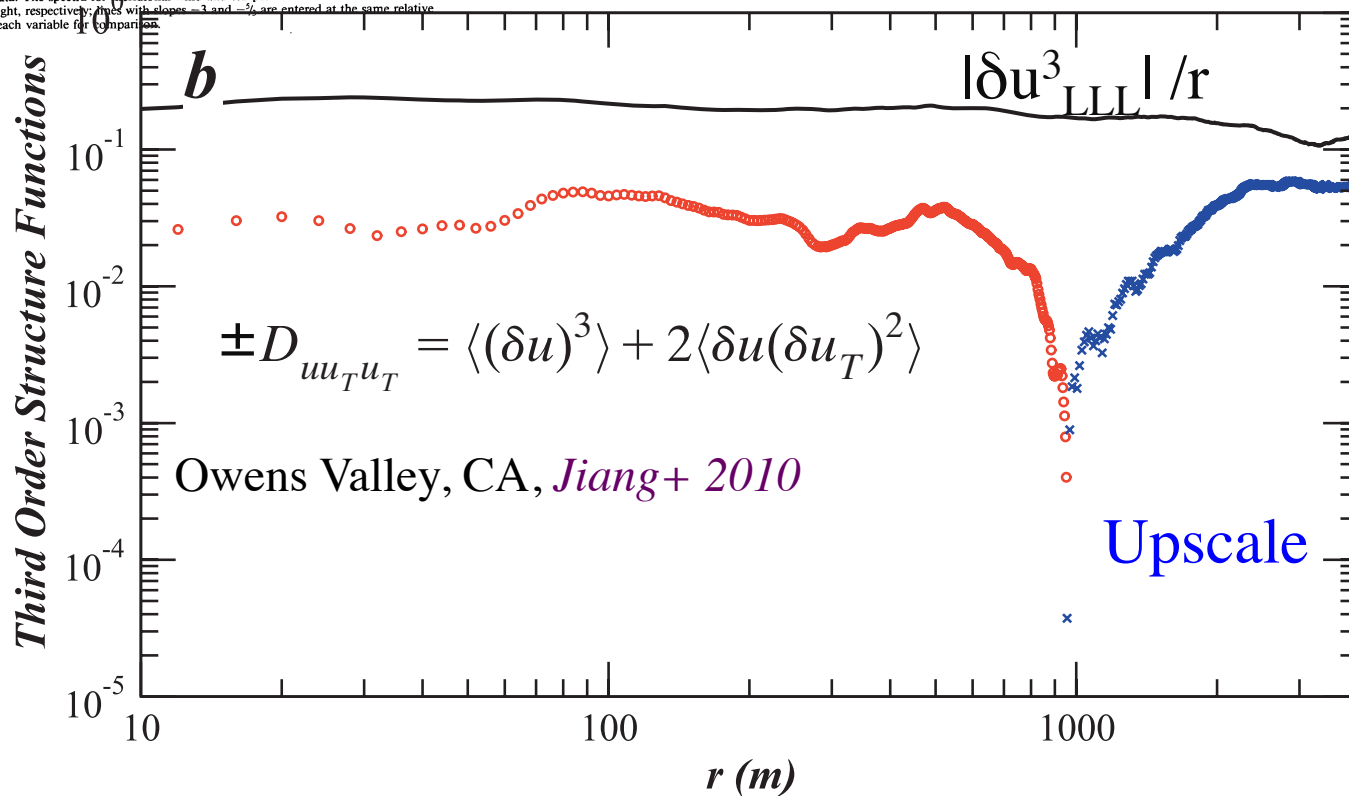


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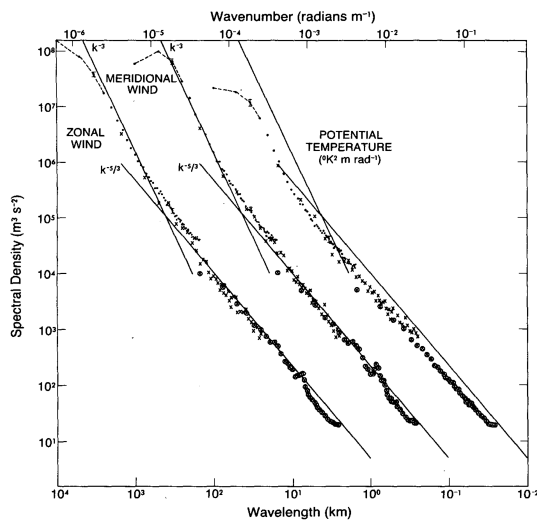
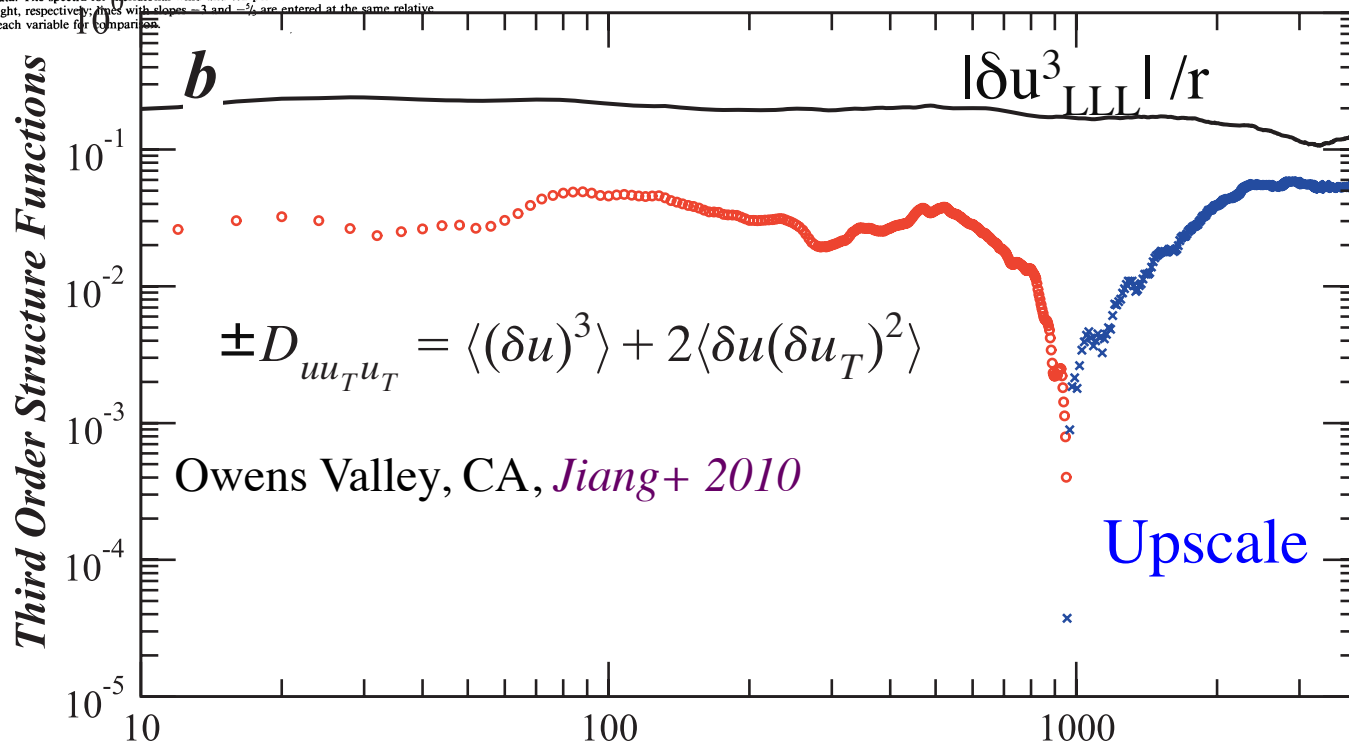


FIG. 3. Variance power spectra of wind and potential temperature near the tropopause from GASP aircraft data. The spectra for meridional wind and temperature are shifted one and two decades to the right, respectively; lines with slopes -3 and $-5/2$ are entered at the same relative coordinates for each variable for comparison.

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Direction of
energy flux?

$\eta < 1 \text{ mm}$

Antarctica: TILDAE exp. $(Marino+ 2017)$

Nastrom + 1985,87
*Lindborg 2007*⁶

Incompressible Boussinesq equations
3D cubic box, periodic boundary conditions

$$\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\omega} \times \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} = -N\rho\hat{e}_z - \nabla\mathcal{P} + \nu\nabla^2\mathbf{u} ,$$
$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = Nw + \kappa\nabla^2\rho ,$$

Parameters, with $f=2\Omega$:

$$Re = \frac{U_0 L_0}{\nu}, \quad Fr = \frac{U_0}{L_0 N}, \quad Ro = \frac{U_0}{L_0 f}, \quad Pr = \frac{\nu}{\kappa}, \quad R_B = Re Fr^2$$

The 2048^3 , 3072^3 and 4096^3 runs:

$k_0 \sim 2.5$, $N/f = 4.95$, $Re = 55000$, $Fr = 0.024$, $Ro = 0.12$, $Pr = 1$, $R_B = 32$

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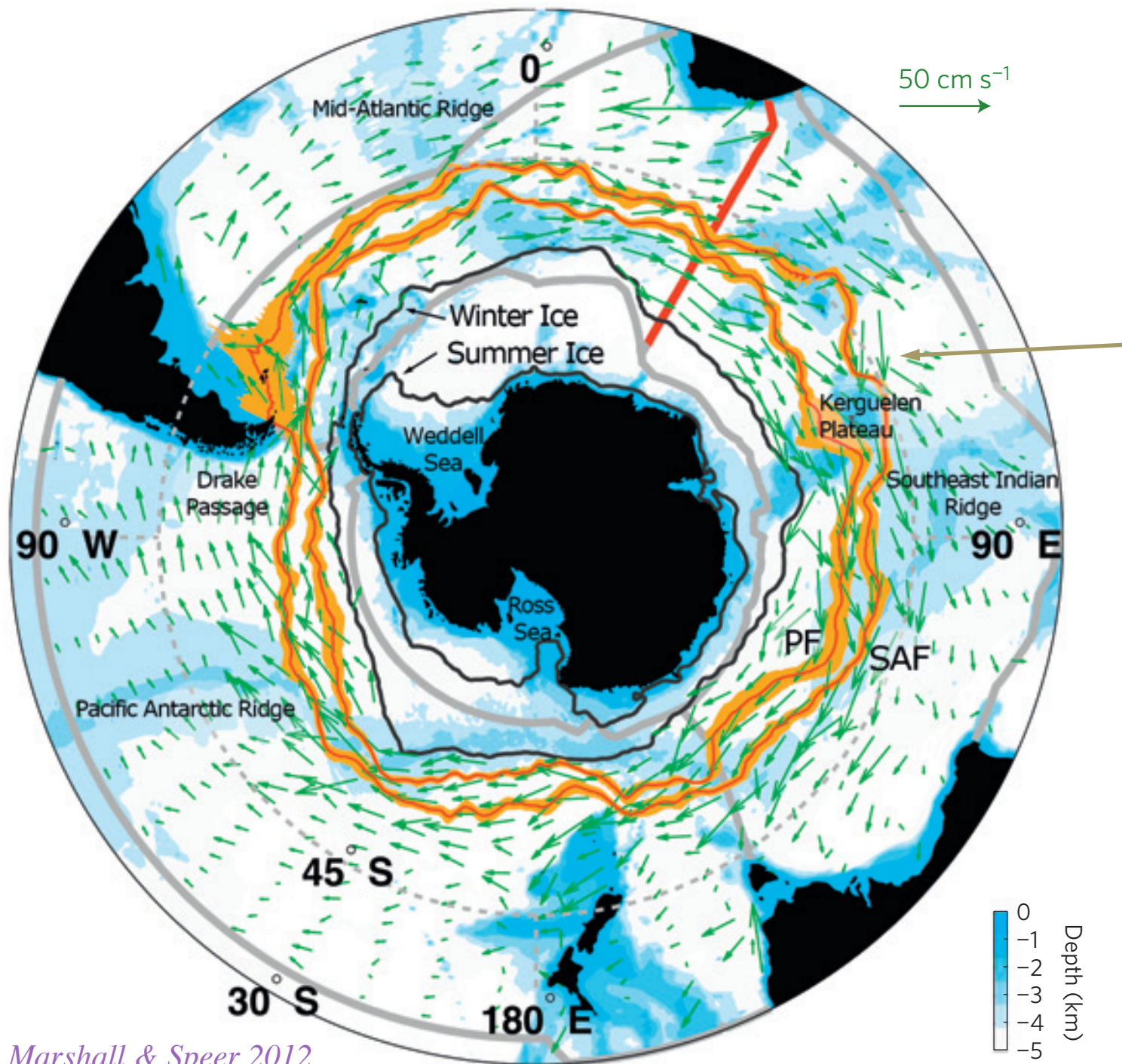
$$[\varrho, \theta] = [L T^{-1}]$$

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*Cubic box
of
turbulence*

*next to
Kerguelen
Plateau
45S, 60E*

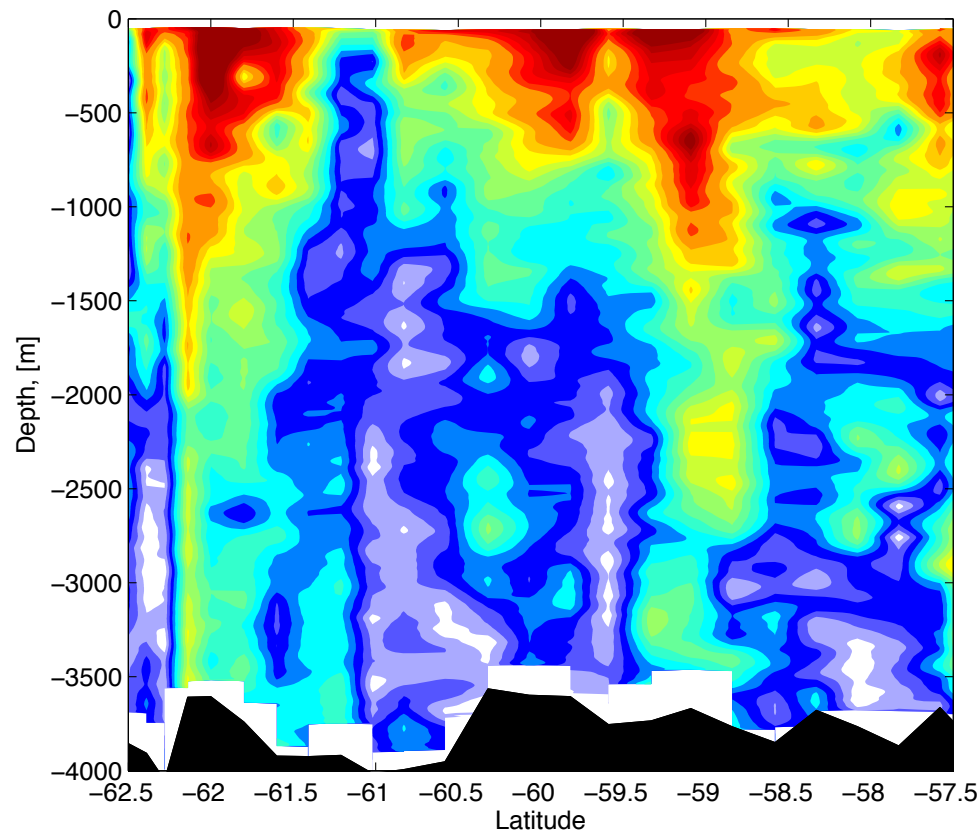


Figure 3-2: Flow speed (m s^{-1}) from the ALBATROSS section, Drake Passage.

Measurements in in the
Southern Antartic Ocean
(*Drake passage*)

← of flow speed

and of buoyancy frequency

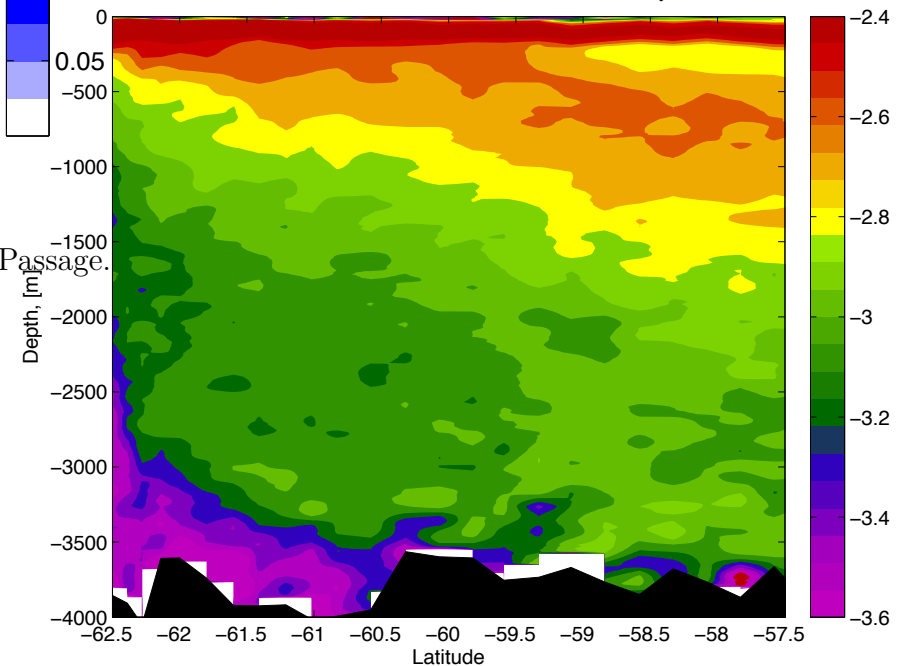
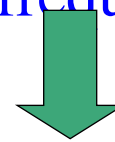
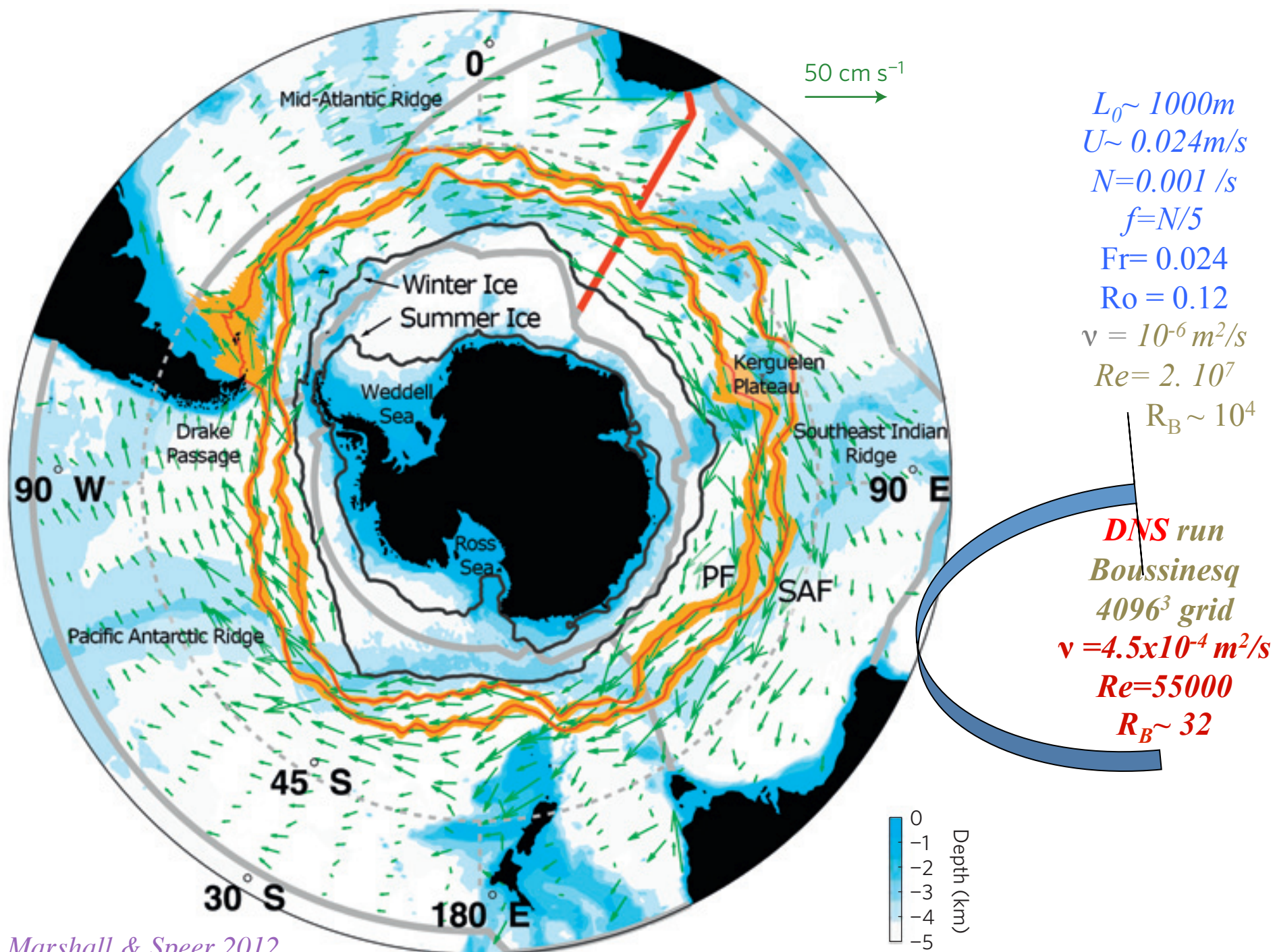
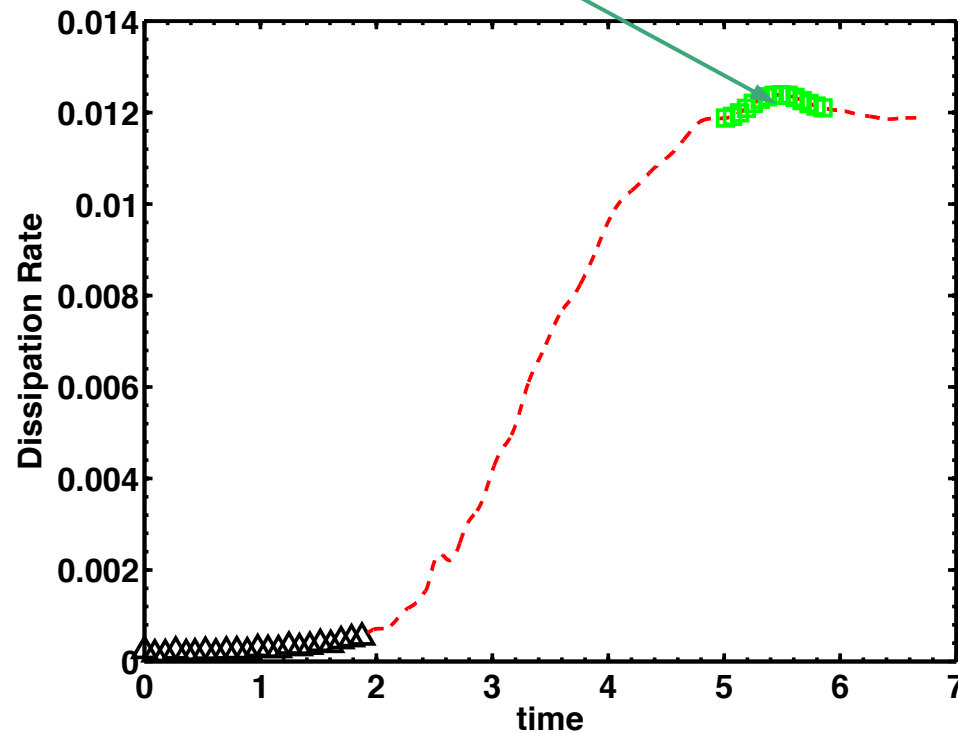


Figure 3-1: Buoyancy frequency (s^{-1}) in logarithmic scale from the ALBATROSS section, Drake Passage.

Nikurashin, 2009



The short 4096^3 run
at peak of dissipation

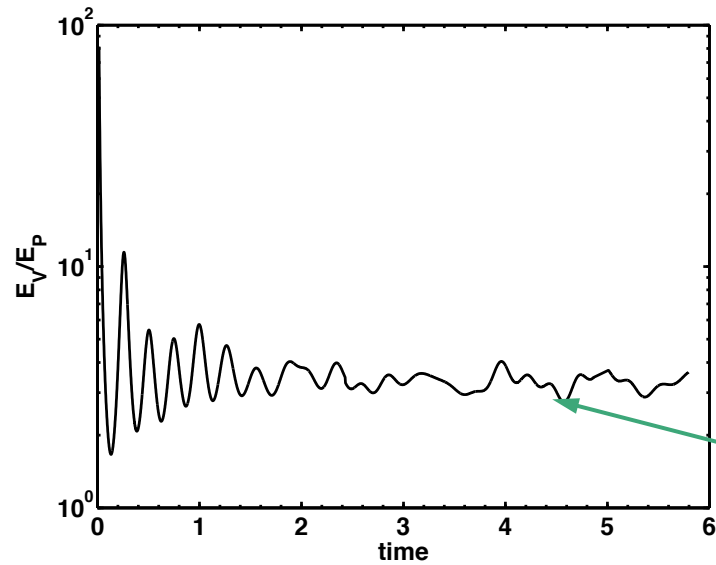


$N/f=4.95$
 $Fr=0.024$, $Ro=0.12$
 $Re=55000$, $R_B=32$

Spin-down
 $K_0=2.5$

Triangles: 1536^3 grid- NSF
- - - : 3072^3 grid- NSF
... Green: 4096^3 grid- DOE

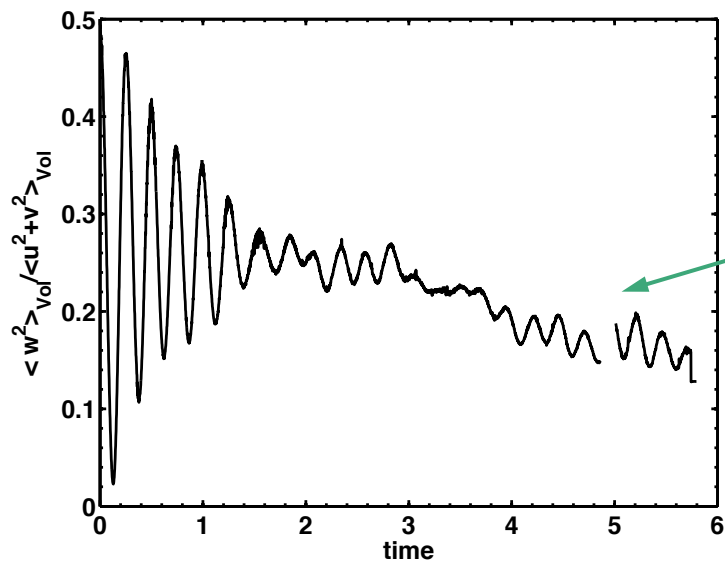
Energy ratios
 $Fr=0.024, R_B=32$



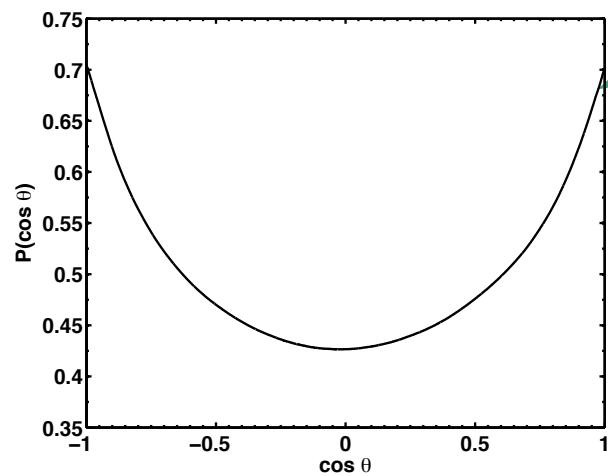
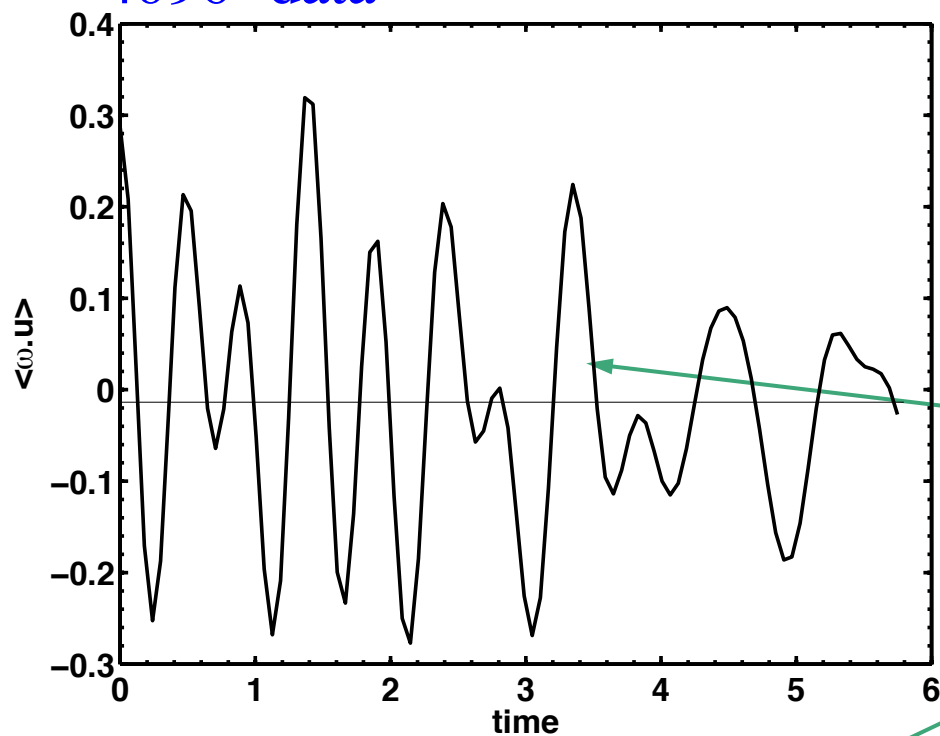
Kinetic to potential: ~ 3

And

Vertical to horizontal: ~ 0.12



4096³ data



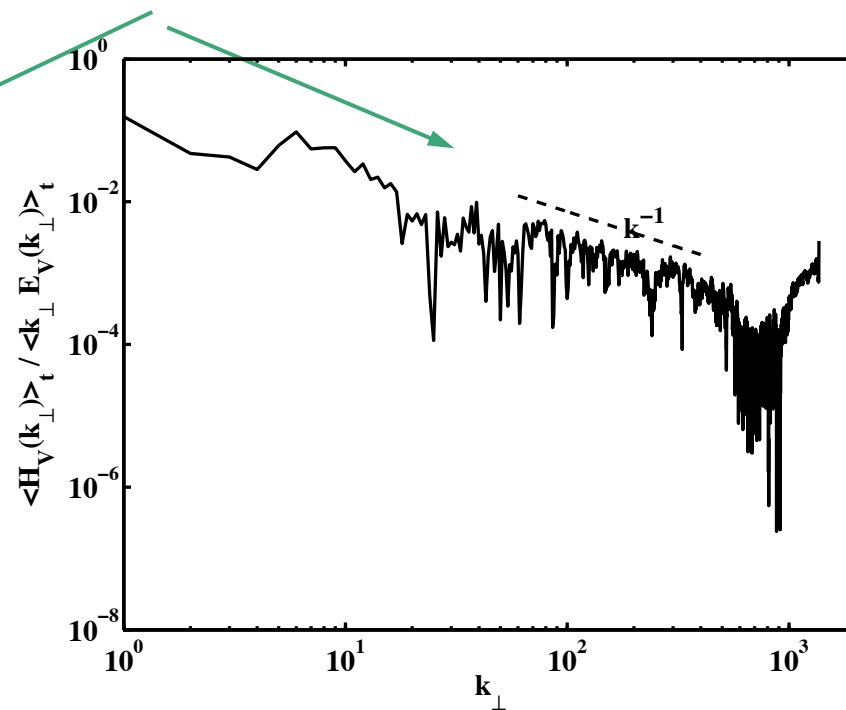
$N/f=4.95$

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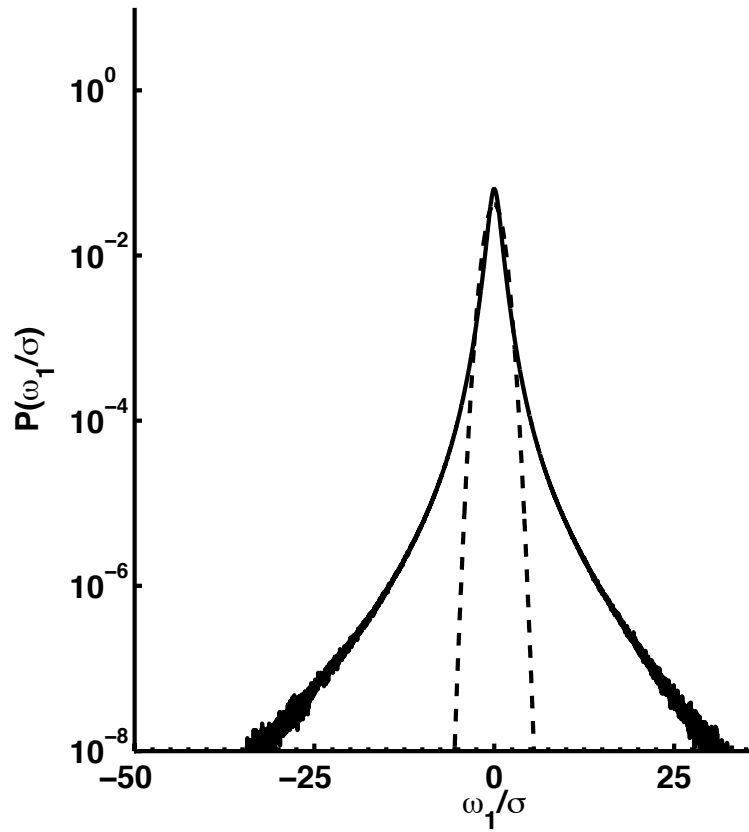
$Re=55000, R_B=32$

$K_0=2.5$, spin-down

Total helicity and
PDF of relative helicity
and rel. helicity spectrum

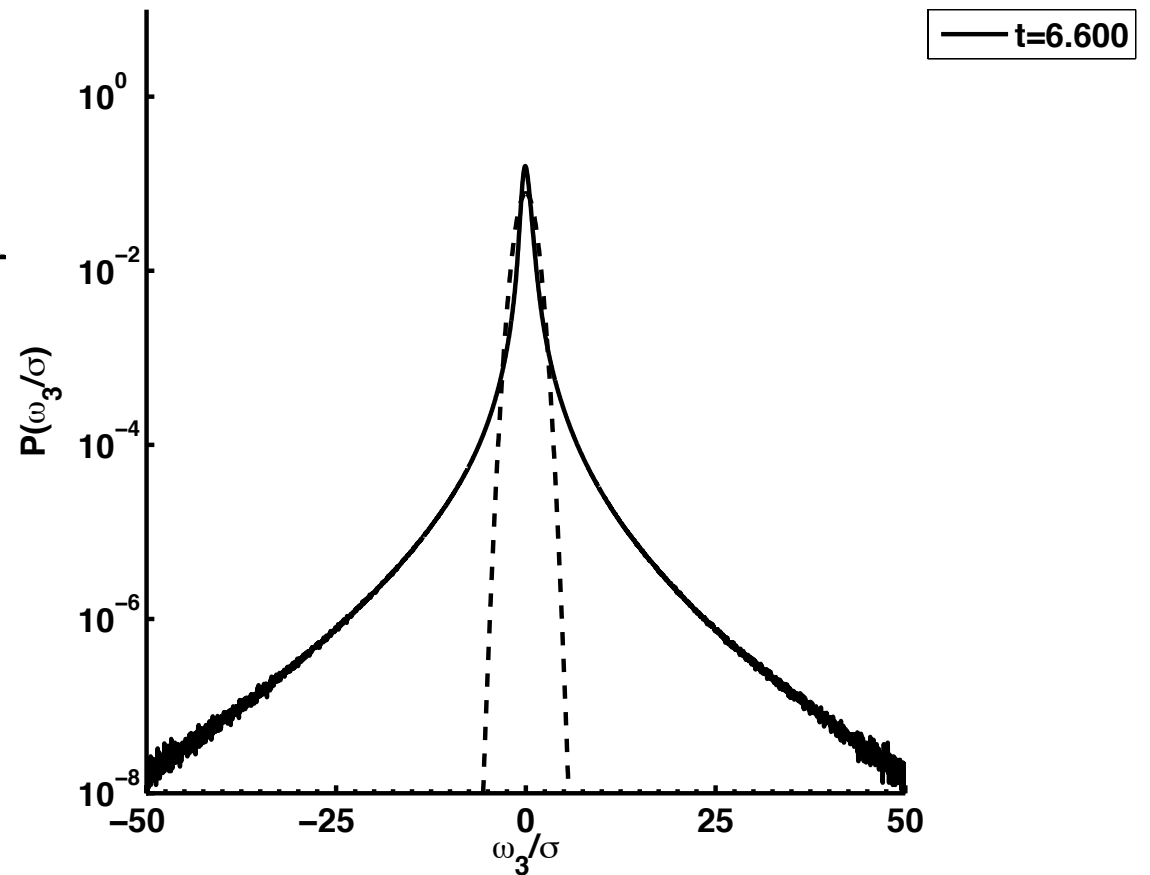


RotStrat2048: ω_1

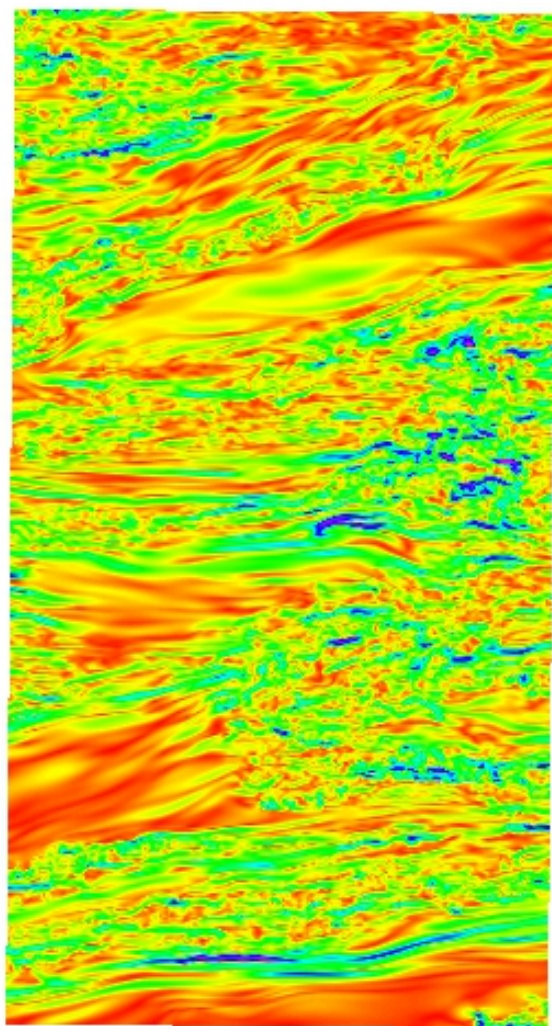


Vorticity components, 2048³ run

RotStrat2048: ω_3



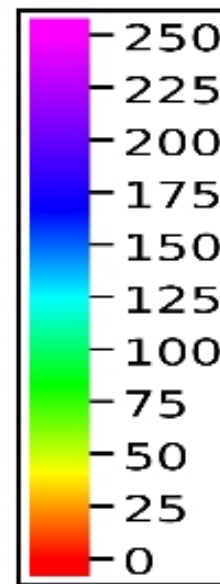
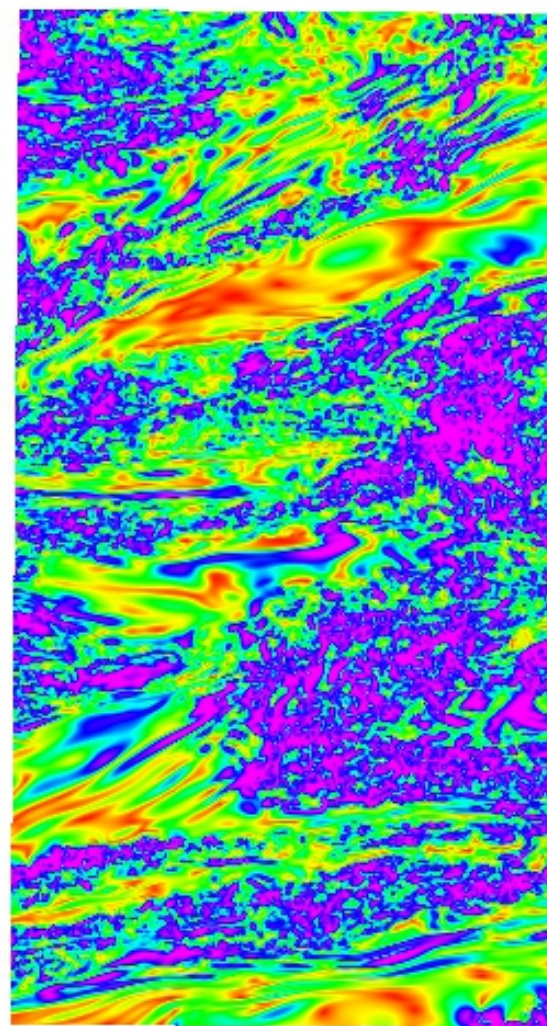
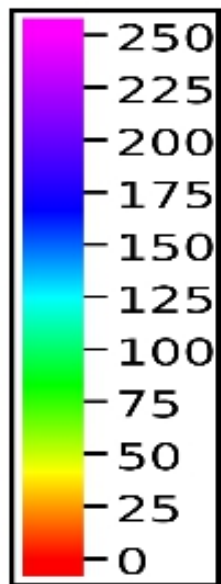
$\omega_{\text{rms}} \sim 17$



4096³ run

0.7 X 0.4

X 0.04
(PVR)

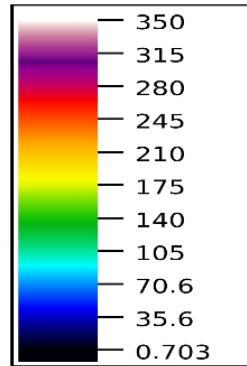
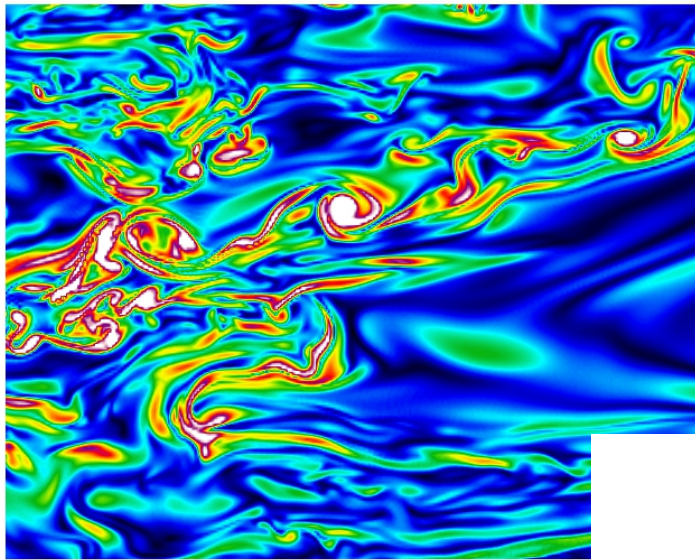


$$|[\partial_z u, \partial_z v]|$$

QG in small scales?

=

$$[N/f]^* |[-\partial_y \theta, \partial_x \theta]|$$



$$N/f=4.95$$

$$Fr=0.024, Ro=0.12$$

$$Re=55000, R_B=32$$

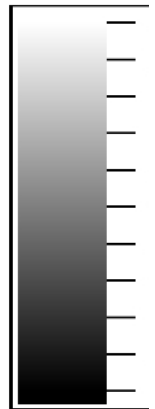
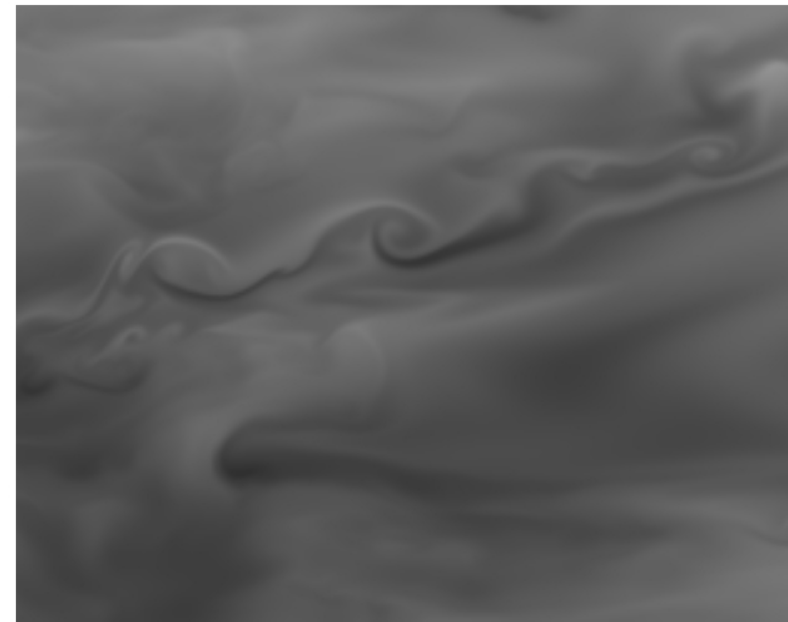
$$K_0=2.5, \text{ spin-down}$$

↑
z

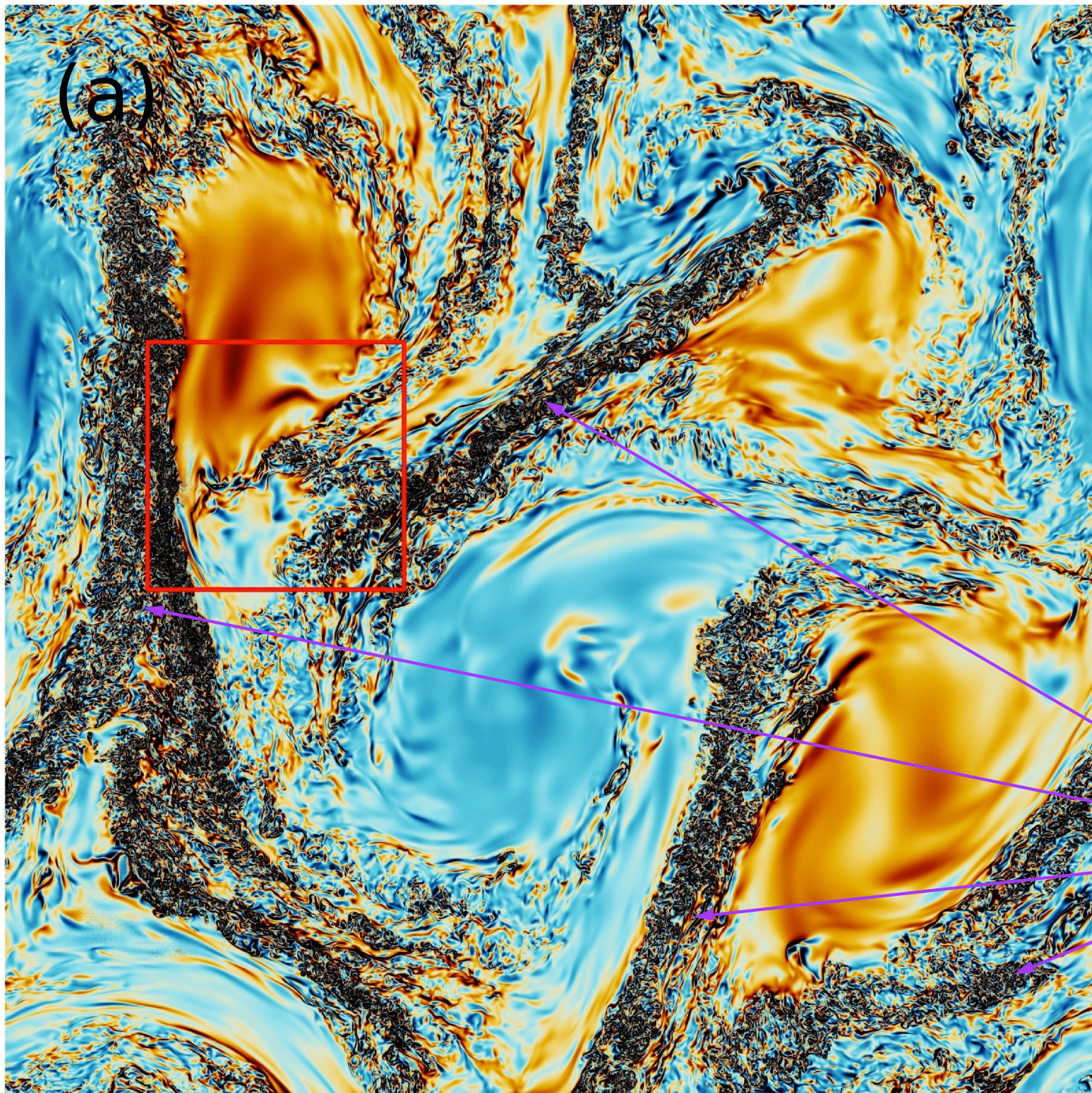
Zoom:

Vertical vorticity ($f \sim 2.7$)

Temperature fluctuations



*Sub-volume $[0.12 \times 0.1 \times 0.01] * [2\pi]^3$*



ω_z

4096³ run
No forcing

Re=55000
N/f~ 5
R_B~ 32

10.0
-10.0

Vorticity
lanes

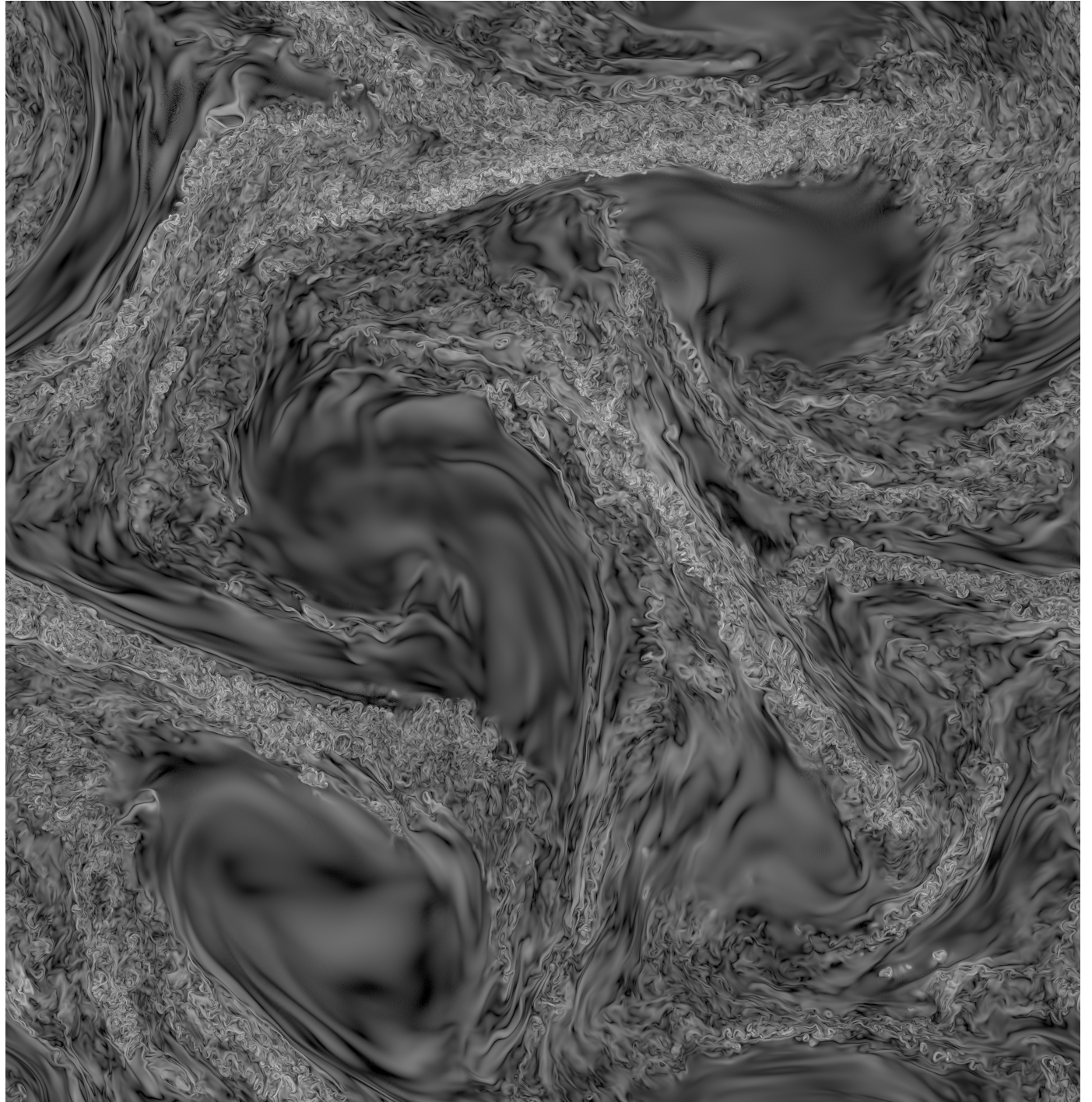
ω_{mag}

Full 4096^2 res.
Log scale

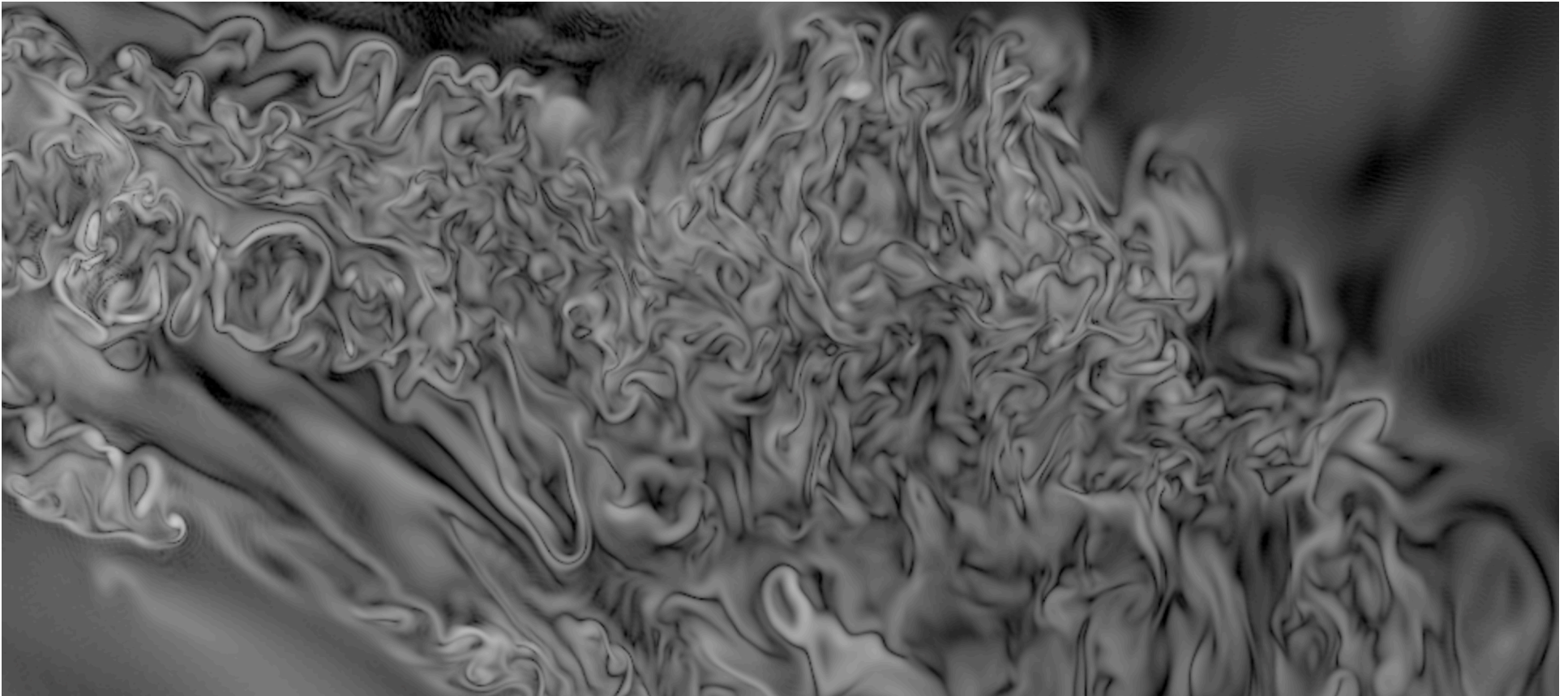
$f=2.7, \omega_{\text{rms}} \sim 17$

$\text{Re}=55000$

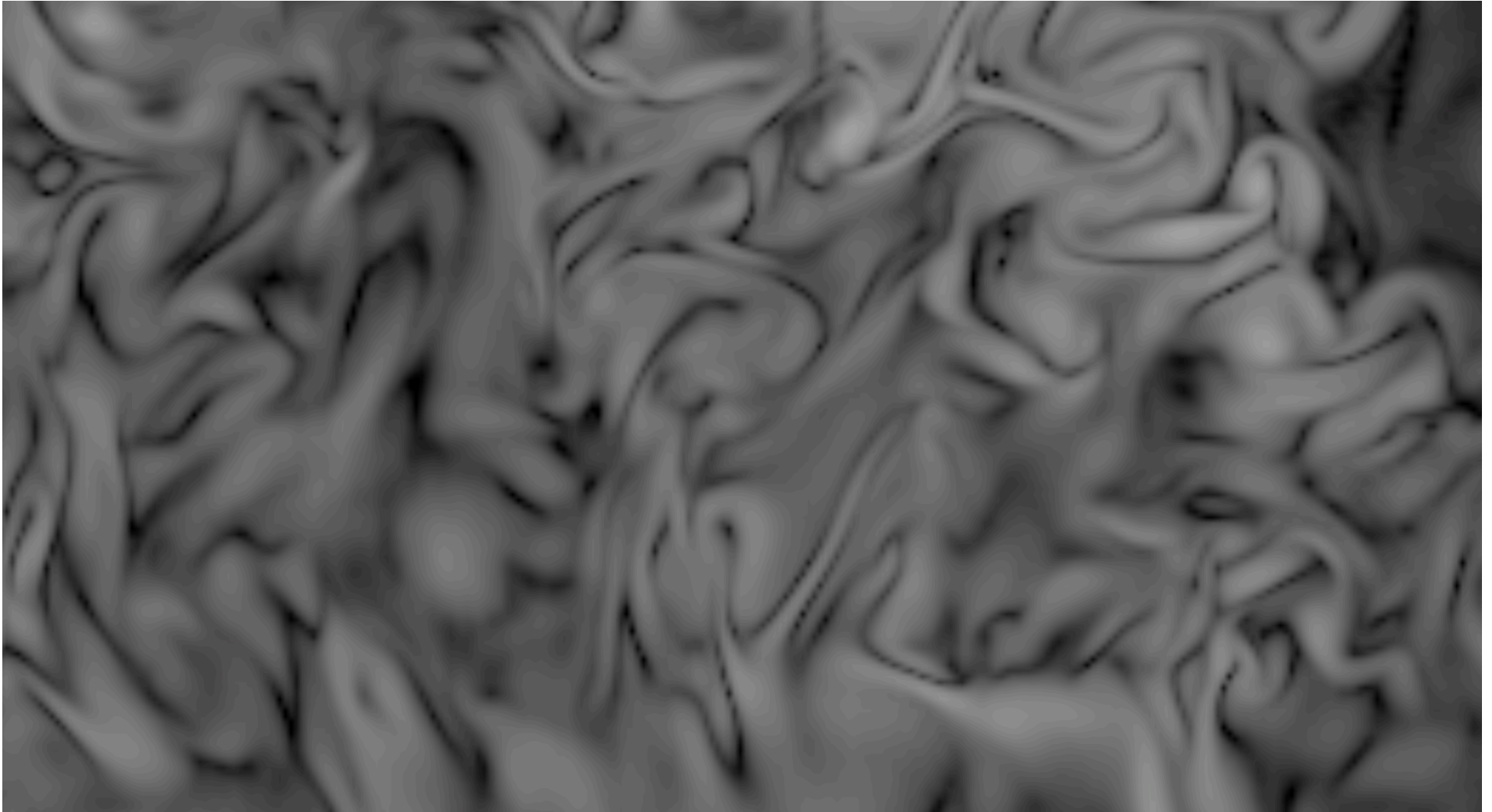
$R_B=32$



Zoom of ω_{mag} at full 4096^2 resolution, log scale, $f=2.7$, $\omega_{\text{rms}} \sim 17$



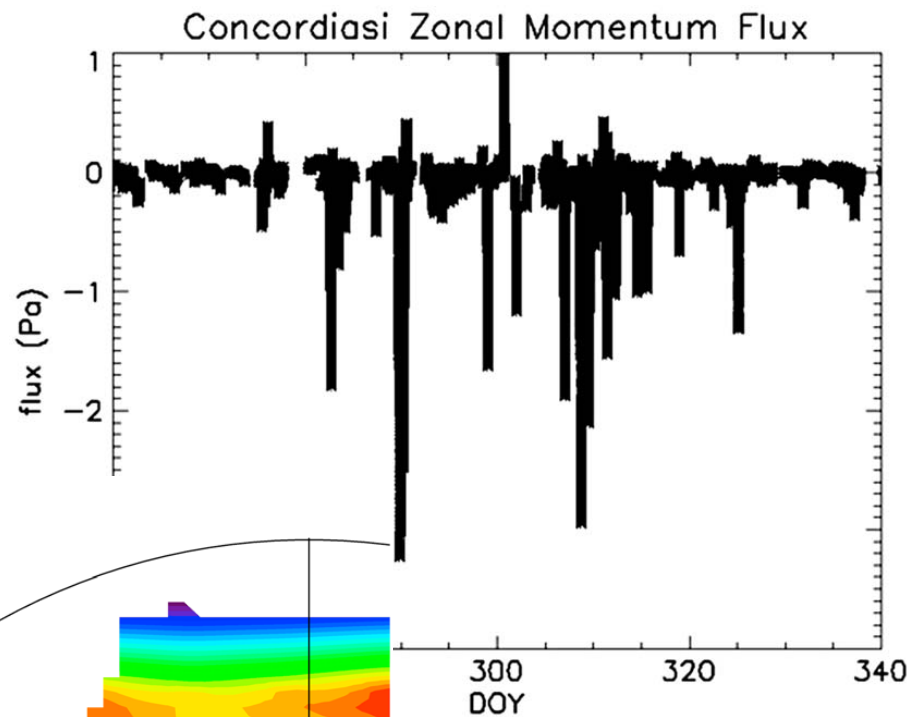
Max. zoom of ω_{mag} at full 4096^2 resolution, log scale, $f=2.7$, $\omega_{\text{rms}} \sim 17$



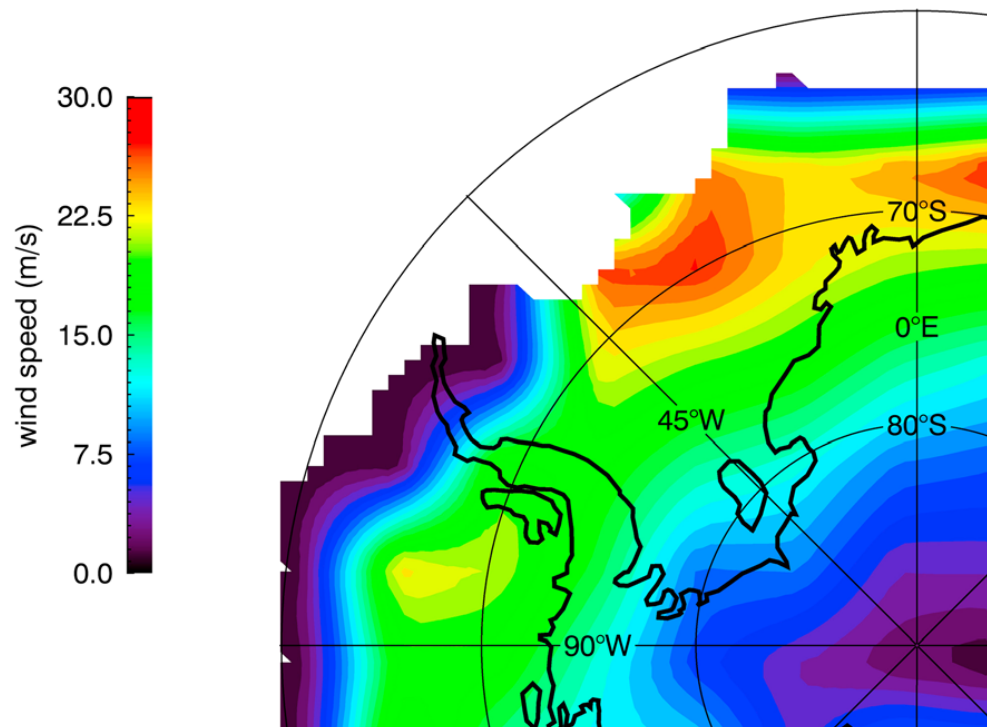
Instability zones: Mountain flux

*Walterscheid et al.
JGR 121, 2016*

$\langle uw \rangle$
30s., 500km²



Mean winds



s of vertical flux of zonal momentum over
as a function of day of year in 2010.

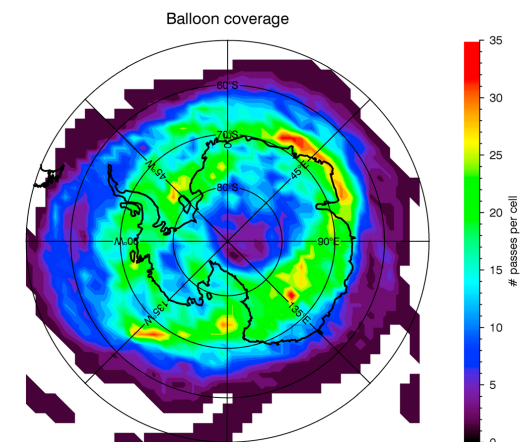


Figure 13. Campaign-averaged zonal wind speed at balloon altitude.

Figure 1. Geographical distribution of the number of Concordiasi balloon passes per 500 km × 500 km cell.

Stably stratified turbulence: Bolgiano-Obukhov 1959 scaling

Main hypothesis: Inertial range with a **constant buoyancy flux** $u\theta^2/\ell$

$E_{V,P}(k) = f(k, \epsilon_P)$ with $\epsilon_P = DE_P/DT$ of dimension m^2s^{-5}

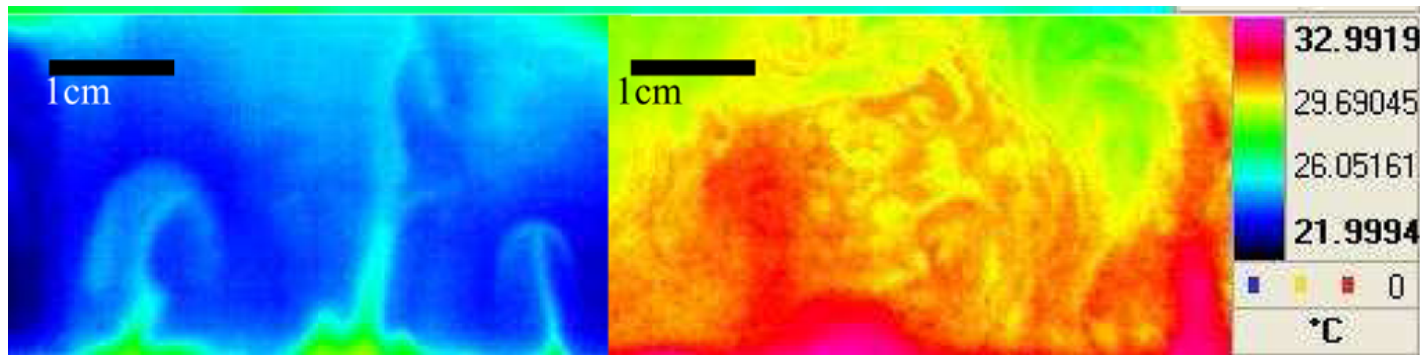
$$\rightarrow E_V(k) = \epsilon_P^{2/5} k^{-11/5}$$

$$\rightarrow E_P(k) = \epsilon_P^{4/5} k^{-7/5}$$

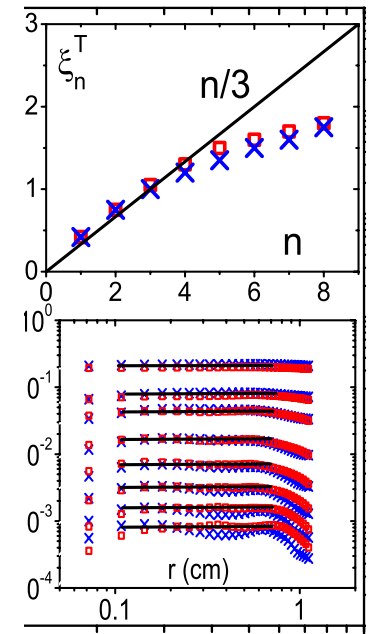
$$\rightarrow U^2/\ell \sim \theta \quad \text{in the momentum equation}$$

Elusive, ... Paradoxical (Lohse & Xia, Ann. Rev. Fluid Mech. 2010), ...

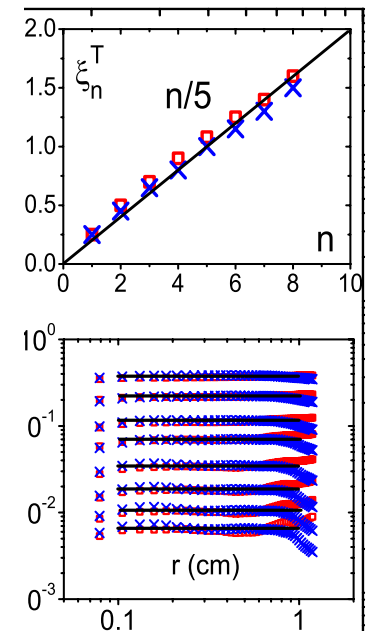
Seychelles et al. (2010) 2D soap bubble experiment



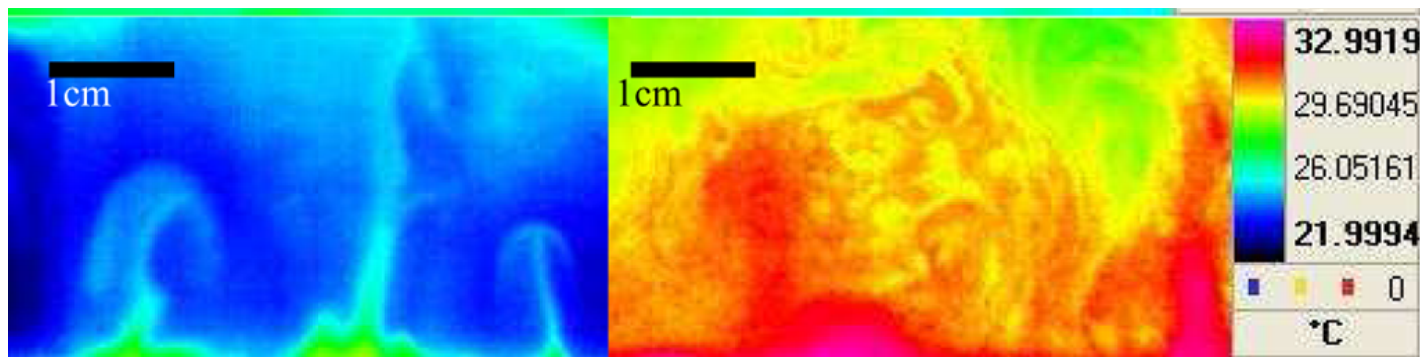
$\Delta T =$ 21C or 50C



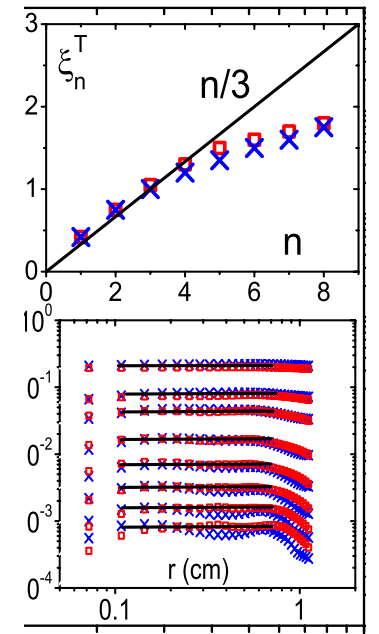
10



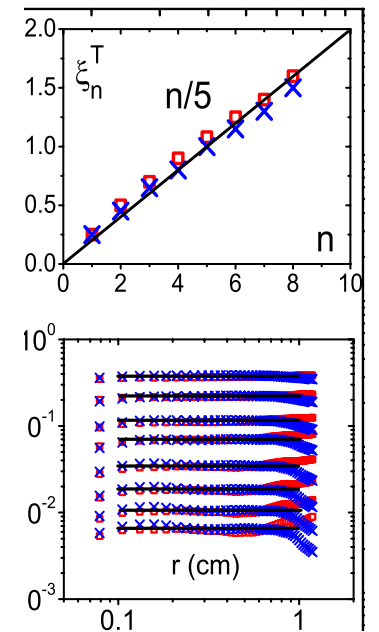
Seychelles et al. (2010) 2D soap bubble experiment



$\Delta T =$ 21C or 50C

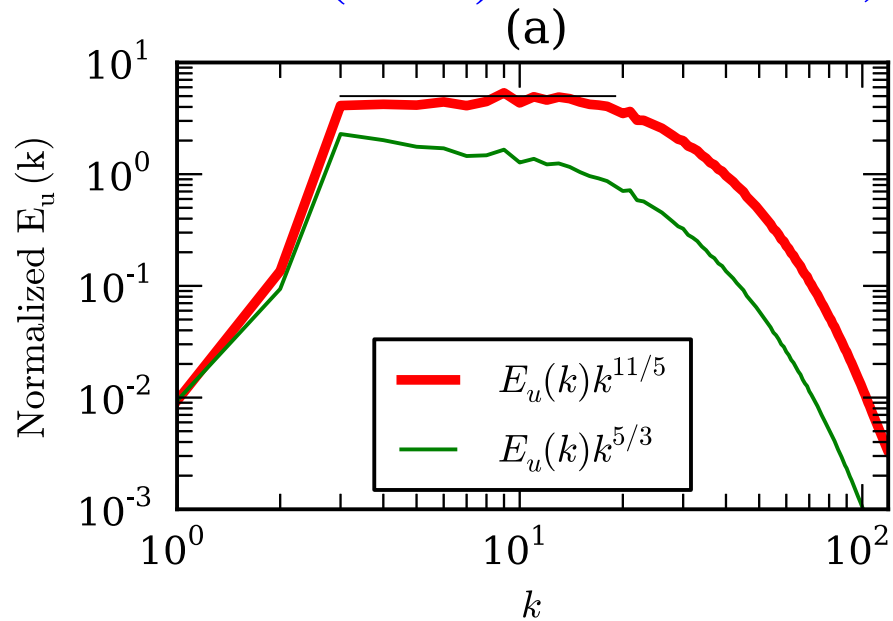


10



Also in shell models of 2D flows (Brandenburg, 1992; Boffetta+ 2012)

Kumar+ (2014) 1024^3 3D run, no rotation



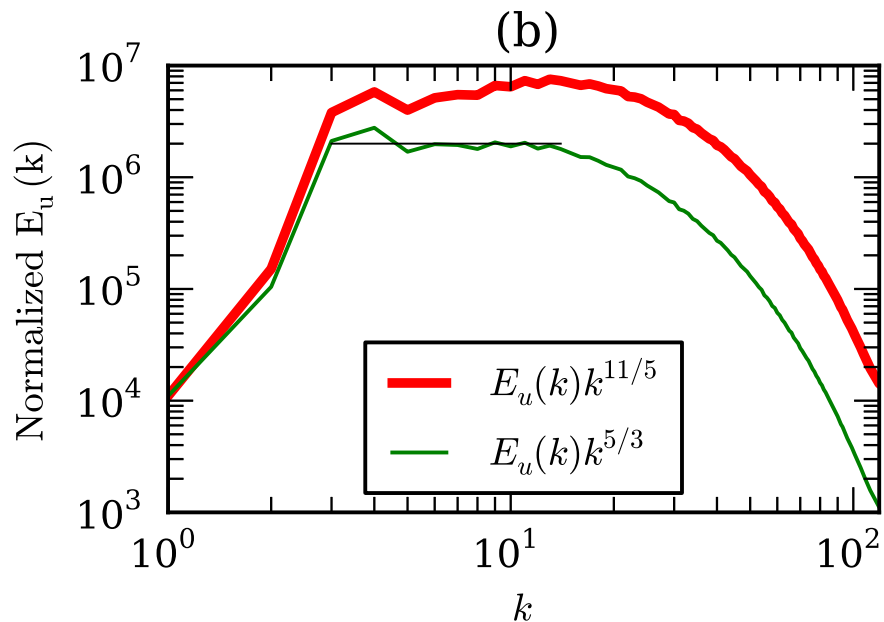
Re=650, large-scale forcing

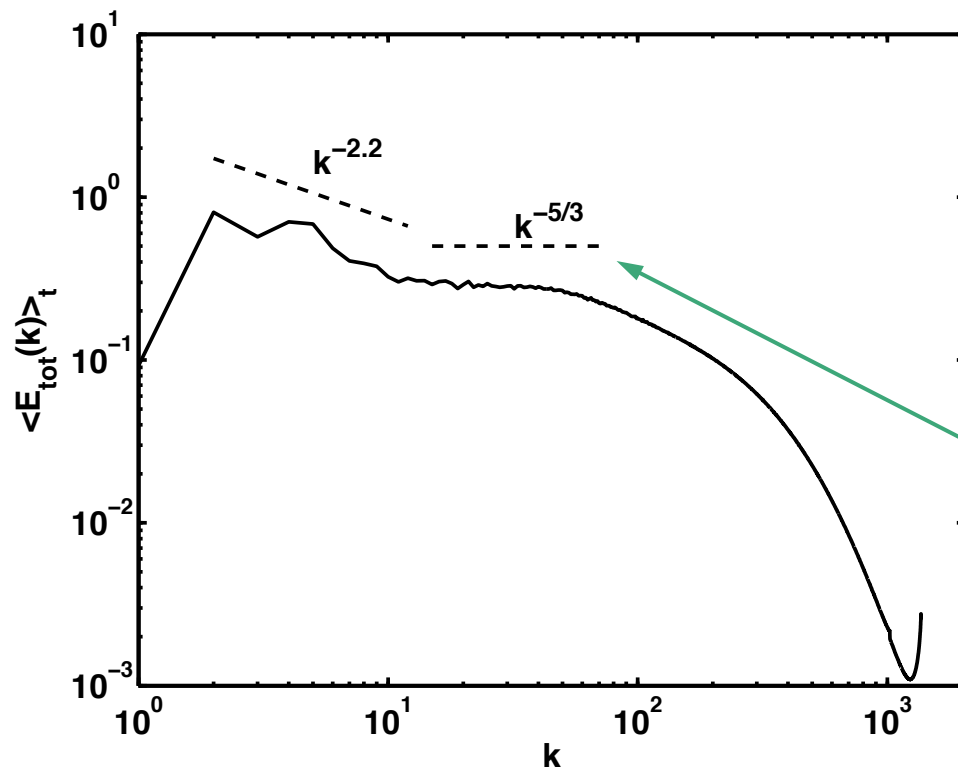
Kinetic energy spectra
compensated by either

BO (11/5), Fr=1.4

or

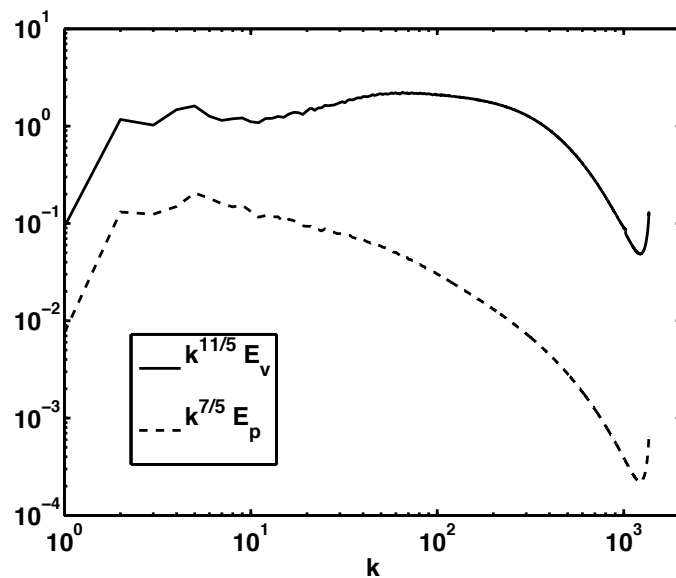
K41 (5/3), Fr=1000





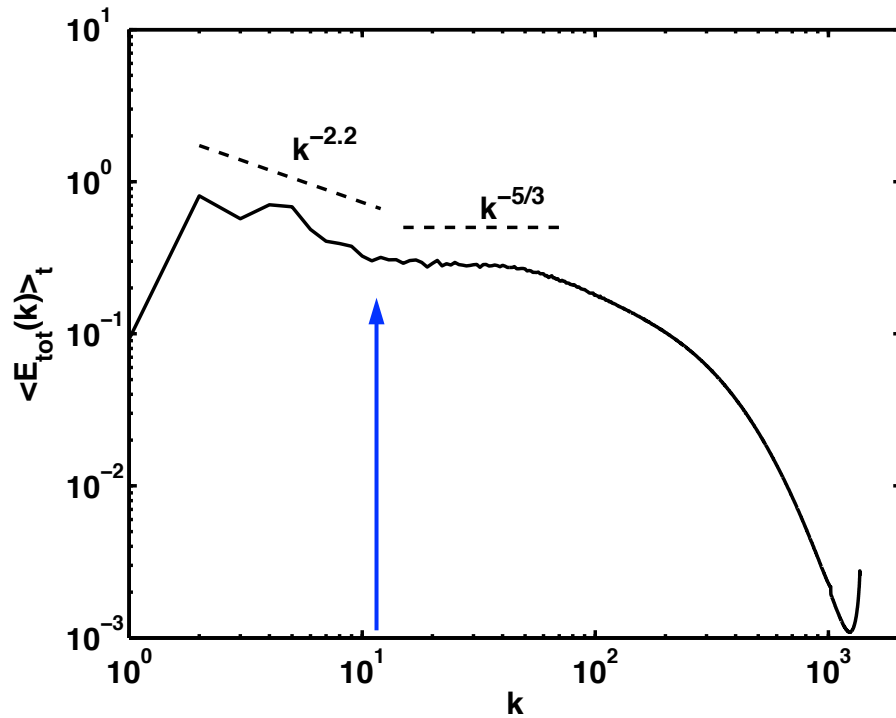
$N/f=4.95$
 $Fr=0.024, Ro=0.12$
 $Re=55000, R_B=32$
 $K_0=2.5$, spin-down

5/3-compensated
 total energy
 isotropic spectrum

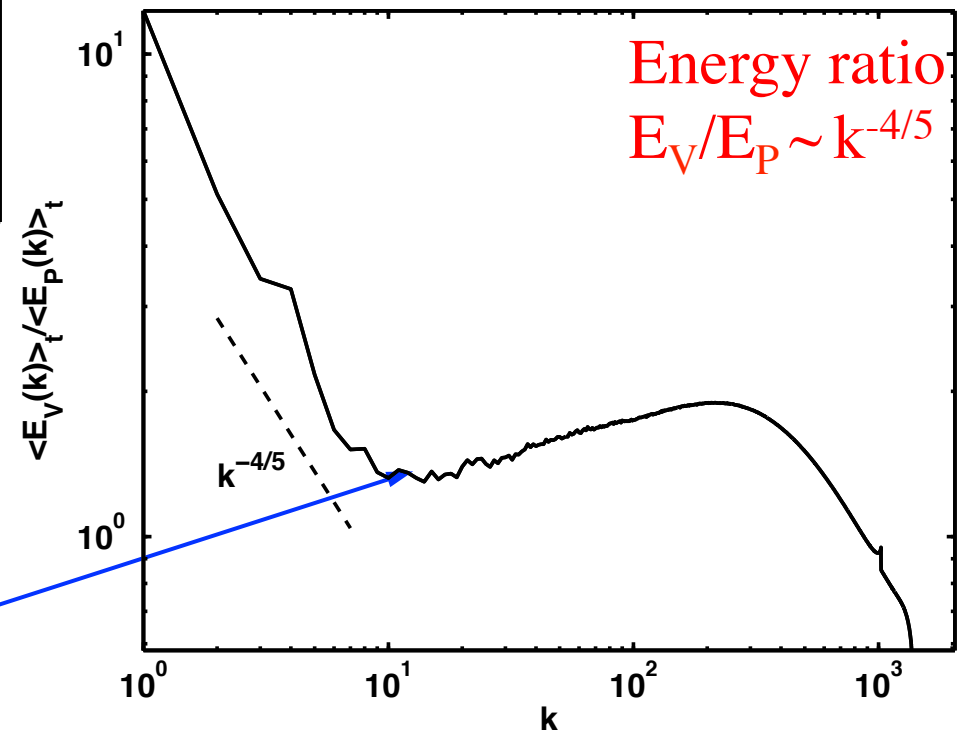


Energy spectra:
 kinetic (___) or
 potential (- - -)
compensated
by 11/5 or 7/5:
 Bolgiano-Obukhov scaling
 in the large scales ($>L_{BO}$)

Total energy spectrum around peak



$$K_{BO} \sim \varepsilon_P^{3/4} \varepsilon_V^{-5/4} \sim 11$$

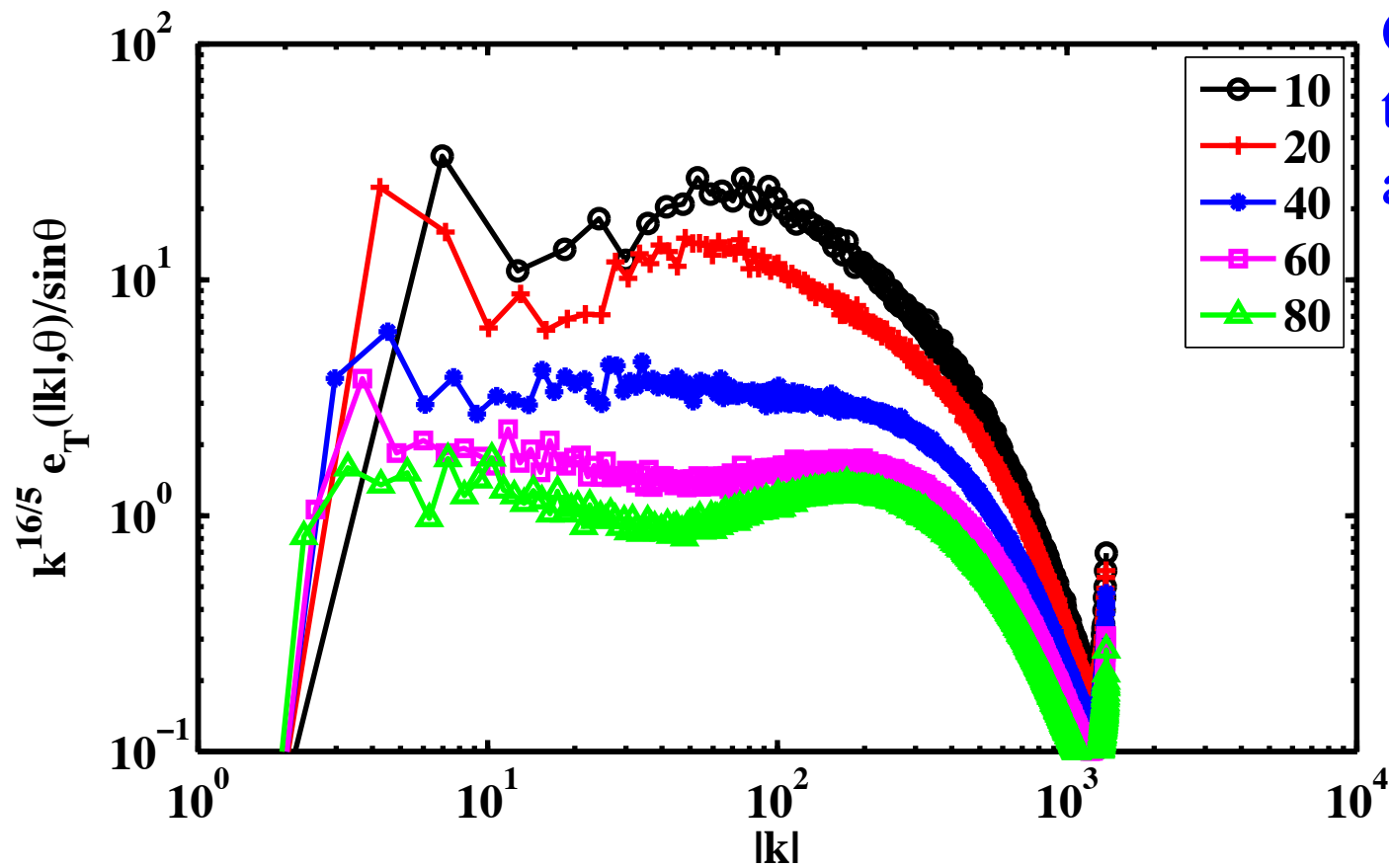


$N/f=4.95$

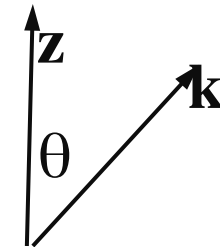
$Fr=0.024, Ro=0.12$

$Re=55000, R_B=32$

$K_0=2.5$, spin-down



**Compensated
total energy
angular spectra**



Vertical buoyancy flux:

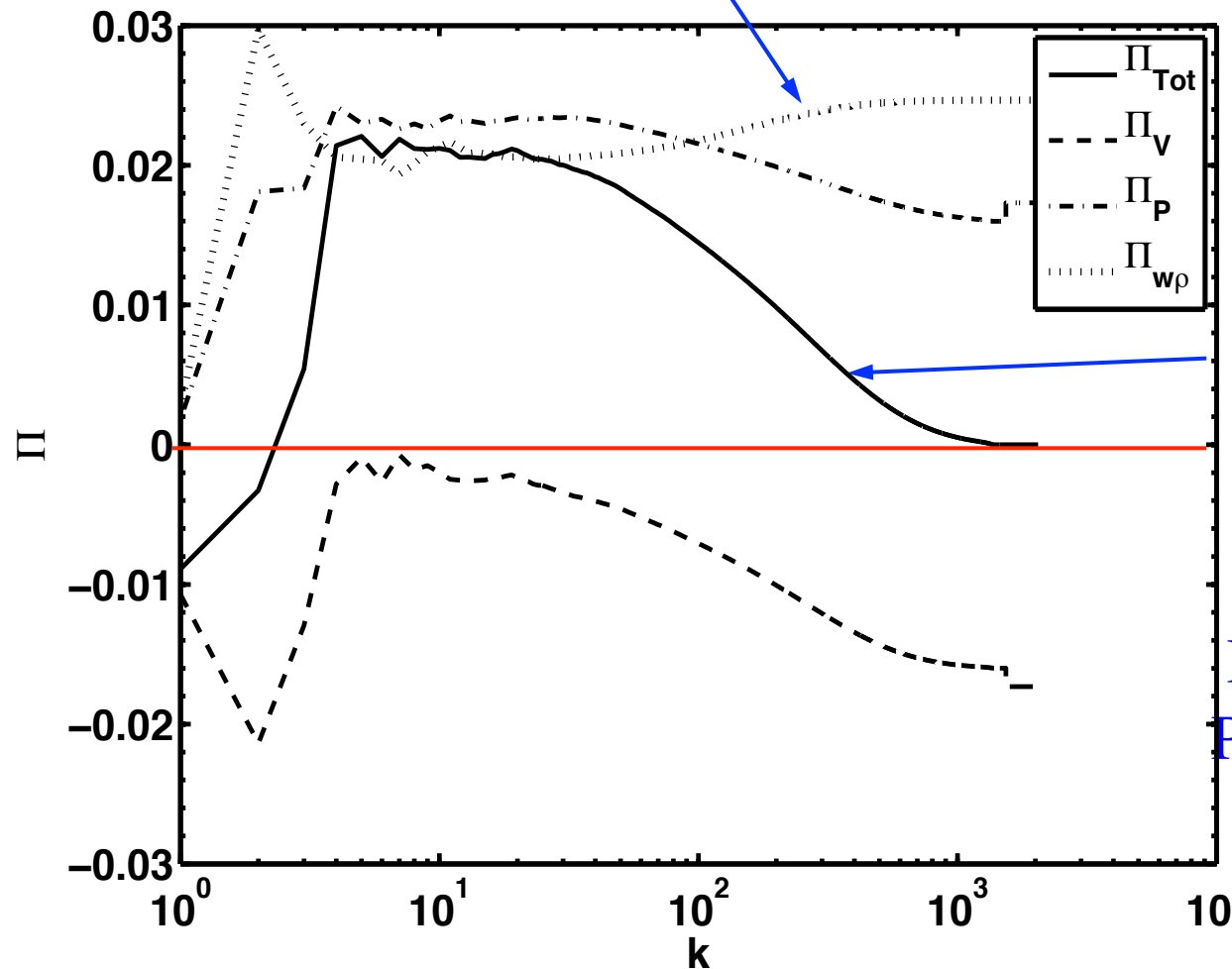
$$\Pi_{w\rho}(k) = \sum_{k'=0}^{k'=k} \sum_{k' < |\mathbf{k}''| < k'+1} \text{Re}(\hat{w}(\mathbf{k}'') \hat{\rho}(\mathbf{k}'')^*) ,$$

4096³ data, N/f=4.95

Fr=0.024, Ro=0.12

Re=55000, R_B=32

K₀=2.5, spin-down



Isotropic energy flux
and its components

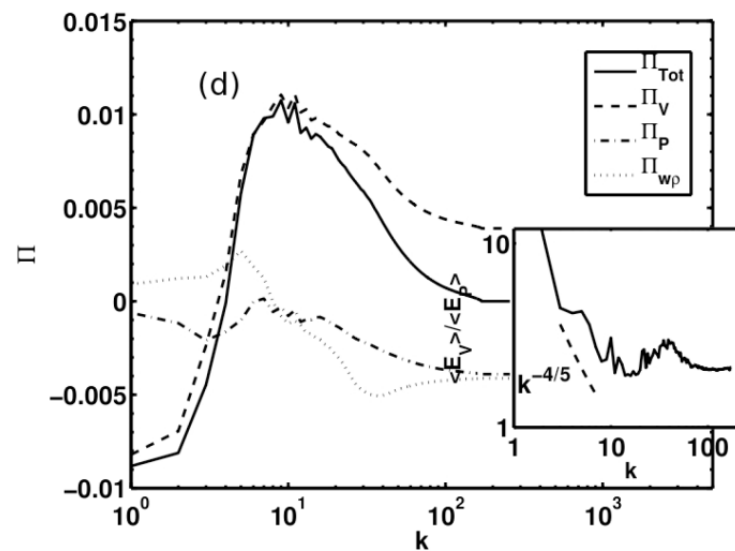
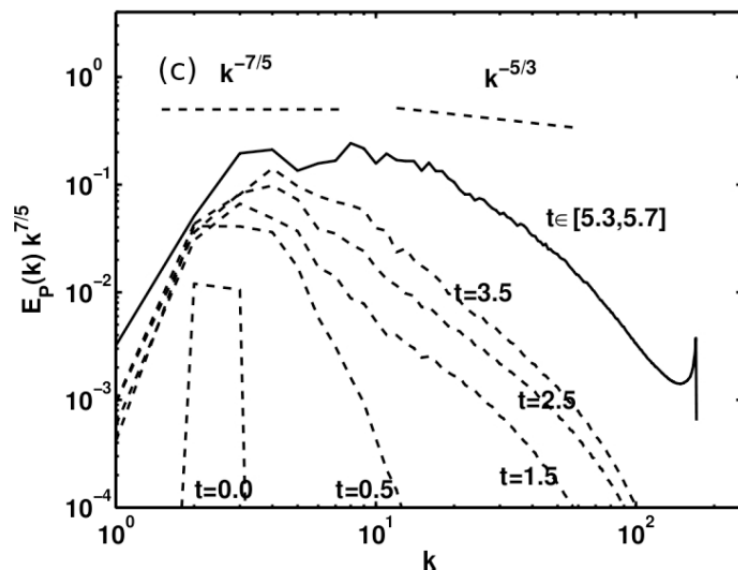
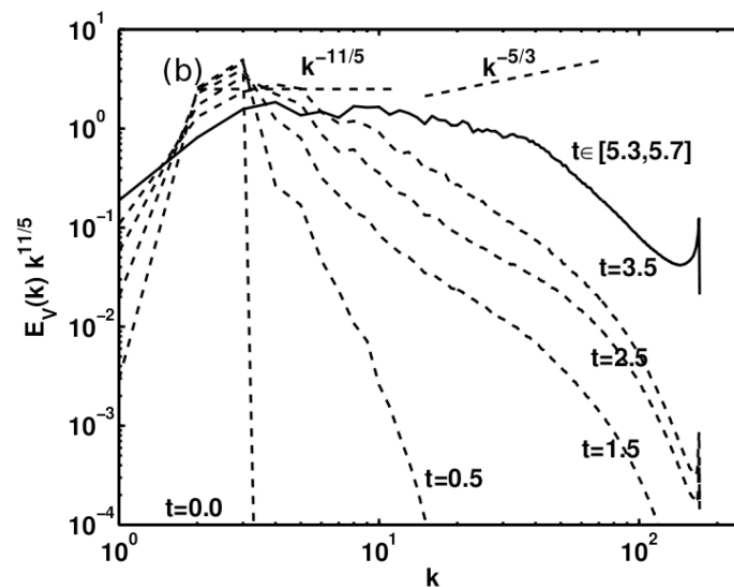
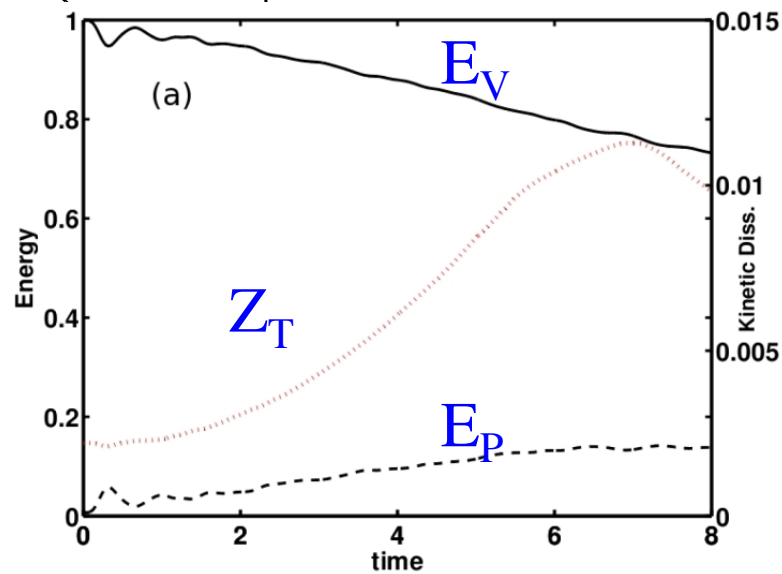
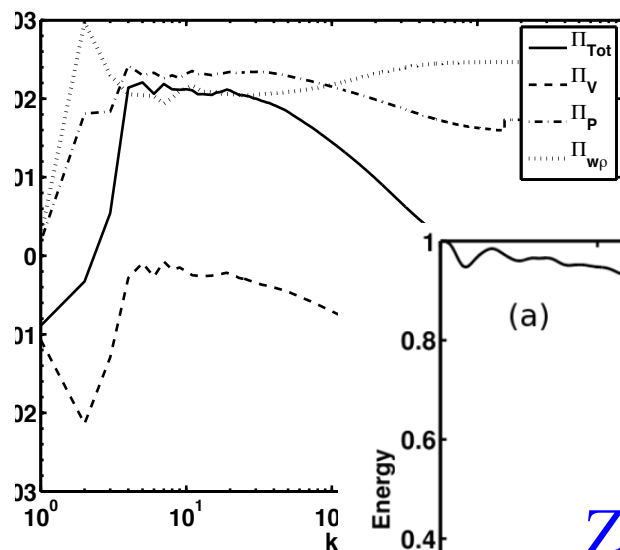
Total (—): $\Pi_T = \Pi_V + \Pi_P$

Kinetic (---): Π_V

Potential (-. -.): Π_P

Buoyancy flux:

Balanced initial conditions, 512^3 res.



Interplay of waves and eddies

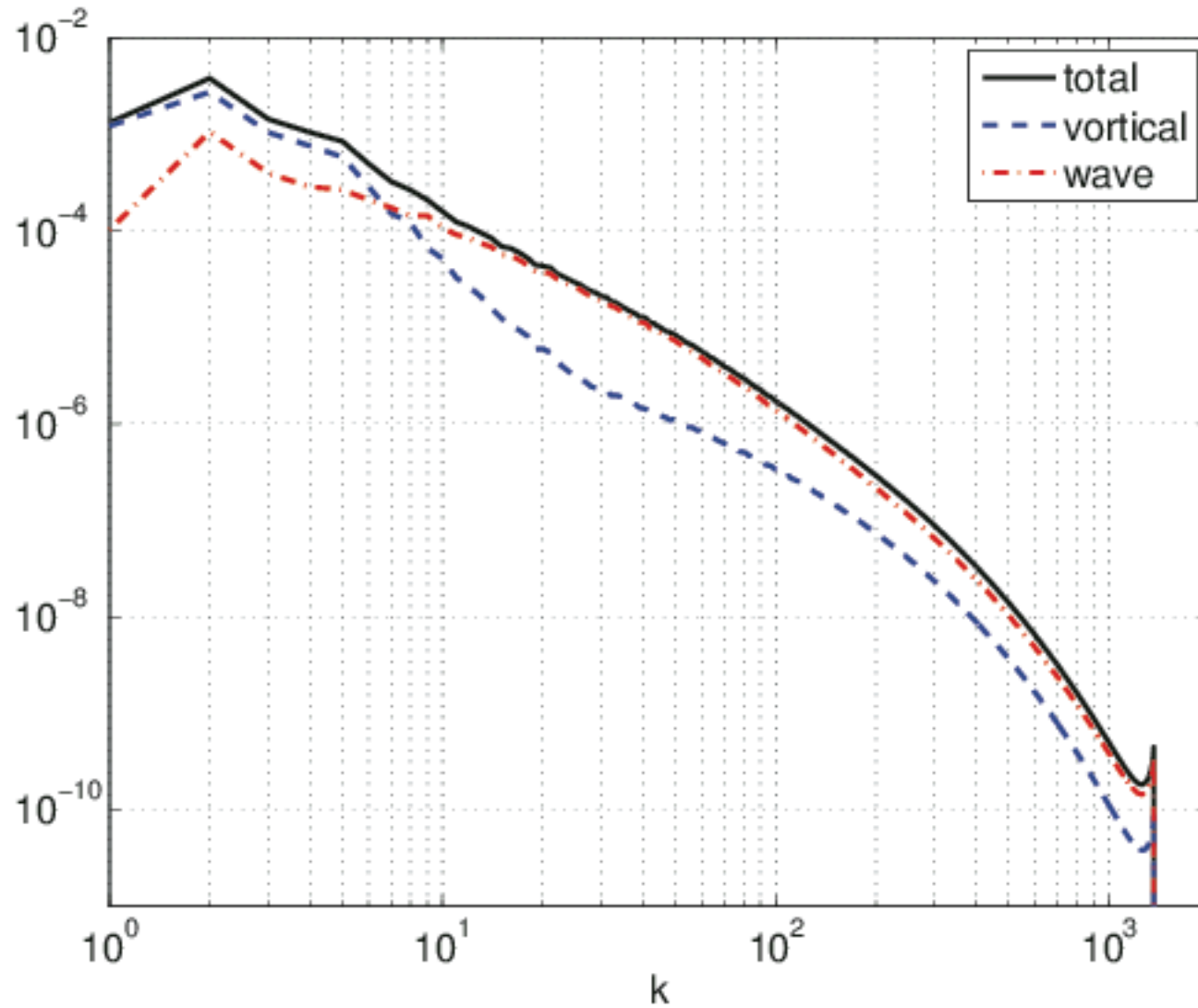
- Field $X(k) = [u_i(k), \theta(k)]$ ($i = 1, 3$) decomposition on 0 & wave (+/-) modes of frequency σ (Leith, 1980; Bartello, 1995; ... Use $\delta = \text{div}_H \mathbf{u}_H$ & ω_z):

$$X(k) = A_0(k)X_0(k) + A_+(k)X_+(k) + A_-(k)X_-(k)$$

*Linear (slow) dynamics is geo. & hydro. balanced, **no vertical velocity** and with all (linear) potential vorticity (since $\sigma \Pi_L = 0$, Smith Waleffe 2002)*

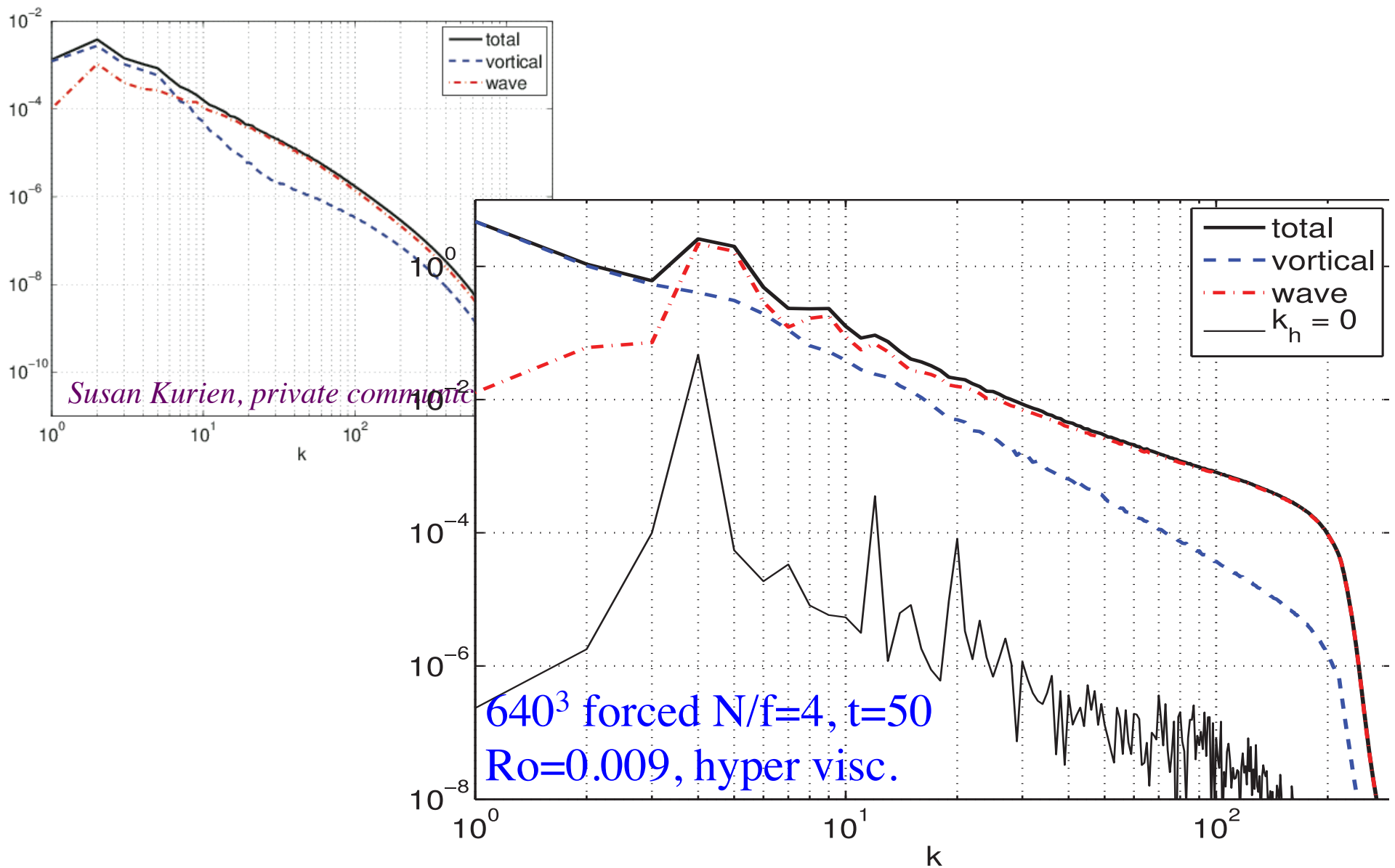
- $E_0 = \langle u_0^2 + \theta_0^2 \rangle / 2 = \sum_k |A_0(k)|^2 = \sum_k E_0(k)$
- $E_W = \langle u_w^2 + \theta_w^2 \rangle / 2 = \sum_k E_W(k) = \sum_k [|A_+(k)|^2 + |A_-(k)|^2] = \sum_k [E_+(k) + E_-(k)]$

4096 normal mode decomposition of total energy spectrum at peak $R_B=32$



Susan Kurien, private communication, 2014

Normal mode decomposition of the total energy spectrum at peak, $R_B=32$

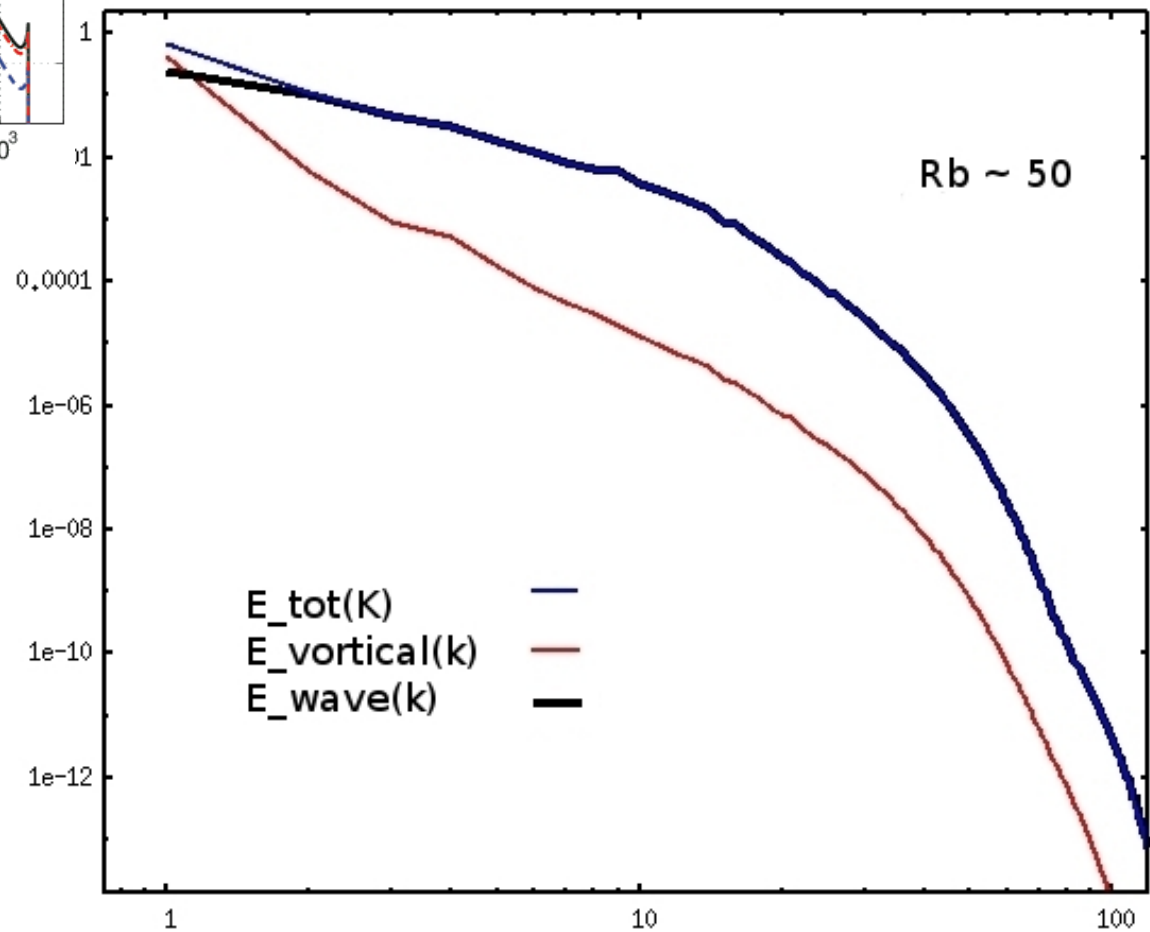
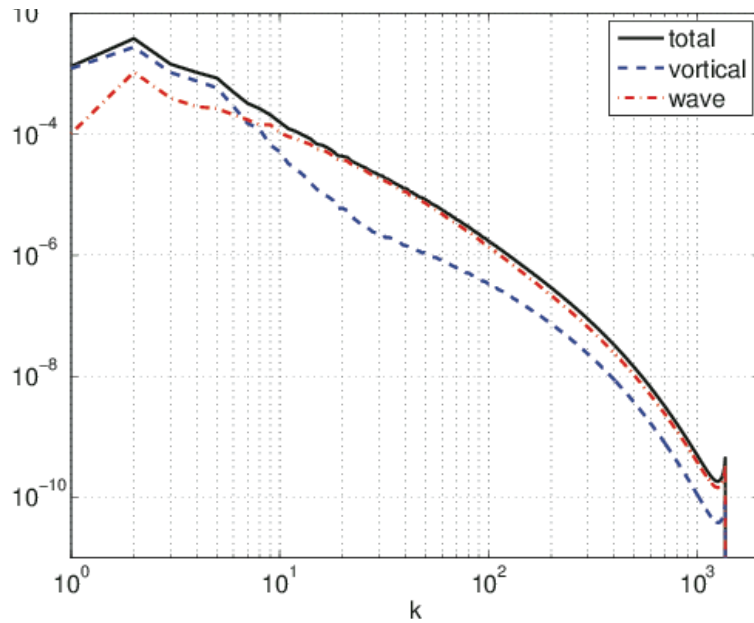


Normal mode decomposition 512³ runs

$Re \sim 10^4$

$Ro=0.038, N/f= \frac{1}{2}$

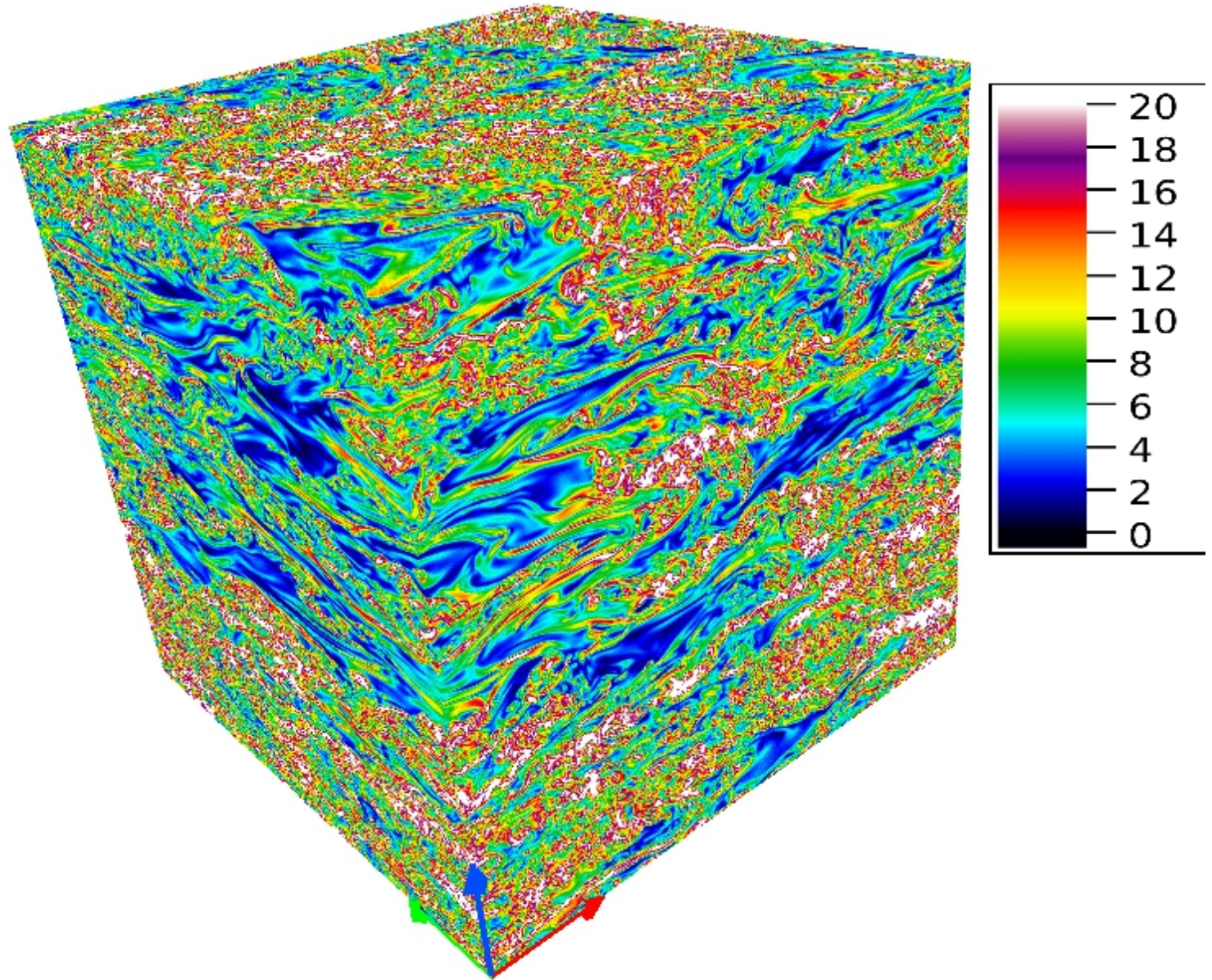
$Fr=0.076, R_B \sim 50$



Mesosphere Lower Thermosphere (MLT) run

$N/f=137$, $Fr = 0.067$, $Ro = 9.2$, $Re \approx 12000$ and $R_B = 53$, 1024^3 grid

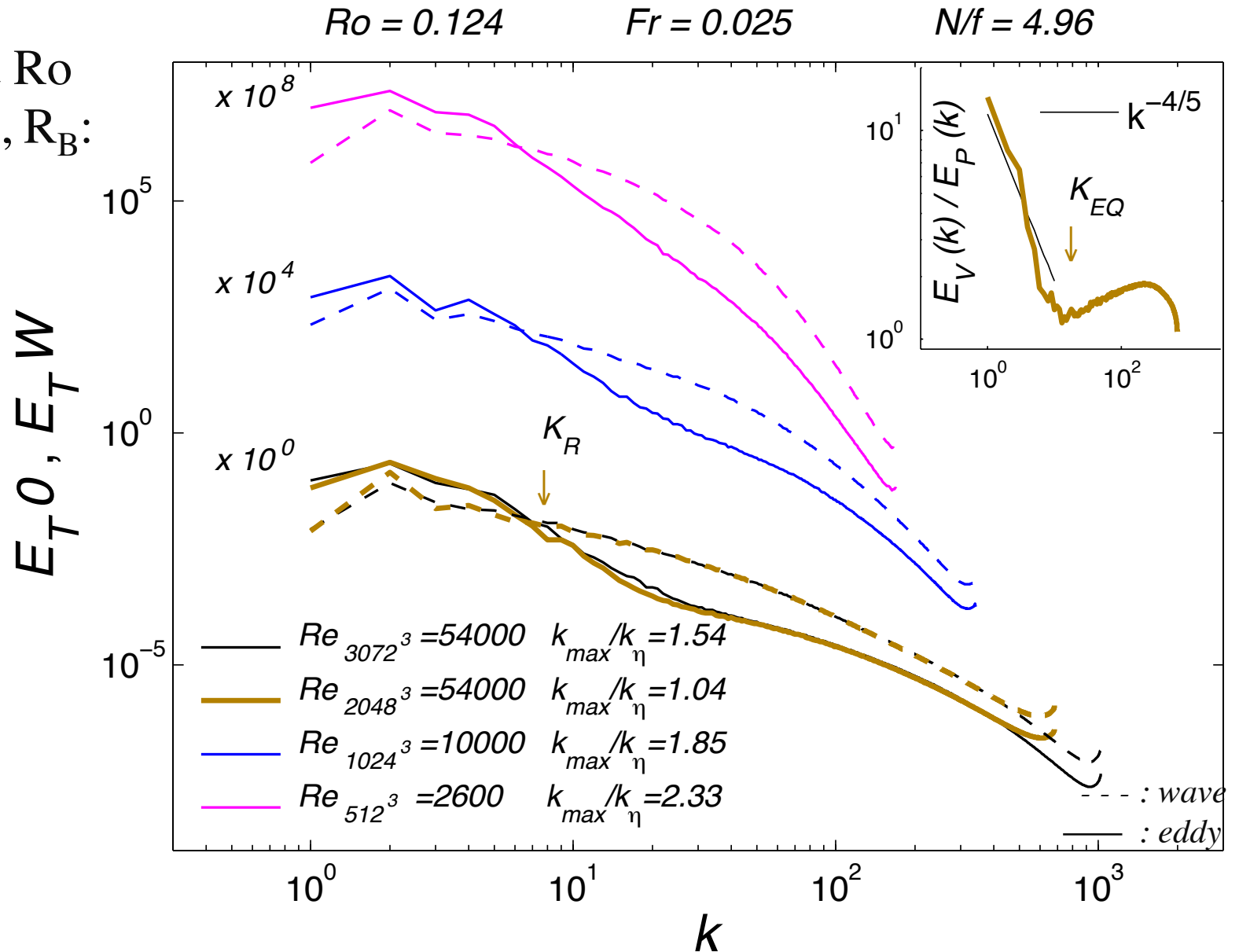
ω_{mag}



Interplay of waves and eddies

Same Fr and Ro
Different Re, R_B :
X 20

Change of
Regime
at K_R



1024³ res.

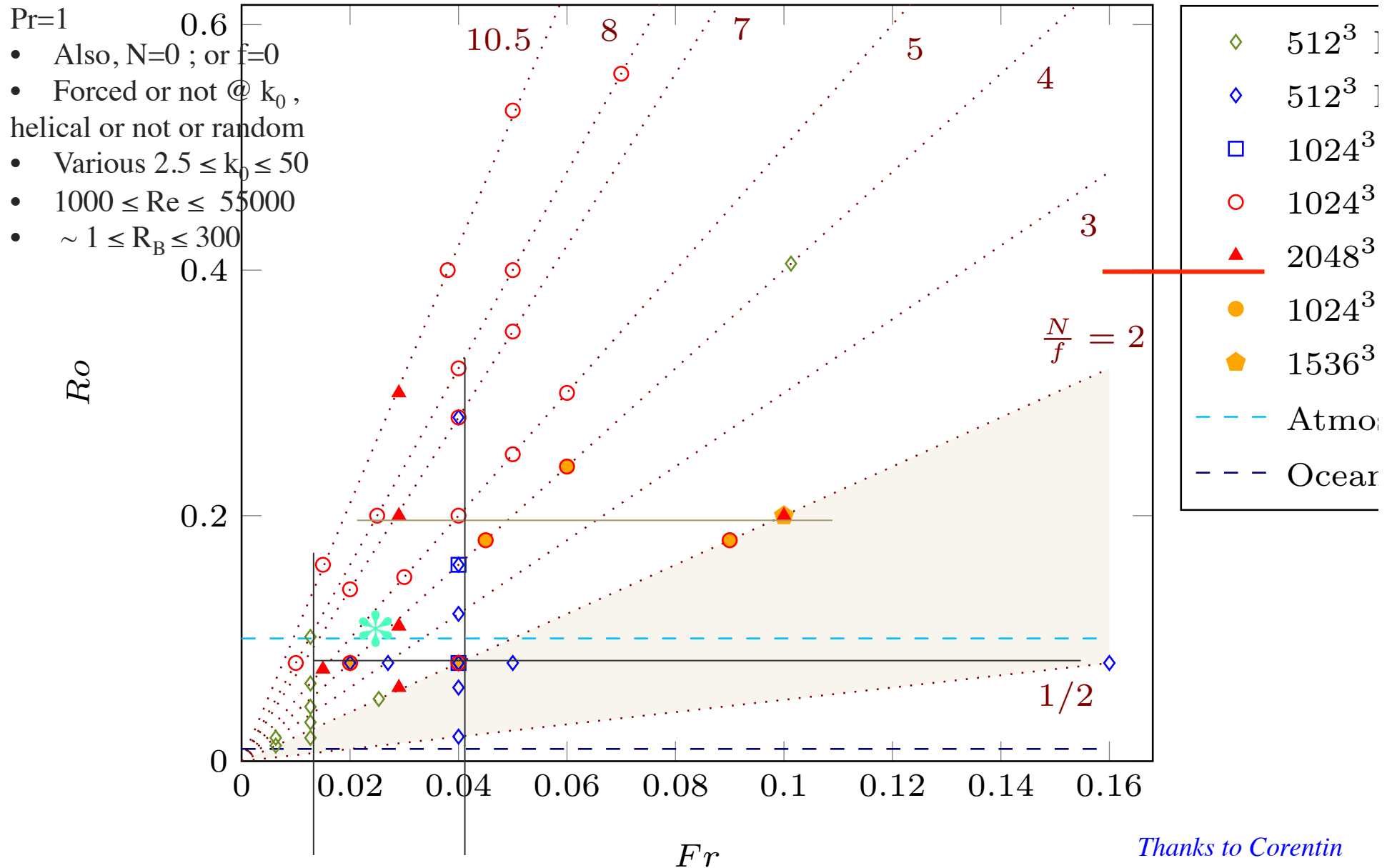
No forcing
 $k_0 \sim 2.5$

$\theta(t=0)=0$

Id	Fr	Ro	Ro/Fr	Re	\mathcal{R}_B	–	Id	Fr	Ro	Ro/Fr	Re	\mathcal{R}_B
1	0.00129	0.12952	100.15	10905	0.01824		34	0.08839	0.61873	7	8525.4	66.61
2	0.002307	0.11535	50	9895.3	0.05267		35	0.09211	4.6056	50	11016	93.47
3	0.006078	0.12038	19.805	10678	0.39452		36	0.09902	9.2829	93.75	7717.1	75.66
4	0.006421	0.63255	98.507	9270.5	0.38225		37	0.09911	6.8828	69.444	7717.9	75.81
5	0.007347	0.22532	30.667	13945	0.75279		38	0.09989	0.49448	4.9502	8200.5	81.83
6	0.01165	0.30505	26.182	14679	1.9927		39	0.10038	9.411	93.75	10747	108.3
7	0.01192	2.9804	250	13504	1.9192		40	0.1007	6.993	69.444	10754	109.05
8	0.01271	0.63538	50	8933.1	1.4425		41	0.10167	7.0606	69.444	16226	167.73
9	0.02098	0.14679	6.9963	11079	4.8773		42	0.13318	0.32923	2.472	7563.2	134.15
10	0.02150	0.46444	21.6	13449	6.2177		43	0.13614	0.67308	4.944	7603.2	140.92
11	0.02253	0.11157	4.95	10977	5.57		44	0.14014	0.98247	7.0106	7442.2	146.16
12	0.02618	4.5819	175	12044	8.26		45	0.14222	1.4063	9.8881	7327.7	148.22
13	0.02800	0.14002	5	10722	8.41		46	0.16318	9.791	60	9576.1	255
14	0.03015	9.4215	312.5	10522	9.56		47	0.18841	37.481	198.94	2520.7	89.48
15	0.03332	4.5814	137.5	13020	14.45		48	0.20152	5.0381	25	8718.2	354.06
16	0.03646	9.1153	250	13217	17.57		49	0.20159	10.08	50	8756.8	355.87
17	0.03774	0.13994	3.708	10534	15.00		50	0.2059	40.96	198.94	6272.1	265.89
18	0.04094	0.60719	14.832	9842.9	16.50		51	0.25825	10.33	40	8575.7	571.95
19	0.04221	0.21106	5	14841	26.44		52	0.34081	0.84249	2.472	5024.4	583.59
20	0.04527	3.0177	66.67	12790	26.21		53	0.37971	11.391	30	7116.6	1026.1
21	0.04737	4.4406	93.75	8881.9	19.93		54	0.3966	42.192	106.38	1983.7	312.03
22	0.04803	4.5028	93.75	12366	28.53		55	0.46877	1.1719	2.5	4500.4	988.95
23	0.04870	4.5657	93.75	18586	44.08		56	0.55387	11.077	20	7138.8	2190
24	0.04897	9.1824	187.5	18550	44.49		57	0.61542	1.5386	2.5	5012.1	1898.3
25	0.04940	9.2627	187.5	12769	31.16		58	0.89372	2.2343	2.5	4707.8	3760.3
26	0.05656	0.28278	5	13730	43.92		59	1.2497	12.497	10	6473.2	10110
27	0.05679	0.1404	2.472	9747.6	31.44		60	2.6918	13.459	5	4019.3	29123
28	0.060595	3.0297	50	11649	42.77		61	5.4829	13.707	2.5	4257.7	1.28×10^5
29	0.062114	0.61418	9.8881	9639.4	37.19		–					
30	0.066925	920.23	13750	11486	51.45		62	0.01212	Inf	Inf	15225	2.24
31	0.06715	9.2332	137.5	11728	52.88		63	0.02678	Inf	Inf	11804	8.46
32	0.073074	3.0448	41.667	12211	65.20		64	0.06692	Inf	Inf	11487	51.44
33	0.08608	0.4304	5	12111	89.74		65	0.20147	Inf	Inf	8796.7	357.07

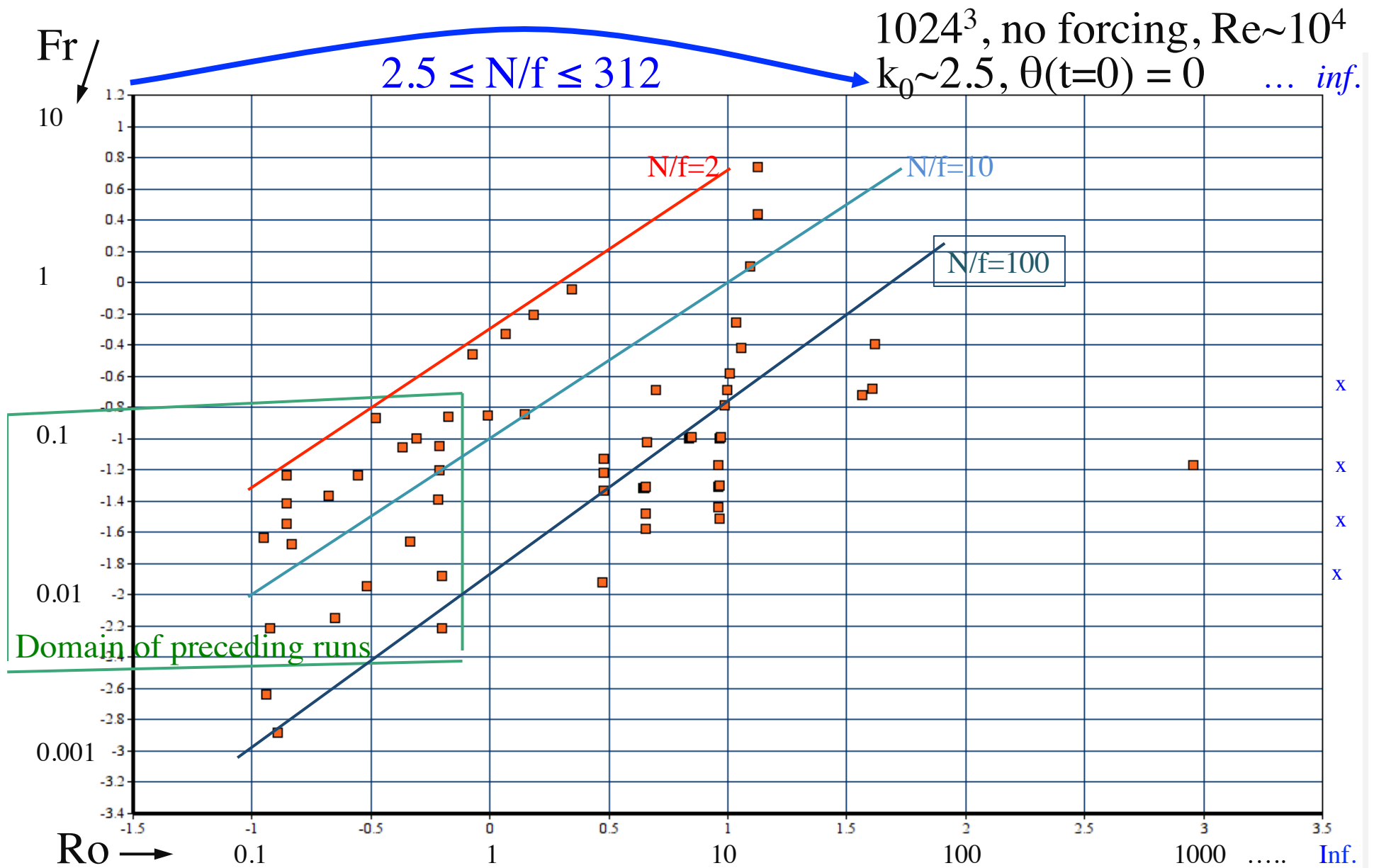
Id	Fr	\mathcal{R}_ω	N/f	Re	k_η/k_M	ϵ_D	r_ϵ	L_{int}	L_B	ℓ_{Oz}	ℓ_{Ze}	Id	Fr	\mathcal{R}_ω	N/f	Re	k_η/k_M	ϵ_D	r_ϵ	L_{int}	L_B	ℓ_{Oz}	ℓ_{Ze}
1	.0013	13.5	100	10905	.53	.27	.018	2.57	.02	.0001*	.100	34	.0884	57.1	7	8525	.73	.20	.09	2.32	1.29	.115	2.14
2	.0023	11.5	50	9895	.50	.20	.02	2.60	.038	.00025*	.090	35	.0921	483	50	11016	.96	.22	.086	2.12	1.22	.109	38.5*
3	.0061	12.4	20	10678	.49	.23	.015	2.64	.10	.00097*	.086	36	.099	815	94	7717	.74	.22	.088	2.09	1.30	.122	110.*
4	.0064	60.9	98	9270	.52	.22	.02	2.40	.097	.001*	1.08	37	.0991	605	69	7718	.75	.22	.088	2.09	1.30	.121	70.*
5	.0073	26.6	31	13945	.64	.23	.035	2.49	.115	.0012*	.206	38	.0999	44.8	4.95	8200	.75	.20	.10	2.28	1.429	.142	1.57
6	.0116	36.9	26	14679	.68	.25	.012	2.56	.187	.002*	.30	39	.1004	976	94	10747	.98	.23	.091	2.07	1.30	.124	113.*
7	.0119	346.	250	13504	.68	.26	.018	2.38	.178	.0026*	10.4*	40	.1007	725	69	10754	.98	.23	.090	2.07	1.31	.124	72.*
8	.0127	60.	50	8933	.53	.22	.023	2.35	.187	.003*	1.126	41	.1017	899	69	16226	1.33*	.236	.090	2.06	1.32	.126	73.*
9	.0210	15.4	7	11079	.53	.23	.021	2.72	.358	.007*	.140	42	.1332	28.6	2.5	7563	.76	.198	.108	2.12	1.77	.213	.828
10	.0215	53.9	22	13449	.69	.24	.02	2.41	.325	.007*	.680	43	.1361	58.7	4.95	7603	.79	.21	.12	2.10	1.80	.231	2.53
11	.0225	11.7	4.95	10977	.54	.204	.026	2.78	.39	.01*	.106	44	.1401	84.8	7	7442	.80	.21	.12	2.05	1.81	.235	4.35
12	.0262	503	175	12044	.73	.243	.026	2.22	.36	.01*	22.3*	45	.1422	120.3	10	7328	.80	.217	.12	2.02	1.81	.236	7.39*
13	.0280	14.5	4.95	10722	.56	.020	.031	2.74	.482	.014*	.158	46	.1632	958	60	9576	1.07*	.22	.136	1.91	1.96	.292	136.*
14	.0301	966	312	10522	.74	.22	.03	2.05	.388	.012*	64.5*	47	.1884	1882	199	2521	.39	.17	.148	1.98	2.35	.392	1100*
15	.0333	523	137	13020	.77	.26	.025	2.31	.534	.014*	22.6*	48	.2015	470	25	8718	1.09*	.21	.153	1.80	2.28	.40	50.*
16	.0365	1048	250	13217	.77	.26	.028	2.33	.53	.017*	67.1*	49	.2016	943	50	8757	.84	.21	.154	1.80	2.29	.40	141.*
17	.0377	14.4	3.7	10534	.59	.20	.039	2.71	.64	.025	.176	50	.2059	3244	199	6272	.84	.19	.163	1.85	2.39	.438	1229*
18	.0409	60.	15	9843	.61	.22	.040	2.52	.648	.026	1.51	51	.2582	957	40	8576	1.13*	.22	.169	1.76	2.86	.598	151*
19	.0422	25.7	4.95	14841	.78	.22	.038	2.65	.703	.028	.315	52	.3408	59.7	2.5	5024	.87	.21	.171	1.53	3.27	.791	3.07
20	.0453	341	67	12790	.81	.25	.038	2.30	.654	.027	14.8*	53	.3797	961	30	7117	1.16*	.22	.187	1.53	3.65	.973	160.*
21	.0474	418	94	8882	.63	.24	.042	2.29	.682	.030	27.7*	54	.3966	1879	106	1984	.40	.17	.176	1.66	4.13	1.09	1199*
22	.0480	501	94	12366	.82	.24	.042	2.27	.685	.031	28.1*	55	.4688	78.6	2.5	4500	.89	.207	.186	1.42	4.18	1.24	4.89
23	.0487	622	94	18586	1.11*	.24	.042	2.26	.690	.031	28.2*	56	.5539	936	20	7139	1.17*	.21	.203	1.55	5.41	1.82	162.*
24	.0490	1251	187	18550	1.11*	.25	.042	2.25	.691	.031	80.2*	57	.6154	109	2.5	5012	.88	.179	.215	1.60	6.19	2.25	8.88*
25	.0494	1047	187	12769	1.26*	.26	.038	2.27	.706	.030	77.5*	58	.8937	153	2.5	4708	.89	.197	.203	1.49	8.35*	3.56	14.1*
26	.0566	33.1	4.95	13730	.85	.20	.059	2.57	.914	.053	.589	59	1.25	1005	10	6473	1.21*	.243	.199	1.39	10.9*	5.46	173.*
27	.0568	13.9	2.5	9748	.63	.185	.052	2.61	.930	.051	.197	60	2.69	853	4.95	4019	.81	.31	.186	1.42	24.0*	17.0*	190.*
28	.0606	327	50	11649	.86	.23	.054	2.19	.835	.047	16.8*	61	5.48	894	2.5	4258	.84	.34	.191	1.45	50.*	51.*	202.*
29	.0621	60.3	9.9	9639	.67	.22	.057	2.48	.968	.057	1.78												
30	.0669	98623	13750	11486	.88	.23	.059	2.16	.910	.057	91863	62	0.012	∞	∞	15225	.69	.278	.012	2.56	.195	.002*	∞
31	.0671	1000	137	11728	.88	.24	.057	2.18	.921	.057	91.7*	63	.0268	∞	∞	11804	.73	.264	.026	2.17	.366	.01*	∞
32	.0731	336	42	12211	.90	.24	.062	2.24	1.03	.069	18.6*	64	.0669	∞	∞	11487	.88	.23	.059	2.16	.910	.057	∞
33	.0861	47.4	4.95	12111	.94	.19	.094	2.37	1.283	.116	1.294	65	.2015	∞	∞	8797	1.09*	.21	.153	1.81	2.29	.40	∞

Present rot-strat data *(Mostly Marino -ENS, & Rosenberg, SciTex)*



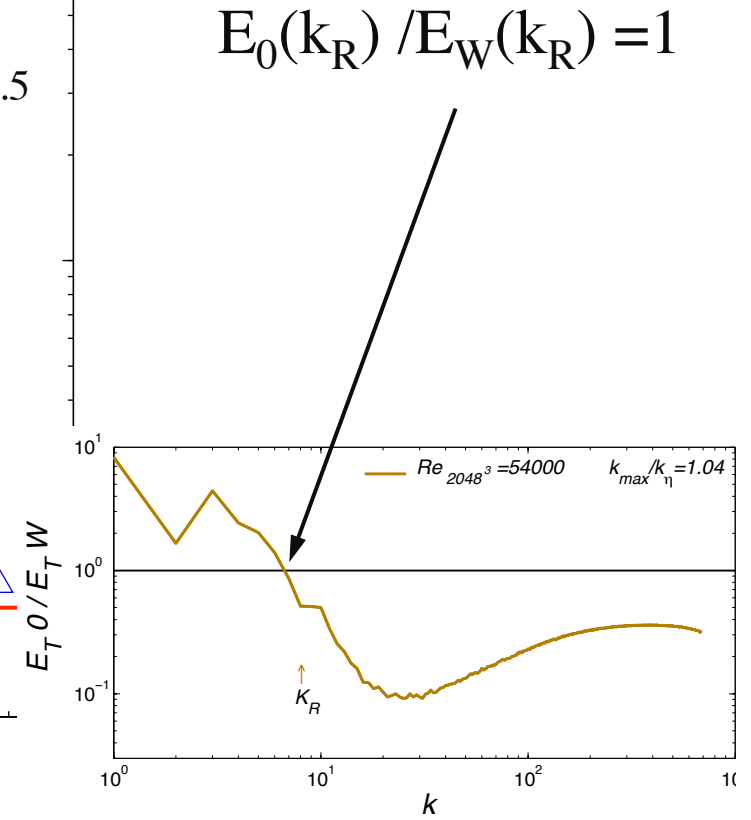
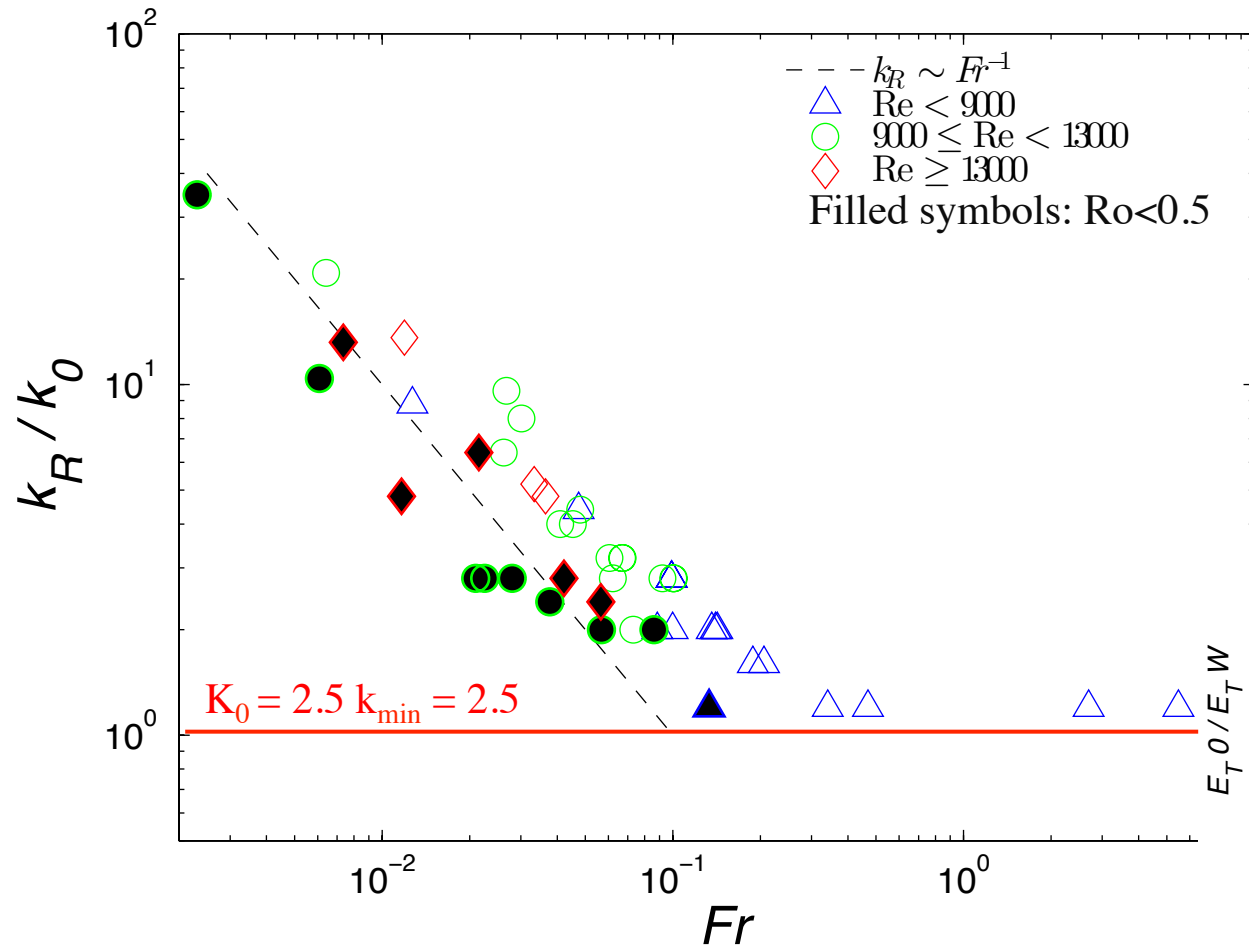
Thanks to Corentin

CPU: NSF (XSEDE & Yellowstone/NCAR); DOE



Thanks to John

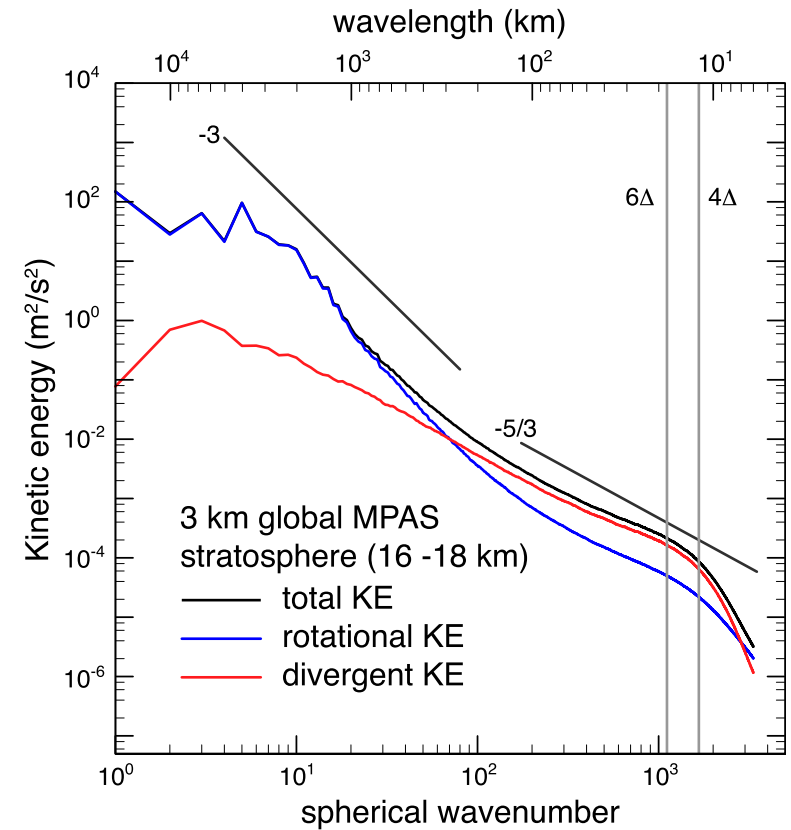
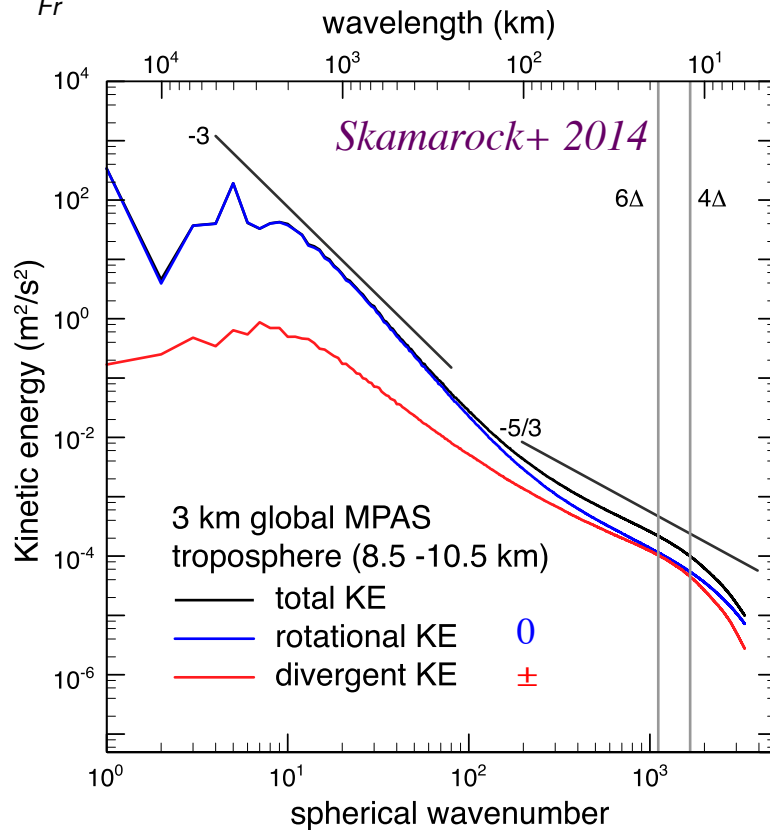
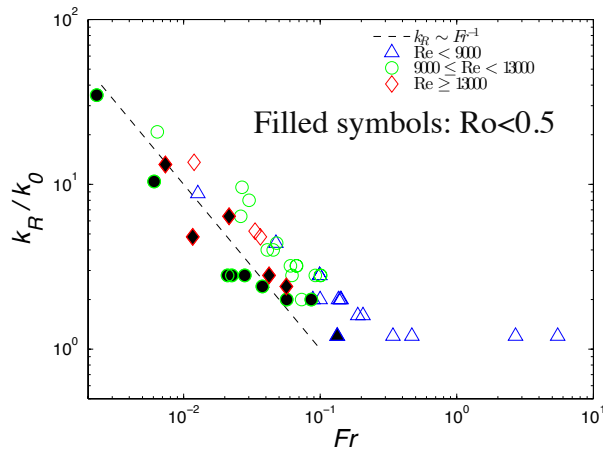
Interplay of waves and eddies



Is $L_R = 2\pi/K_R \sim 1/N \sim L_B = U_0/N$?

Interplay of waves and eddies

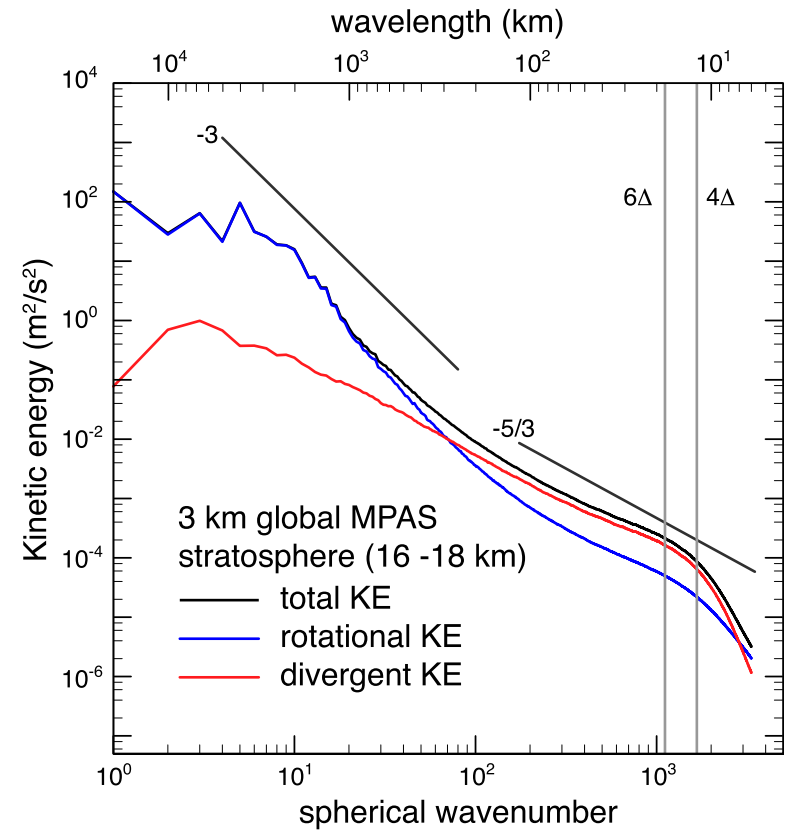
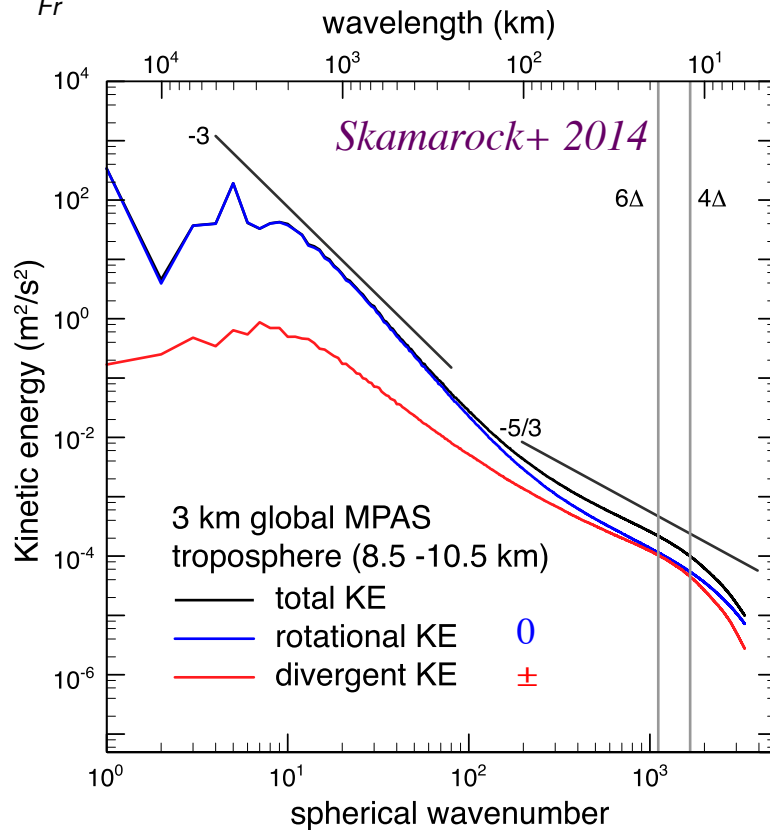
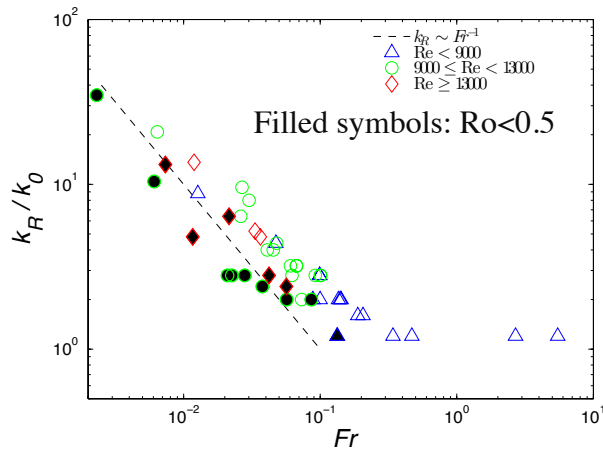
This result of Skamarock+ is compatible with N decreasing with altitude



Interplay of waves and eddies

This result of Skamarock+ is compatible with N decreasing with altitude

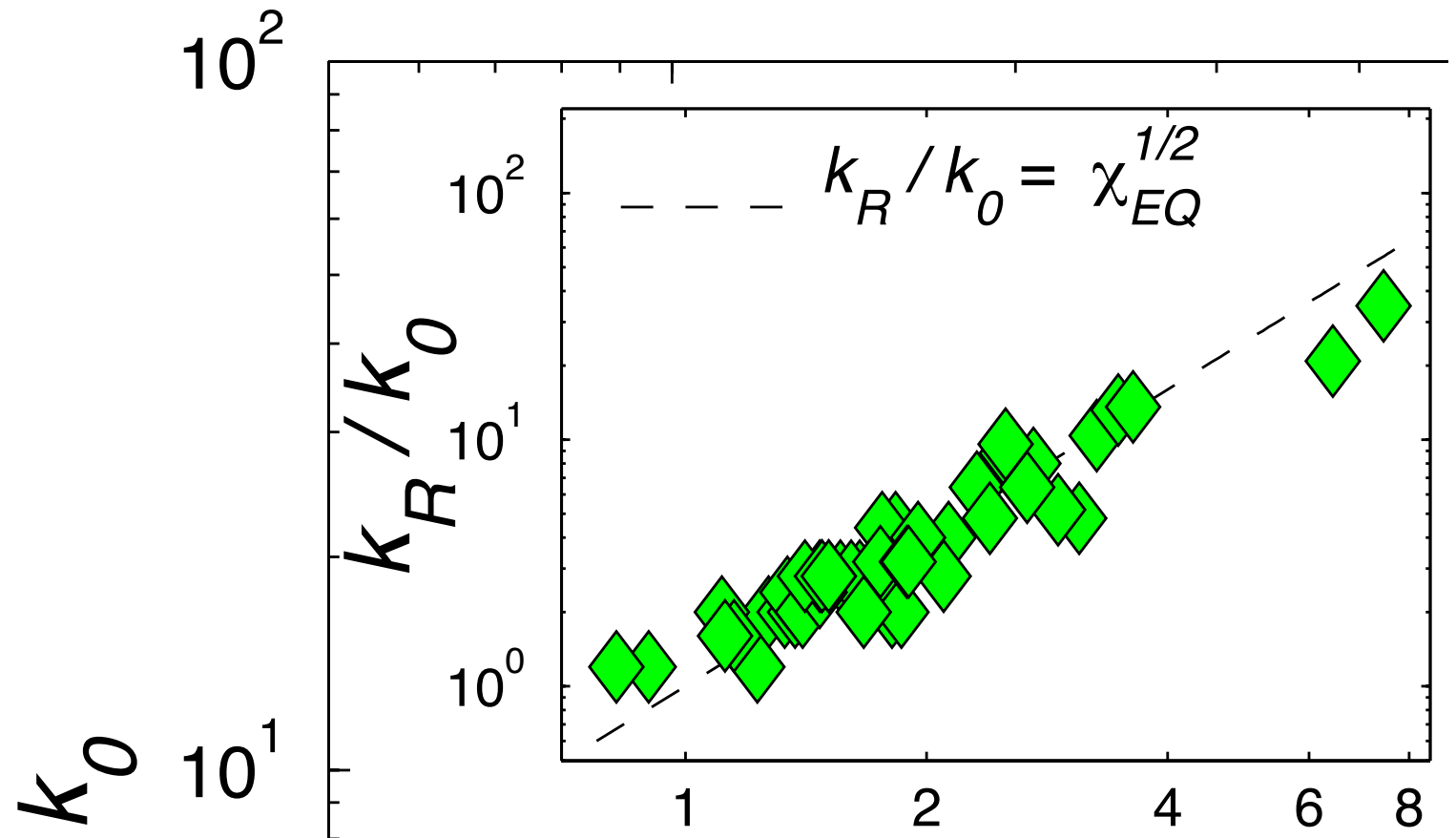
Consequences for modeling?



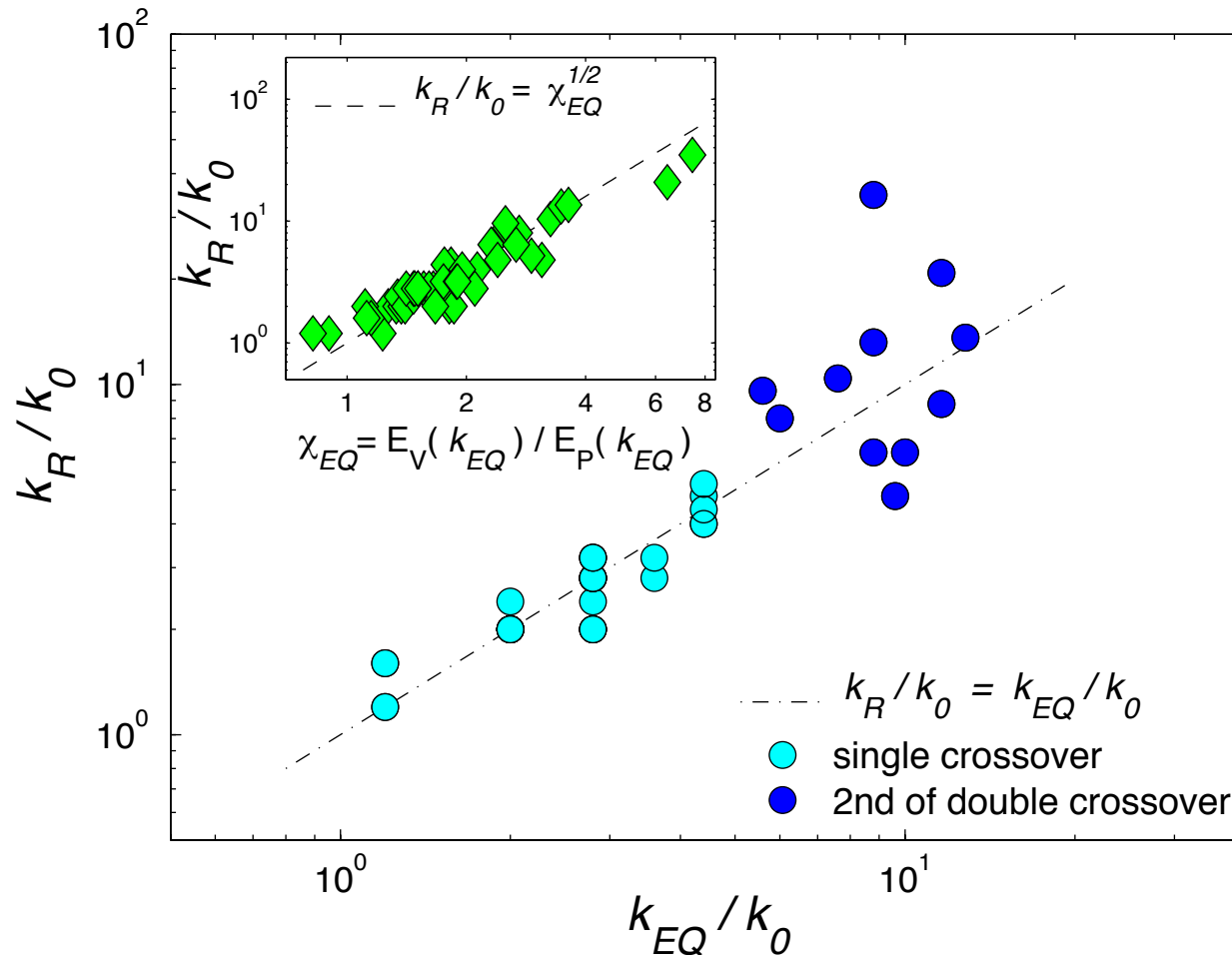
Interplay of waves and eddies

$$\chi_{EQ} = E_V(k=k_{eq}) / E_P(k=k_{eq})$$

$$\text{Data} \rightarrow K_R/k_0 \sim u_{rms}/\theta_{rms} \\ \sim 1/Fr$$



Interplay of waves and eddies



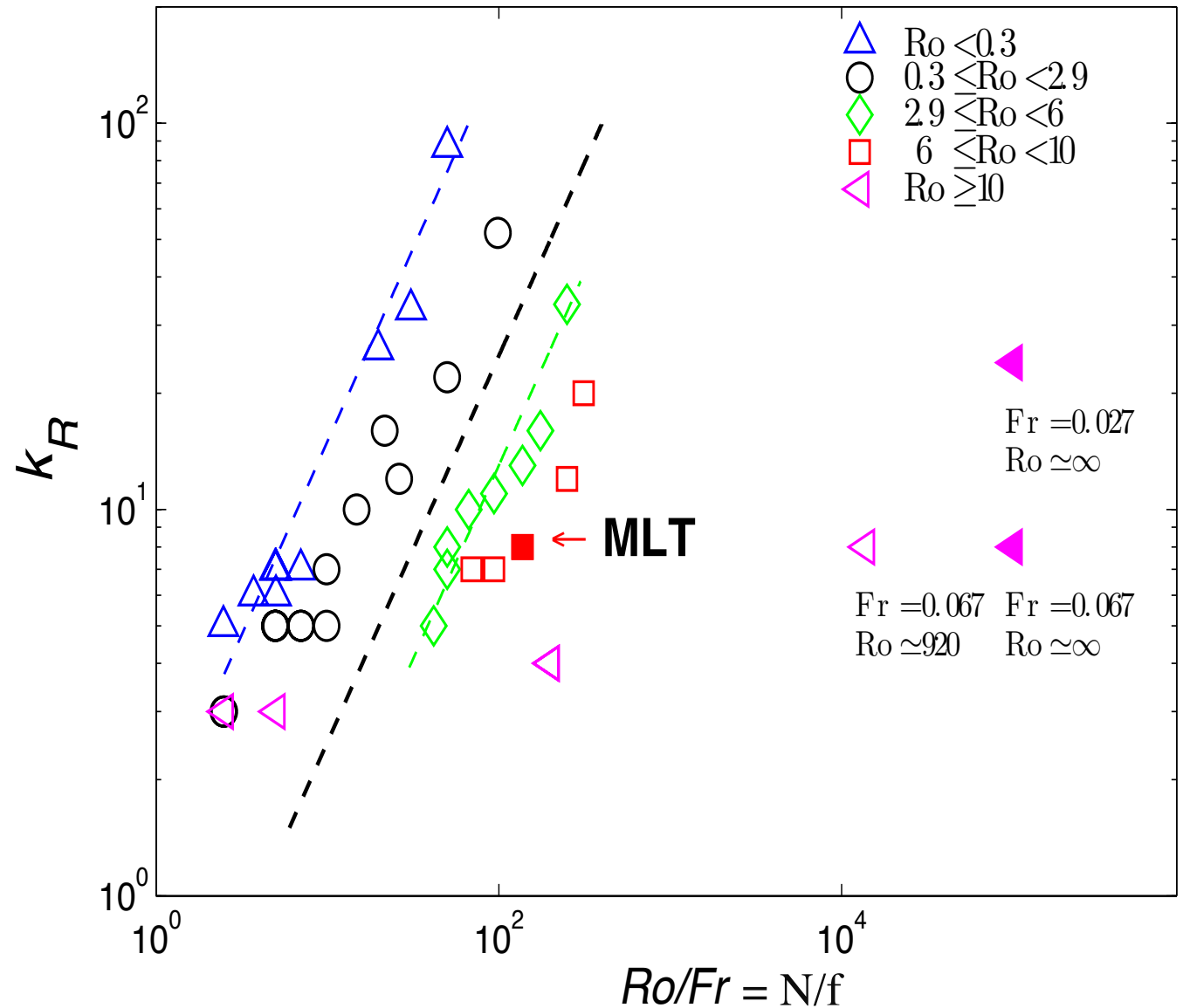
$$K_R/k_0 \sim u_{\text{rms}}/\theta_{\text{rms}} \\ \sim 1/\text{Fr}$$

K_R : change in wave/vortex dominance

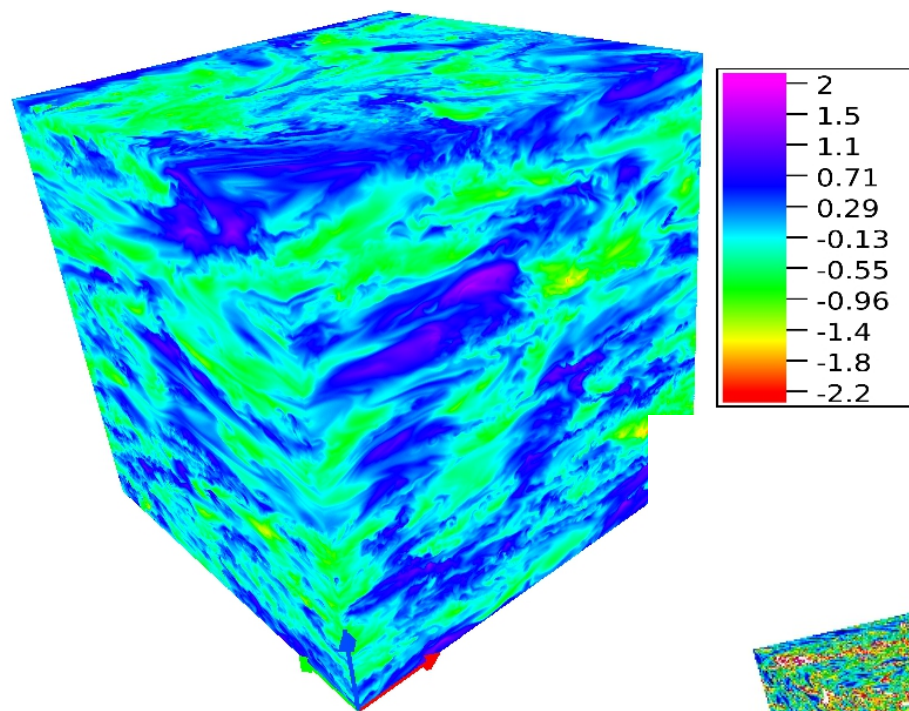
$\sim K_{EQ}$: wvnb at which minimum of $E_V(k)/E_P(k)$, χ_{EQ} , is reached

Interplay of waves and eddies

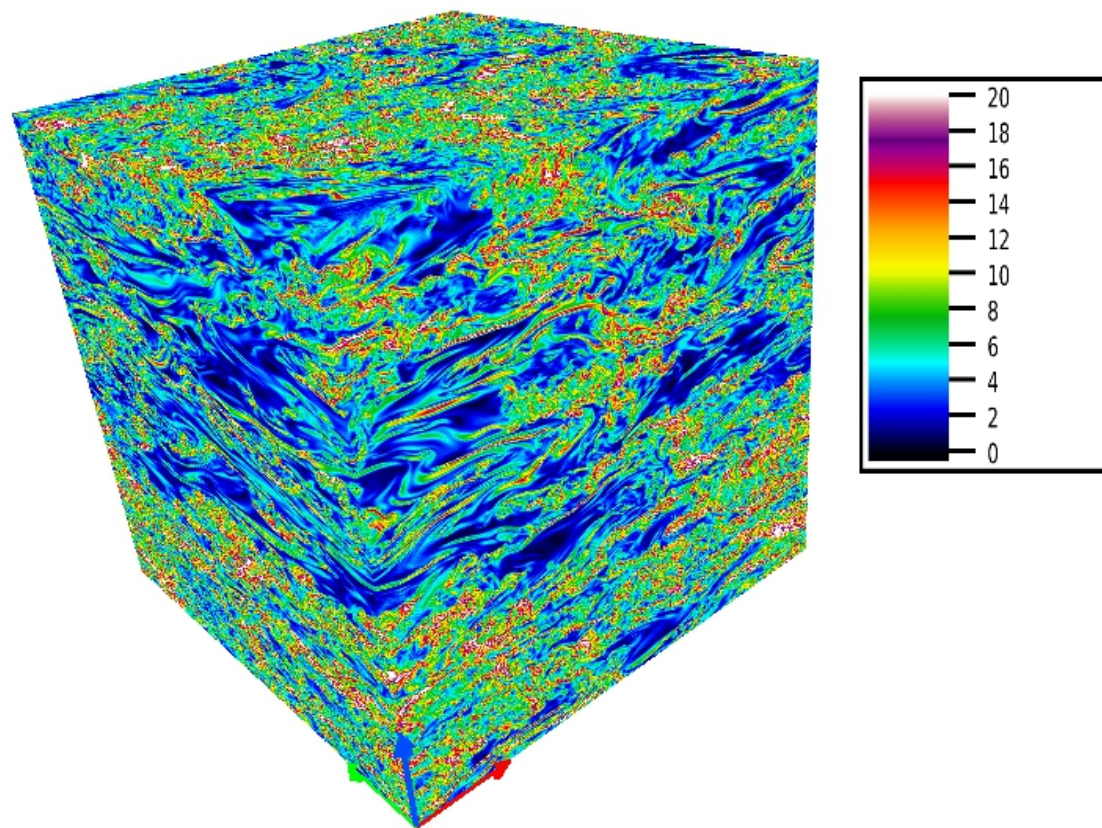
$$K_R \sim a(Ro)/Fr$$

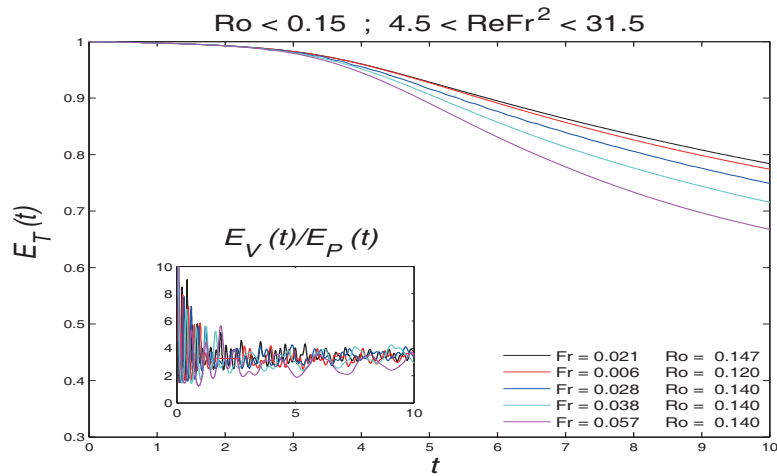


Energy pathway to dissipation and mixing

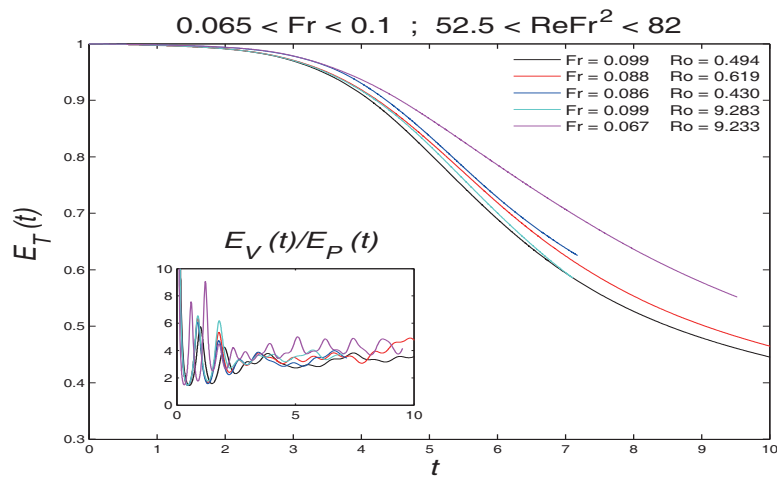


← Temperature and its gradient
 $N/f=137$, $R_B=53$

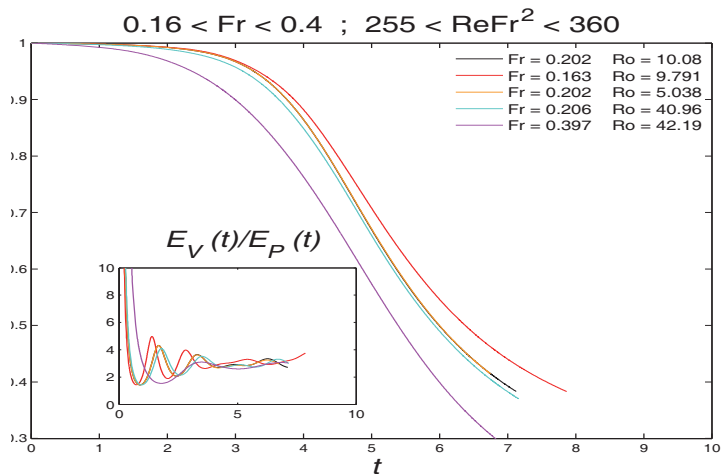


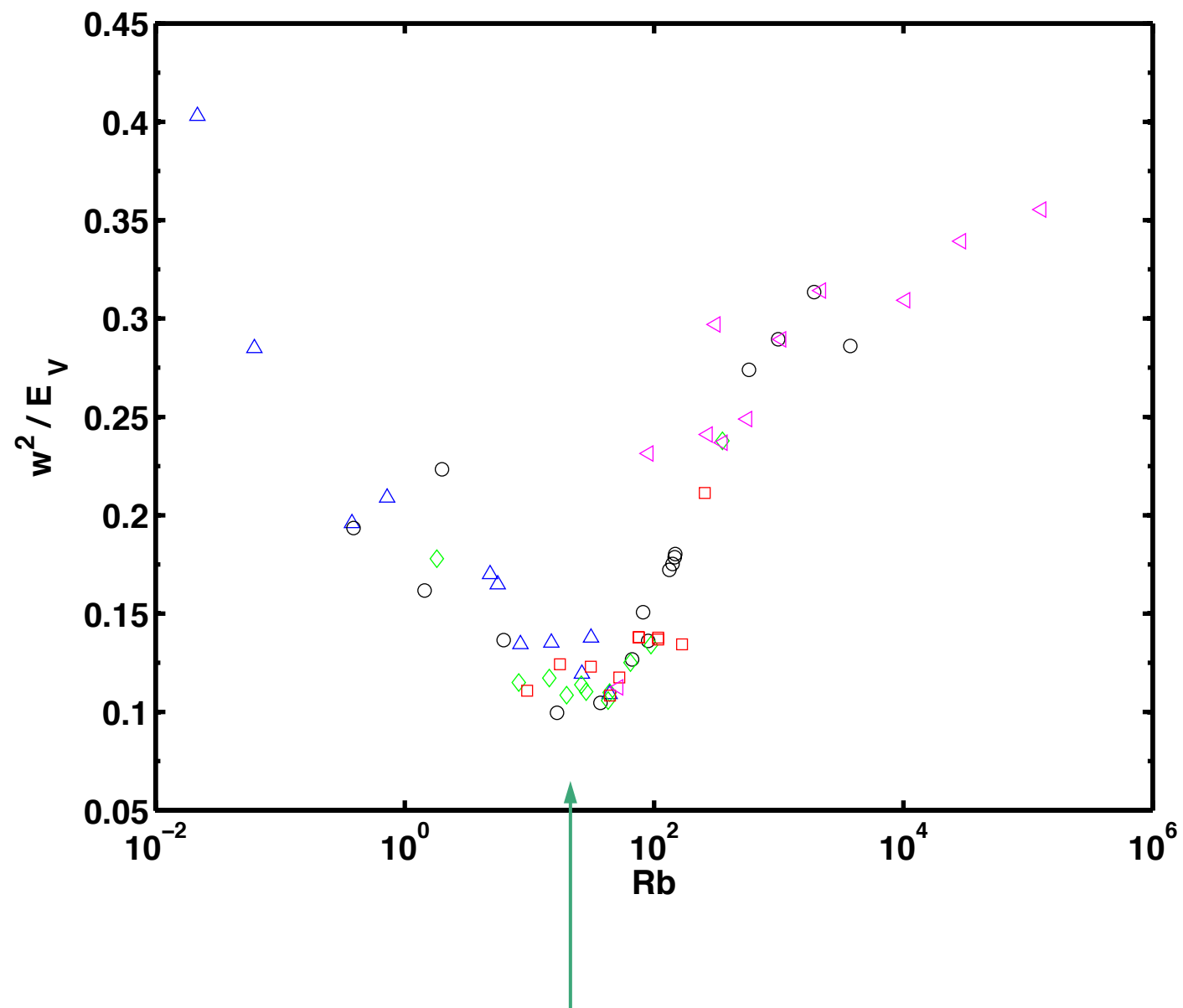


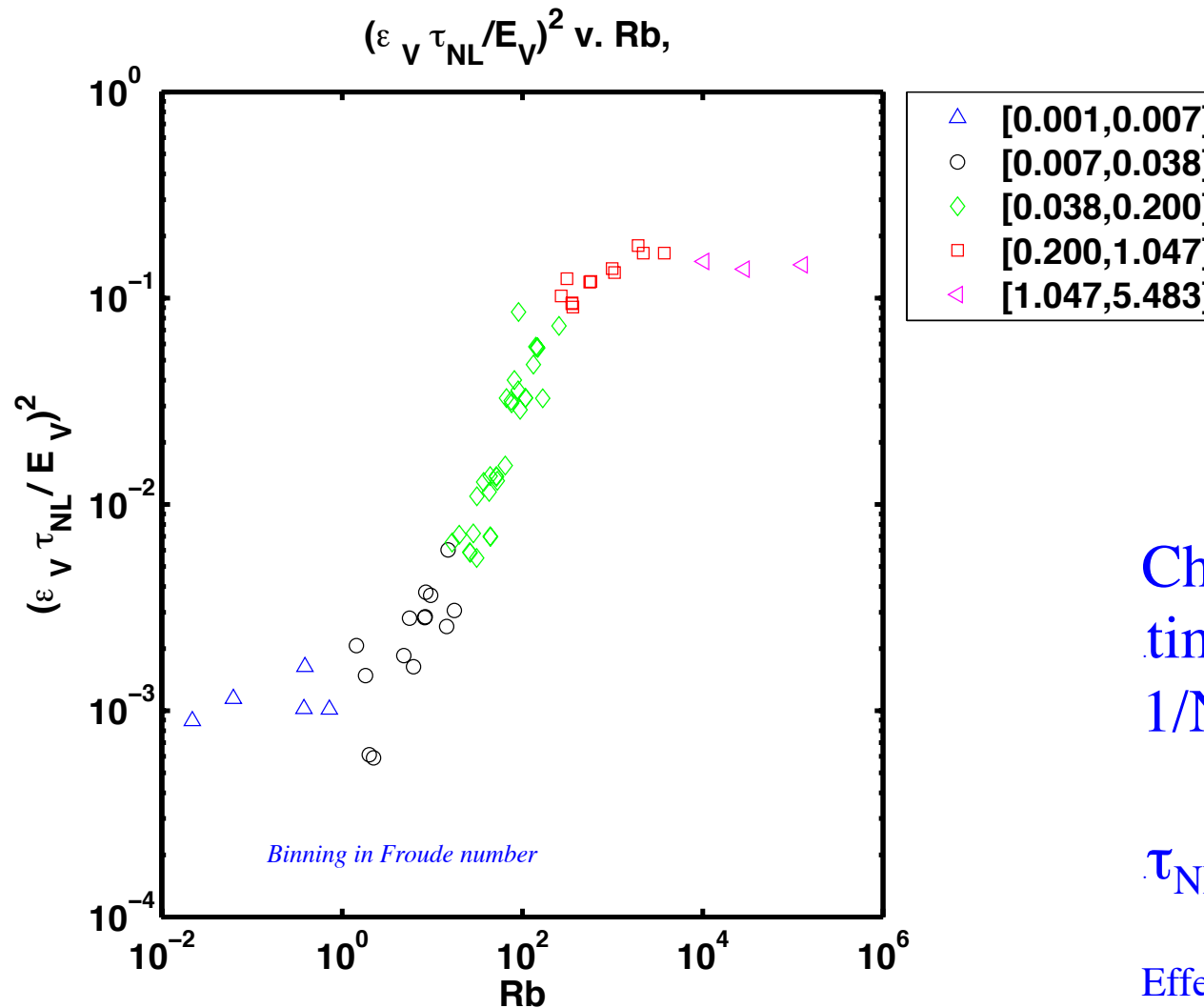
- The kinetic to potential energy ratio ~ 3 , is insensitive to parameters



- Peak of dissipation occurs at approx. the same time except at very low Fr
- The rate at which energy dissipates increases with R_B







Characteristic
times:
 $1/N$, $1/f$

$$\tau_{NL} = L_0/U_0$$

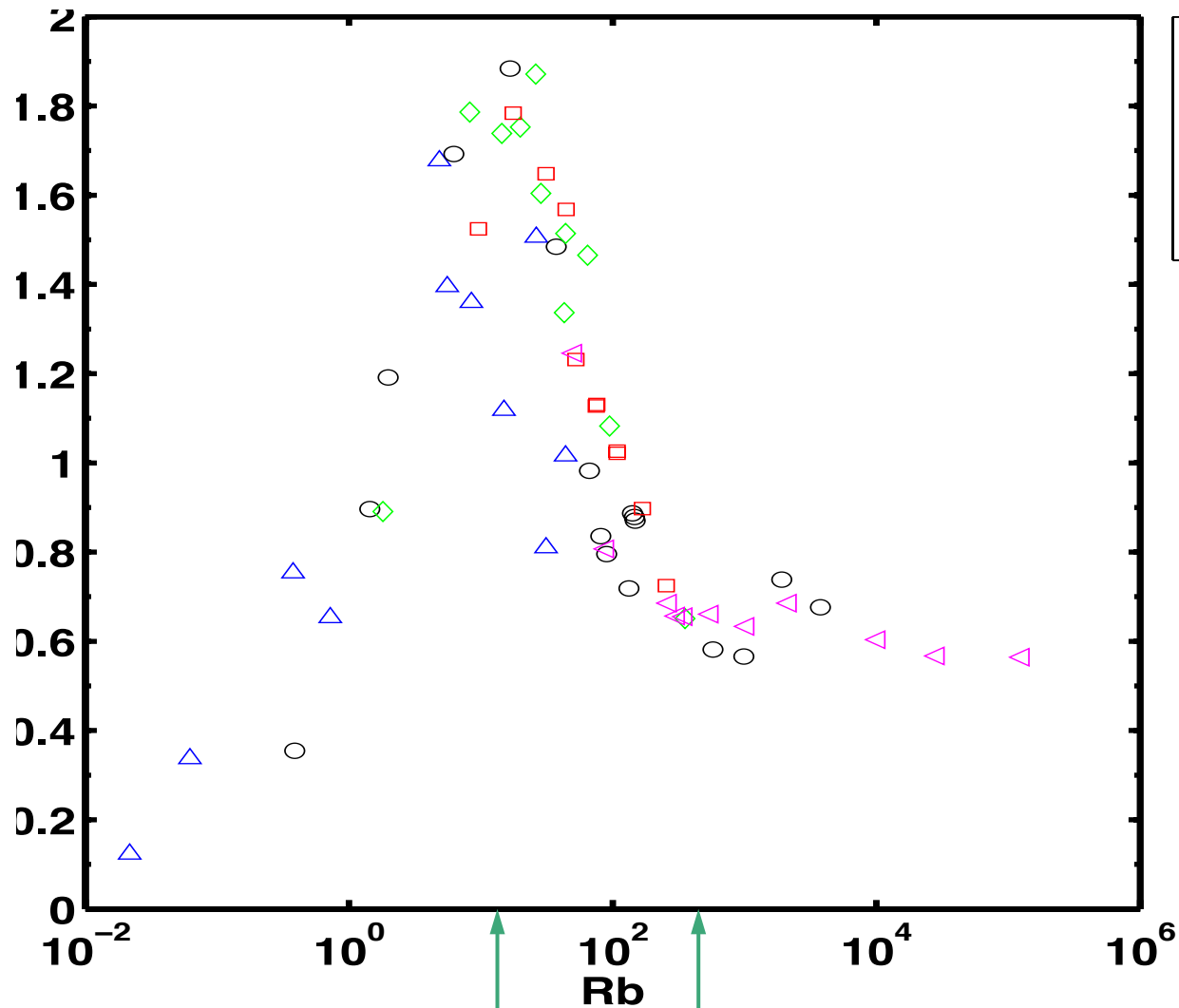
Effective dissipation:

$$T_v = E_v/\epsilon_v ,$$

$$T_p = E_p/\epsilon_p$$

$$[\tau_{NL}/T_v]^2 : \quad 0.001 \quad \rightarrow \quad \rightarrow \quad 0.1$$

→ Effective dissipation (much) slower than nonlinear times

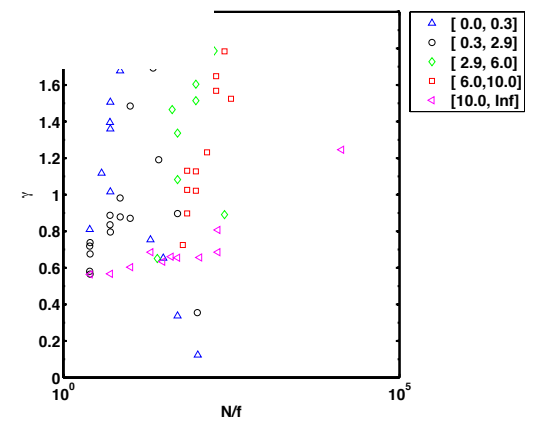


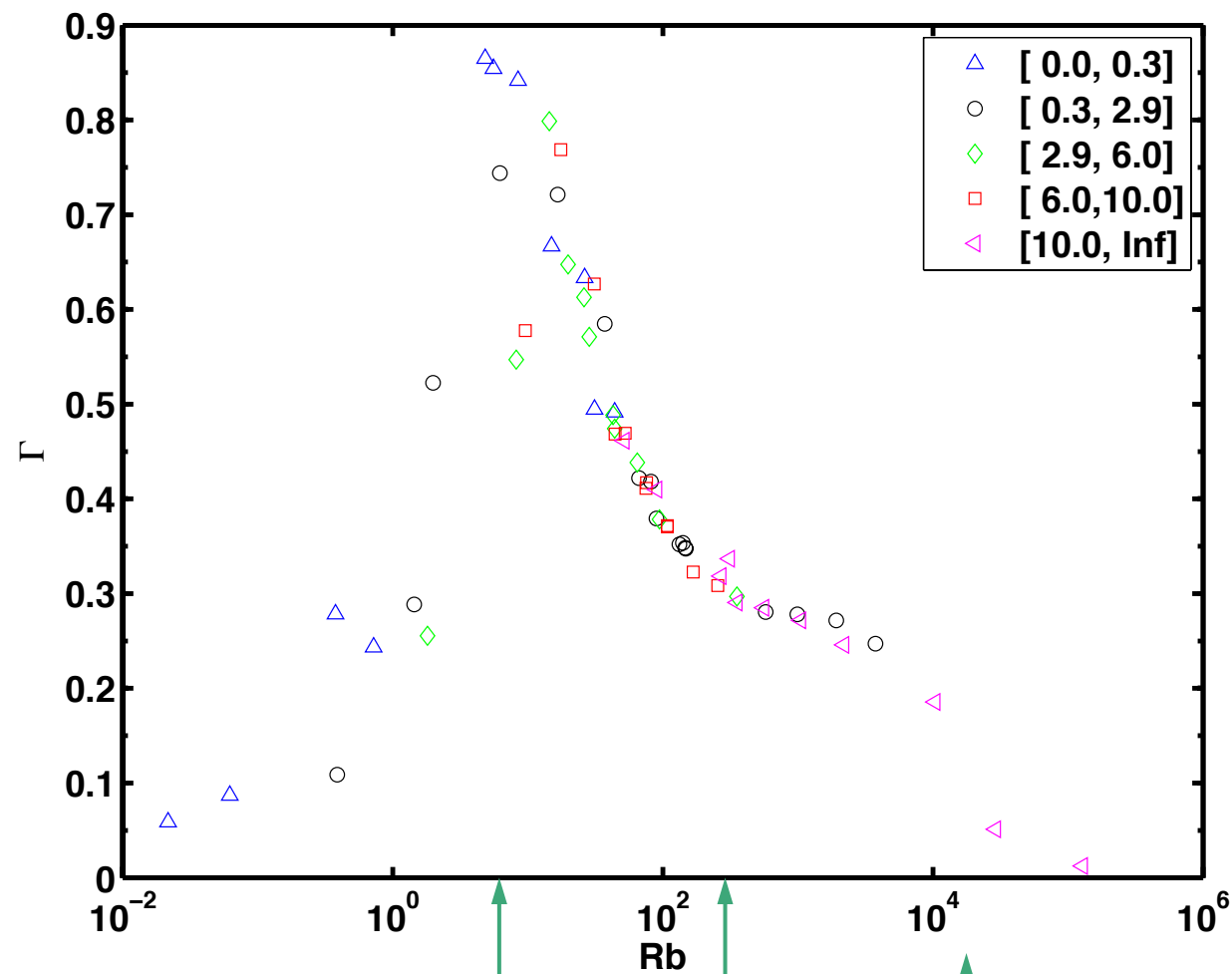
$$\gamma = T_v / T_p$$

$$= [E_v / E_p] [\epsilon_p / \epsilon_v]$$

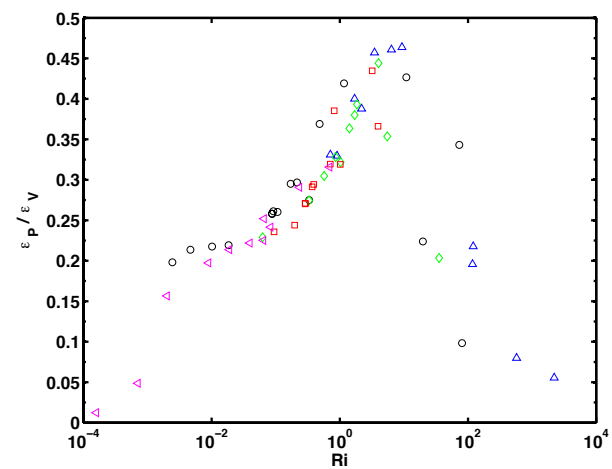
w. binning in Ro

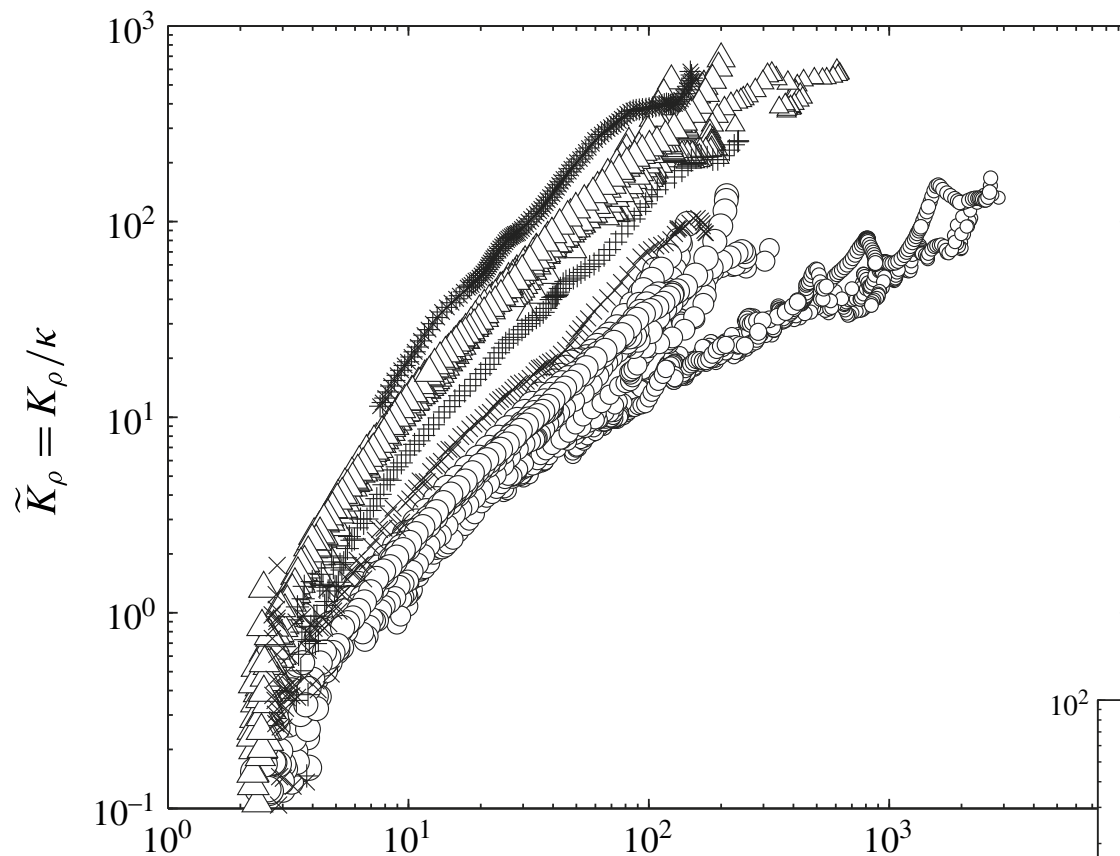
**Role of
Reynolds?
and of
 $E_p(t=0)=0$?**



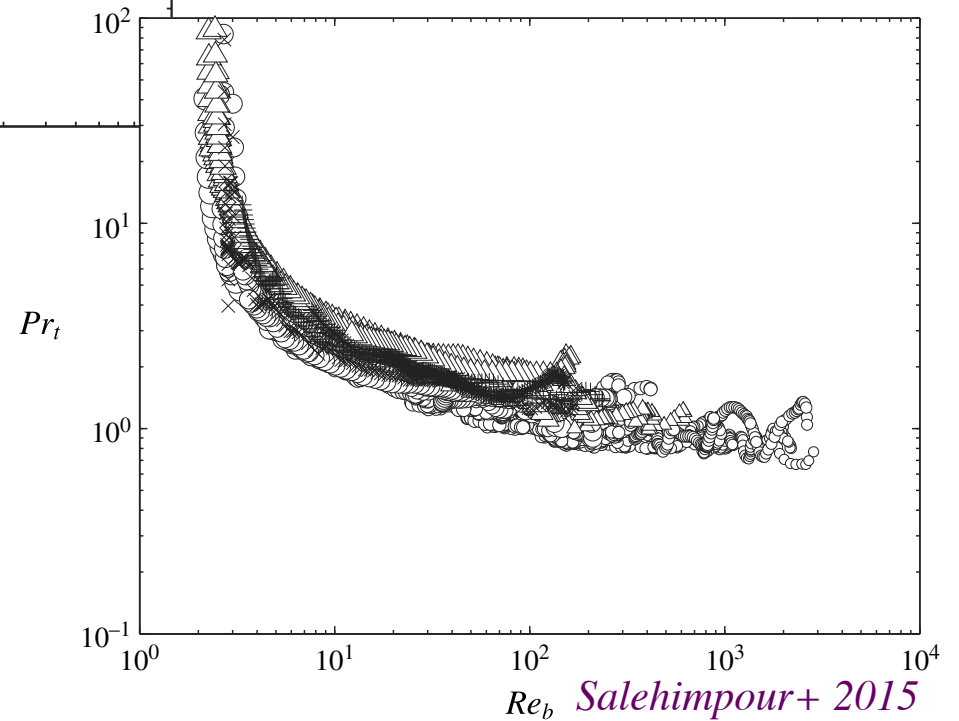
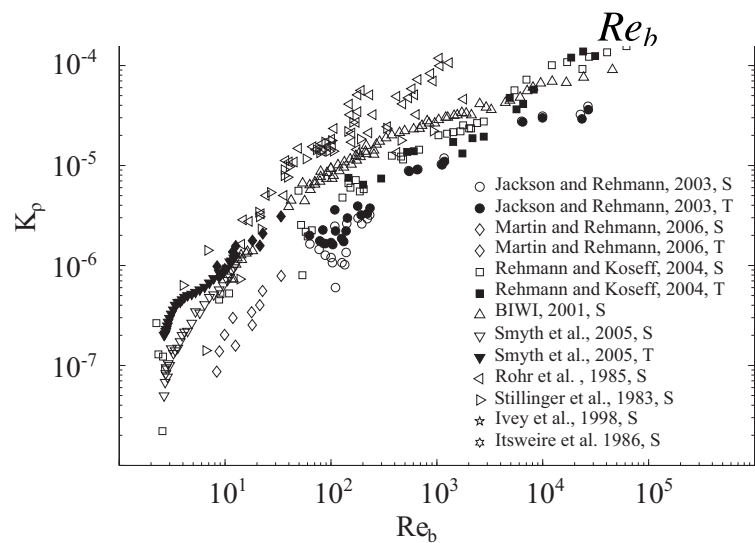


$$\Gamma = \varepsilon_p / [\varepsilon_v - \varepsilon_p]$$





Normalized anomalous
diffusivity and turbulent
Prandtl number as a fn of Re_b



Salehimpour+ 2015

Conclusions and questions

- Large resolutions allow for scale separation and thus to distinguish between (*multiple*) regimes within a flow
 - Evidence for Bolgiano-Obukhov scaling at large scale and complex interplay between velocity and buoyancy modes & fluxes
 - Local instabilities and strong local variations (dissipation, PV, ...)
 - Waves and eddies partition and local small-scale dynamics
 - Role of rotation, inverse cascade, of walls and B.C. *in the BO scaling?*
 - Role of forcing (3D vs. 2D, vortices vs. waves, ...), of large-scale friction, of non-local interactions & large-scale instabilities?
- ^ How much in **RST** and how is it structured and characterized: Intermittency in velocity itself? Generalized model for intermittency of V in **RST**? Helicity?
- ^ Role of temperature fluctuations, balanced or not, in I.C. or forcing?
- ^ Reynolds number in mixing: does mixing saturate?
- ^ What models for vorticity lanes in **RST**?
- ^ How many transitions in $R_B = ReFr^2$? ($4k, Fr \sim .15, Ro = .7, R_B \sim 1200$; or $Fr \sim .25, Ro = .5, R_B \sim 3100$)

Some questions in **R**otating **S**tratified **T**urbulence

^ How much in **RST** and how is it structured and characterized:

- Intermittency in velocity components themselves?
- Generalized model for intermittency of V in **RST**?
- Helicity?

^ Role of

^ temperature fluctuations, balanced or not, in I.C. or forcing?

^ Reynolds number in mixing: does mixing saturate?

^ What models for vorticity lanes in **RST**?

^ How many transitions in $R_B = ReFr^2$? ($4k$, $Fr \sim .15$, $Ro = .7$, $R_B \sim 1200$; or $Fr \sim .25$, $Ro = .5$, $R_B \sim 3100$)

“In this unfolding conundrum of life and history there is such a thing as being too late ... We may cry out desperately for time to pause in her passage, but time is adamant to every plea and rushes on. Over the bleached bones and jumbled residue of numerous civilizations are written the pathetic words: “Too late”. “

Martin Luther King Jr, 1967

After Clive Hamilton, Utopias in the Anthropocene, American Sociological Association, Denver 2012

Geophysical High Order Suite for Turbulence *(D. Gomez & P. Mininni)*

- Pseudo-spectral DNS, periodic BC cubic (*also 2D*), single/double precision; Runge-Kutta for incompressible Navier-Stokes, SQG & Boussinesq. Includes rotation, passive scalar(s), MHD + Hall term
- *GHOST*, from laptop to high-performance, parallelizes linearly up to 130,000 processors, using hybrid MPI/Open-MP *(Mininni et al. 2011, Parallel Comp. 37)*
- LES: alpha model & variants (Clark, Leray) for fluids & MHD
- Helical spectral (EDQNM) model for eddy viscosity & eddy noise
- Lagrangian particles (*w. A. Pumir*)
- Gross-Pitaevskii & Ginzburg-Landau (*PM+M. Brachet*)
- **Data, forced:** 2048^3 Navier-Stokes and 1536^3 & 3072^3 with rotation, both w. or w/o helicity. Rotating stratified turbulence w. 2048^3 grids forced at intermediate scale
- **Data, spin-down MHD:** 1536^3 random + 6144^3 ideal & 2048^3 w. T-Green symmetry
- Decaying rotating stratified flow, $N/f \sim 5$, $Re = 5.5 \cdot 10^4$, 2048^3 , 3072^3 & 4096^3 grids.
- Decaying rotating stratified flow, $2.5 < N/f < 300$, Re up to $1.8 \cdot 10^4$, R_b up to 10^5 , 1024^3 grid.

Some of our recent references

- D. Rosenberg, , R. Marino, C. Herbert & A. Pouquet, Variations of characteristic time scales in rotating stratified turbulence using a large parametric numerical study, EuroPhys. J. E **39**, 49001 (2016)
- R. Marino, D. Rosenberg, C. Herbert & A. Pouquet, Interplay of waves and eddies in rotating stratified turbulence and the link with kinetic-potential energy partition, EuroPhys. Lett. **112**, 44006 (2015)
- D. Rosenberg, A. Pouquet, R. Marino & P. Mininni, Evidence for Bolgiano-Obukhov scaling in rotating stratified turbulence using high-resolution direct numerical simulations, Phys. Fluids **27** (2015)
- C. Herbert, A. Pouquet & R. Marino, Restricted Equilibrium and the Energy Cascade in Rotating and Stratified Flows. J. Fluid Mech. **758**, 374 (2014)
- R. Marino, P. Mininni, D.L. Rosenberg & A. Pouquet, Large-scale anisotropy in stably stratified rotating flows, Phys. Rev. E **90**, 023018 (2014)
- C. Rorai, P. Mininni & A. Pouquet, Turbulence comes in bursts in stably stratified flows, Phys. Rev. E **89**, 043002 (2014)
- A. Pouquet & R. Marino, Geophysical turbulence and the duality of the energy flow across scales, Phys. Rev. Lett. **111**, 234501 (2013)
- R. Marino, P. Mininni, D. Rosenberg & A. Pouquet, Inverse cascades in rotating stratified turbulence: fast growth of large scales, EuroPhys. Lett. **102**, 44006 (2013)
- P. Mininni & A. Pouquet, Inverse cascade behavior in freely decaying two-dimensional fluid turbulence, Phys. Rev. E **87**, 033002 (2013)
- P. Mininni, D. Rosenberg & A. Pouquet, Isotropization at small scales of rotating helically driven turbulence, J. Fluid Mech. **699**, 263 (2012)
- P. Mininni, D. Rosenberg, R. Reddy & A. Pouquet, An hybrid MPI-OpenMP scheme for scalable parallel pseudo-spectral computations for fluid turbulence, Parallel Computing **37**, 316 (2011)
- P. Mininni & A. Pouquet, Rotating helical turbulence. Part I. Global evolution and spectral behavior. Phys. Fluids **22**, 035105; and Part II, 035106 (2010)