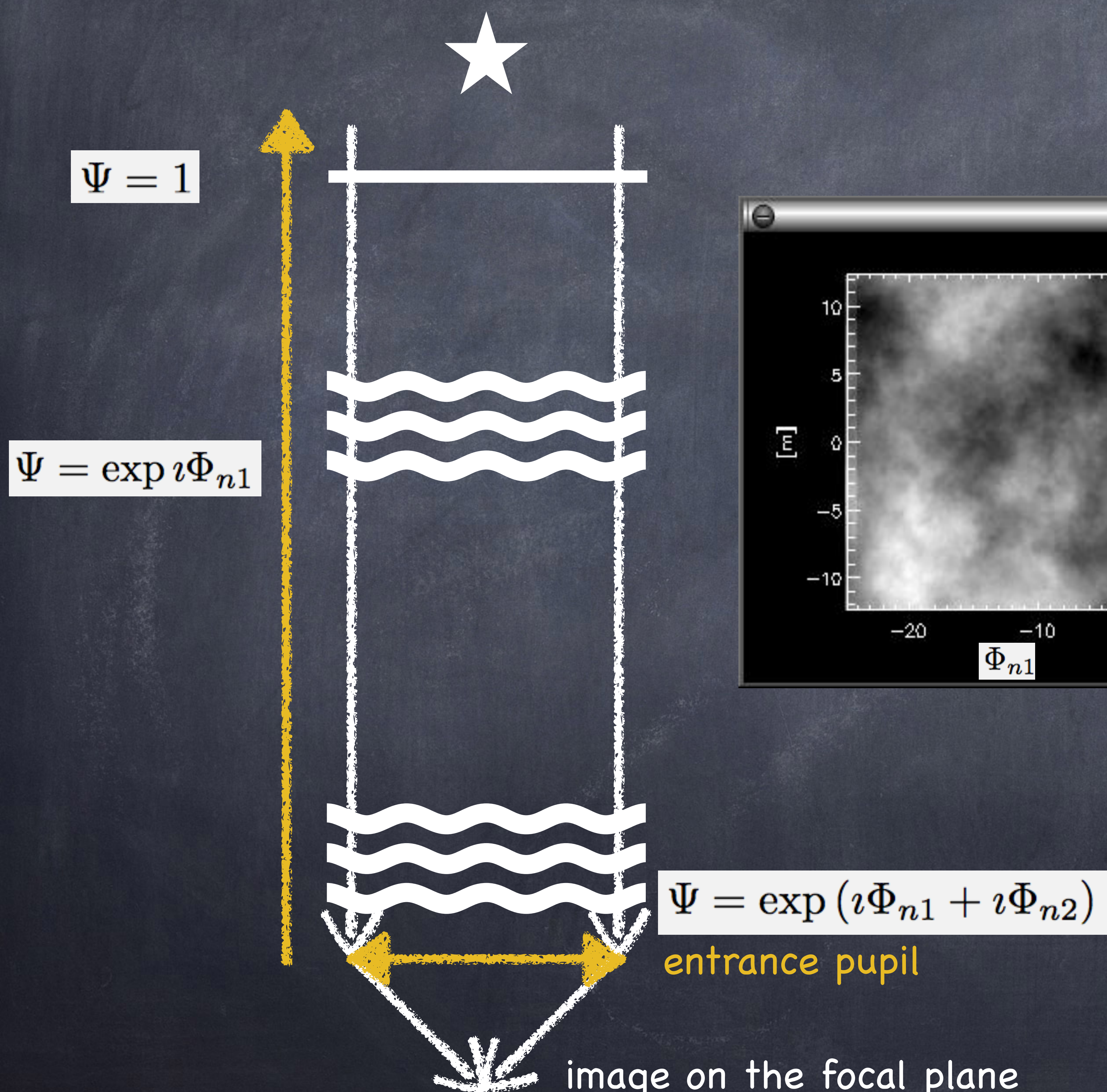
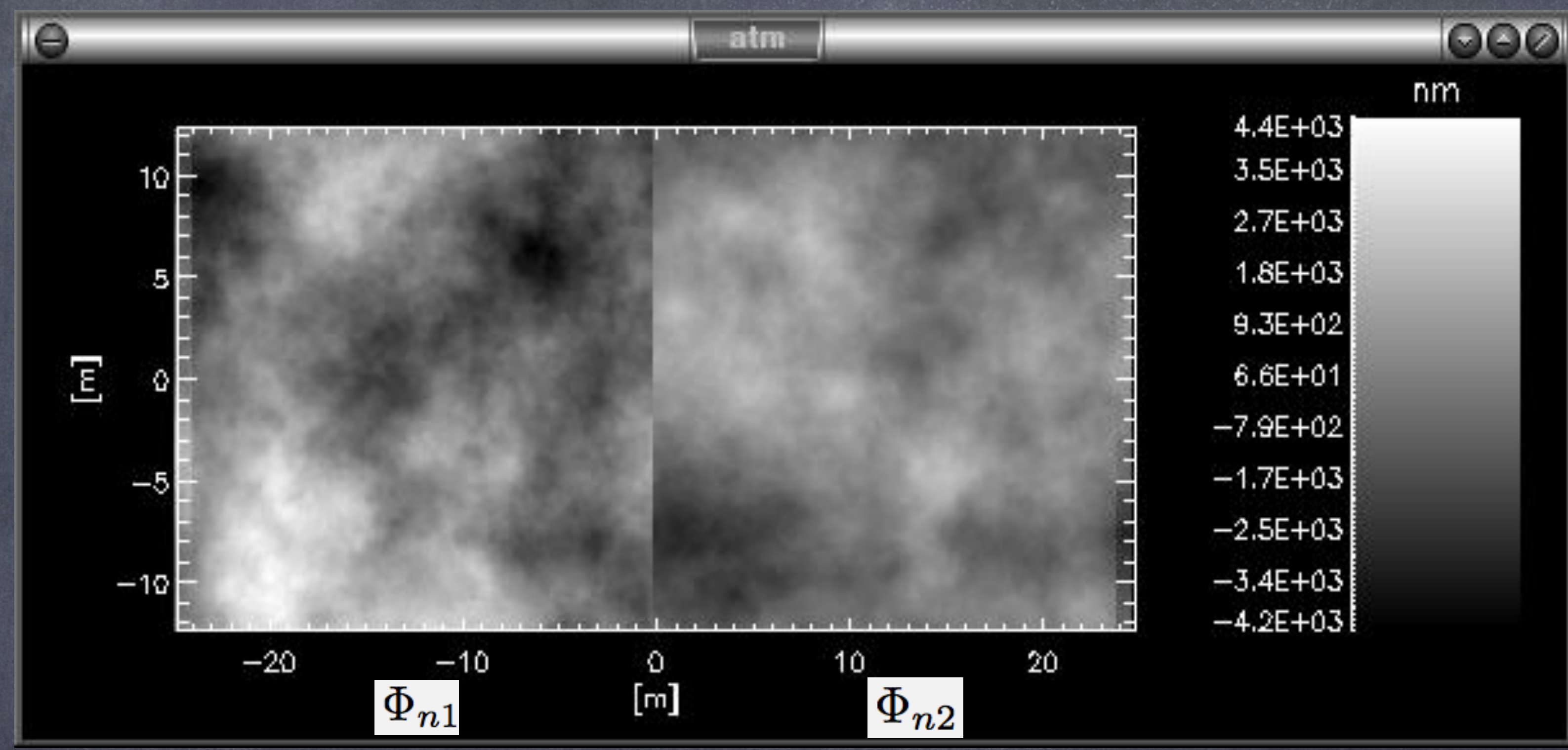
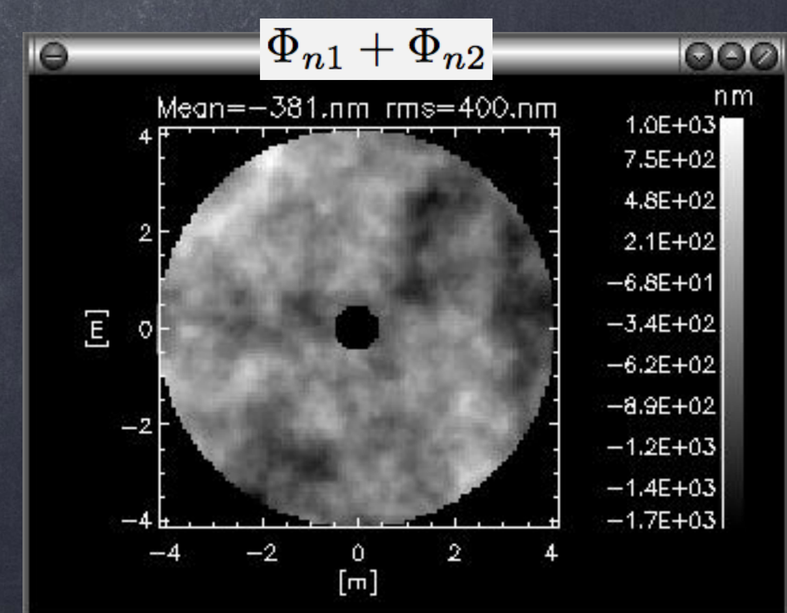
Images & turbulence — 06



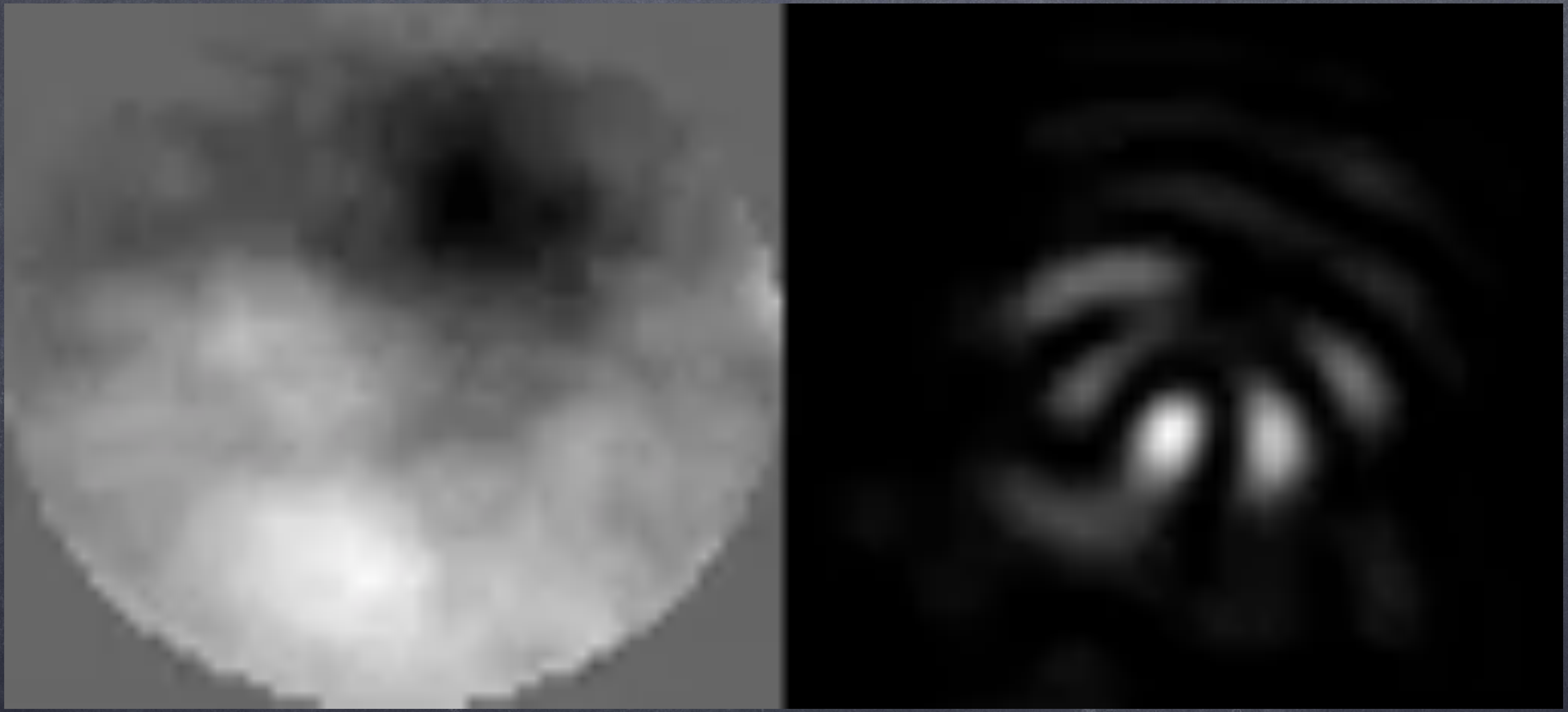




Images & turbulence — 07

entrance pupil

image on the focal plane



remembering eq. 2.17 from the course of Éric Aristidi:

$$I(x,y) = \frac{1}{\lambda^2 F^2} \left| \hat{f}_0 \left(\frac{x}{\lambda F}, \frac{y}{\lambda F} \right) \right|^2$$

directly coming from (eq. 2.16):

$$f_F(x,y) = \frac{e^{ikF}}{i\lambda F} \, e^{\frac{i\pi\rho^2}{\lambda F}} \, \hat{f}_0 \left(\frac{x}{\lambda F}, \frac{y}{\lambda F} \right)$$

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The wavefront is, modulo $\lambda/2\pi$, proportional to the phase $\Phi(r)$ of the wave $\Psi(r)$ which has went through the turbulent atmosphere before reaching the telescope:

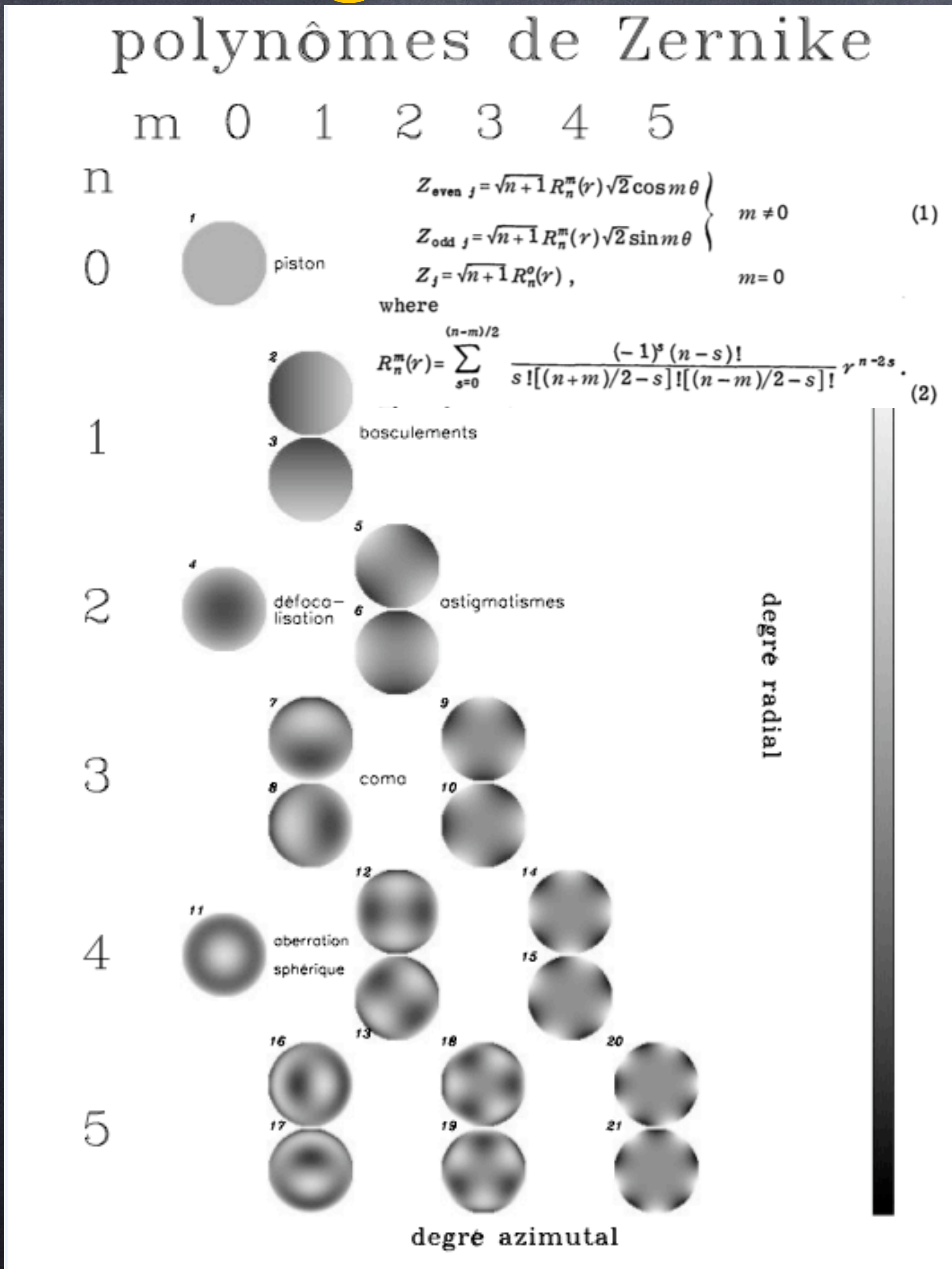
$$\Psi(\vec{r}) = A(\vec{r})\,\exp\{\imath\Phi(\vec{r})\}$$

Note that this phase can be decomposed following a base of polynomials, for example Zernike ones:

$$\Phi(\vec{r}) = \sum_i a_i\, Z_i(\vec{r})$$

Images & turbulence — 09

entrance pupil



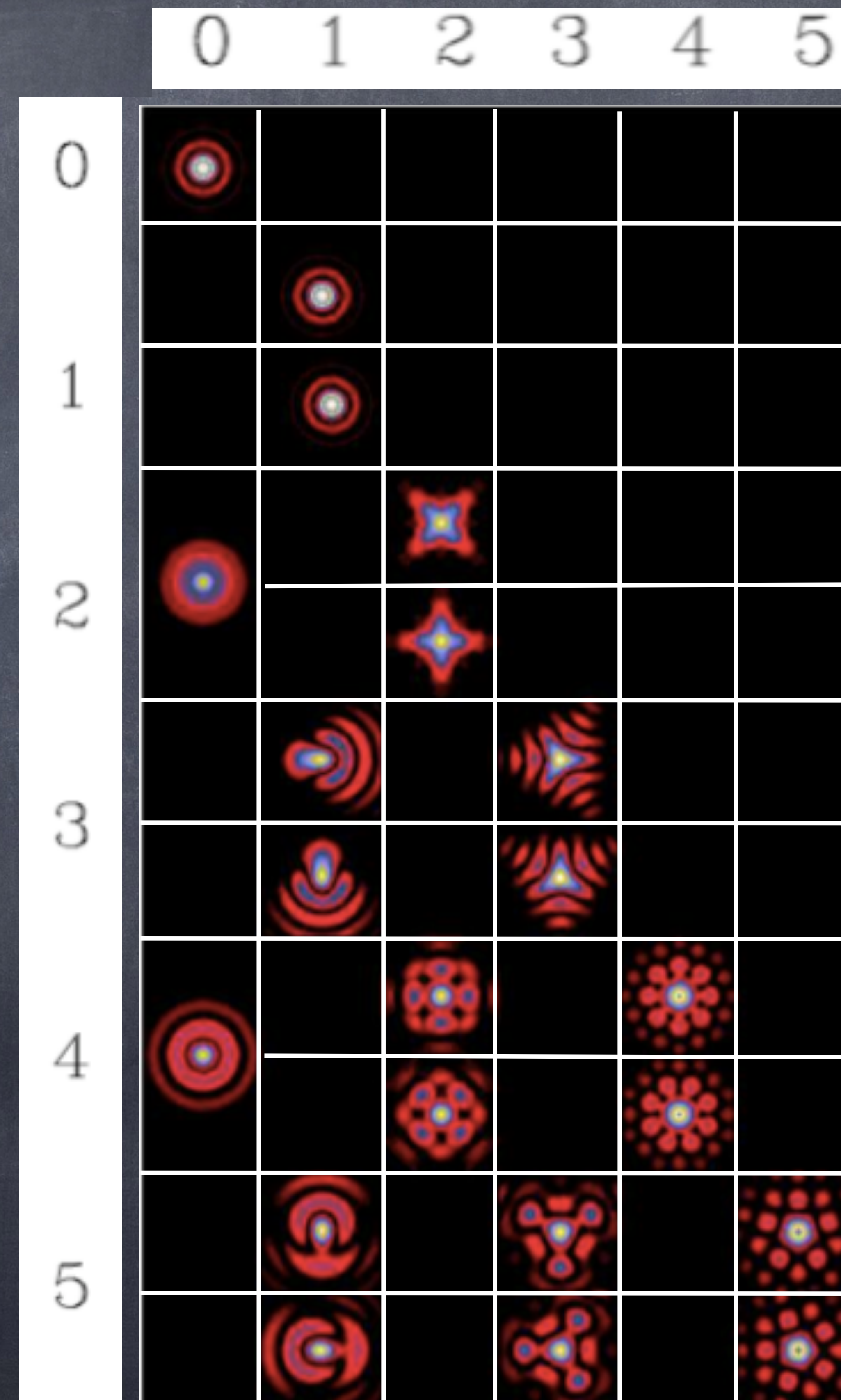


image on the focal plane

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turbulence intensity [$m^{1/3}$]

$$r_0 = 0.185\ \lambda^{\frac{6}{5}}\ \cos(\gamma)^{\frac{3}{5}}\ \left[\int_0^\infty C_n^2(z)\ dz\right]^{-\frac{3}{5}}$$

dimension of r_0 ? value in band H knowing r_0 at 500nm (10cm) ?...

$$\tau_0 = 0.36\,\frac{r_0}{\bar{v}}$$

$$\bar{v} = \left(\frac{\int C_n^2(h) v(h)^{\frac{5}{3}} dh}{\int C_n^2(h) dh}\right)^{\frac{3}{5}}$$

$$f_G = 3.185\,\frac{\bar{v}}{r_0}$$

$$\epsilon_0 = 0.98\,\frac{\lambda}{r_0}$$

$$N_s \simeq 0.34\left(\frac{D}{r_0}\right)^2$$

$$\theta_0 = 0.314\ \frac{r_0}{\bar{h}}$$

$$\bar{h} = \left(\frac{\int C_n^2(h) h^{\frac{5}{3}} dh}{\int C_n^2(h) dh}\right)^{\frac{3}{5}}$$

Number of speckles for r_0=10cm and D=1m ?...

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r_0 in band H knowing r_0 at 500nm ?...

$$r_0 = 0.185\ \lambda^{\frac{6}{5}}\ \cos(\gamma)^{\frac{3}{5}}\ \left[\int_0^\infty C_n^2(z)\ dz\right]^{-\frac{3}{5}}$$

$$r_0^{\mathrm{H}=1.65\,\mu\mathrm{m}} = r_0^{500\,\mathrm{nm}} \left(\frac{1.65}{0.5}\right)^{\frac{6}{5}} \simeq 0.42$$

Number of speckles for r_0=10cm and D=1m ?...

$$N_S^{500\,\mathrm{nm}} \simeq 0.34\ \left(\frac{1.0}{0.1}\right)^2 \simeq 34$$

$$N_S^{\mathrm{H}} \simeq 0.34\ \left(\frac{1.0}{0.42}\right)^2 \simeq 2$$

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$$\Phi_\varphi(\vec{\nu}) = 0.0228\ r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{6}}$$

Power Spectral Density (PSD) of the phase, function of the spatial frequency

Kolmogorov/von Kármán model

- Kolmogorov : outerscale of turbulence $\mathcal{L}_0$ is infinite.
- One can refine the model by considering also ℓ_0.
- ∃ other models with a finite $\mathcal{L}_0$ (and a non-zero ℓ_0).

Energy cascade: wind shear => turbulent energy injected into the system via a large eddy ($\mathcal{L}_0$) which splits into smaller and smaller eddies (ℓ_0), and is finally viscously dissipated. Interval [ℓ_0, $\mathcal{L}_0$] = **inertial range**.

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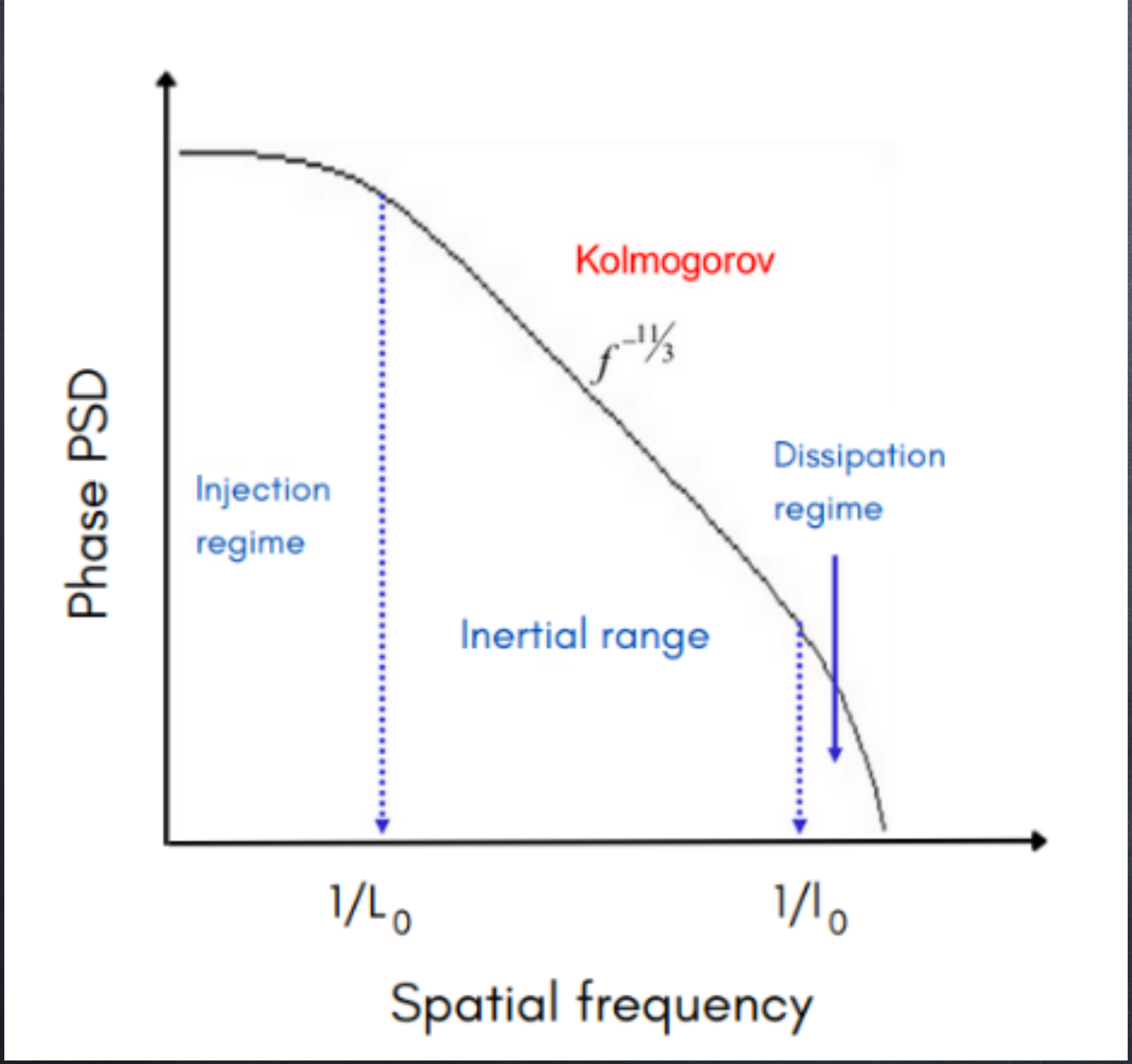
$$\Phi_\varphi(\vec{\nu}) = 0.0228\ r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{6}}$$

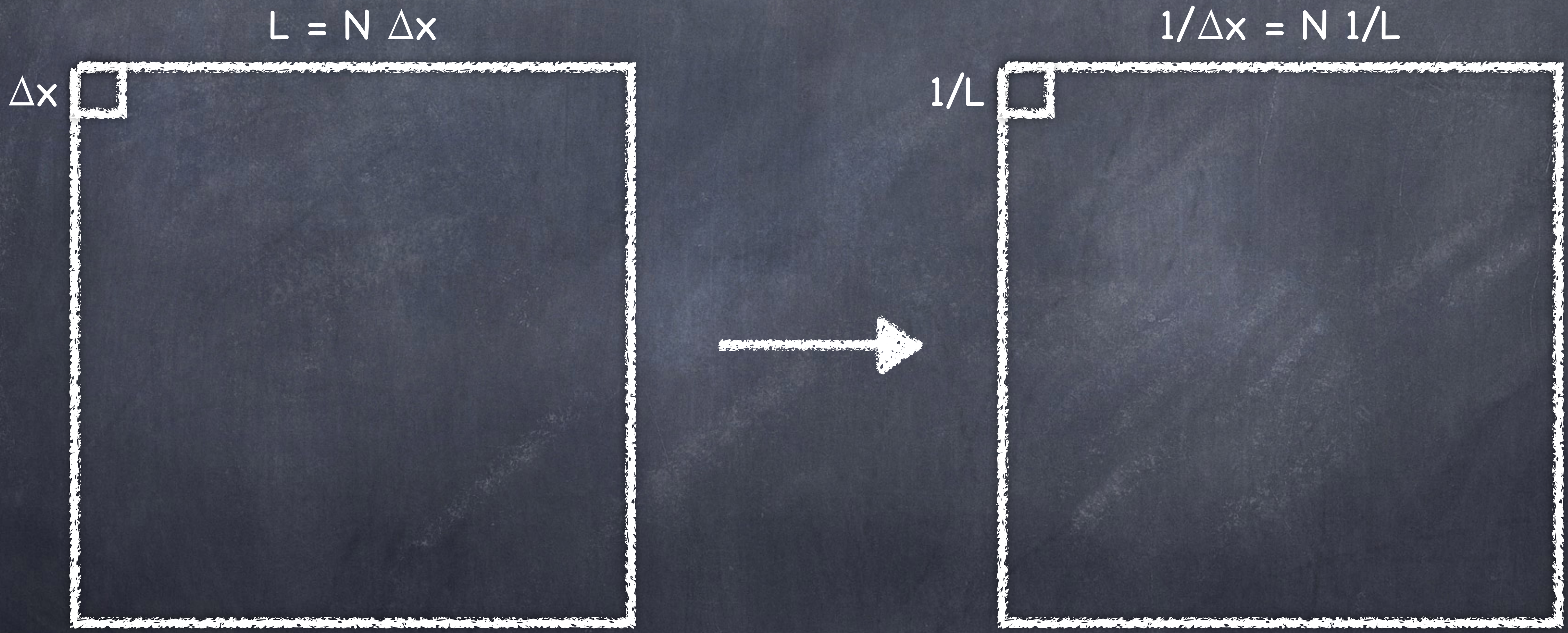


Energy cascade:

wind shear => turbulent energy injected into the system via a large eddy ($\mathcal{L}_0$) which splits into smaller and smaller eddies (ℓ_0), and is finally viscously dissipated.

Interval [ℓ_0, $\mathcal{L}_0$] = inertial range.

(A reminder of discrete Fourier transform (DFT)...)



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$$\Phi_\varphi(\vec{\nu}) = 0.0228\ r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{6}}$$

Which, numerically written, and by considering wavefronts made of 'dim' pixels corresponding to 'L' meters, becomes:
(re-writing - "de-dimensionalizing" - the equation with L_0=L_0 L/L and ν=ν L/L…)

```
freq = findgen(dim)
dsp  = .0228*(L/r0)^(5/3.)*L^2*(freq^2+(L/L0)^2)^(-11./6)
```

And which (with the right frequency scale) can be plot with:

```
plot_oo, 1./L*findgen(dim), dsp, XR=[1/L/1.2,dim*1/L*1.2], /XS
```

=> make a function that computes PSD(L_0, r_0, dim, L) and plot it for different [r_0, L_0]… [with, for example: dim=1000, L=100., r0=0.1, L0=100.,10.,1.]

(IDL stuff — 3)

Example of a function that computes the sum of two parameters:

```
function sum2par, par1, par2
result=par1+par2
return, result
end
```

Compile and run the function (written, e.g., in a file sum2par.pro):

```
idl > .r sum2par
—> % Compiled module: SUM2PAR
idl > res = sum2par(2,1)
idl > print, res                    (or simply: idl> res)
—> 3
```

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```
function dsp_theo, dim, L, r0, L0
;
; dim = array linear dimension [px]
; L   = array physical length [m]
; r0  = phase screen Fried parameter [m]
; L0  = phase screen outerscale [m]
; use: dsp=dsp_theo(dim,L,r0,L0)
; to be plotted afterwards with:
; plot_oo, 1./L*findgen(dim), dsp, XR=[1/L/1.2,dim*1/L*1.2], /XS, $
;          YR=[0.1, 1E7], TIT='PSD(L0)', XTIT='freq. [1/m]', YTIT='PSD'
; oplot  , 1./L*findgen(dim), dsp, LINE=1
; playing, e.g., with L0=100.,10.,1., or r0=.05, .1, .2
;
freq = findgen(dim)
dsp  = .0228*(L/r0)^(5/3.)*L^2*(freq^2+(L/L0)^2)^(-11./6)

return, dsp
end
```