Imaging through turbulence

(image formation, atmospheric turbulence, intro to adaptive optics)

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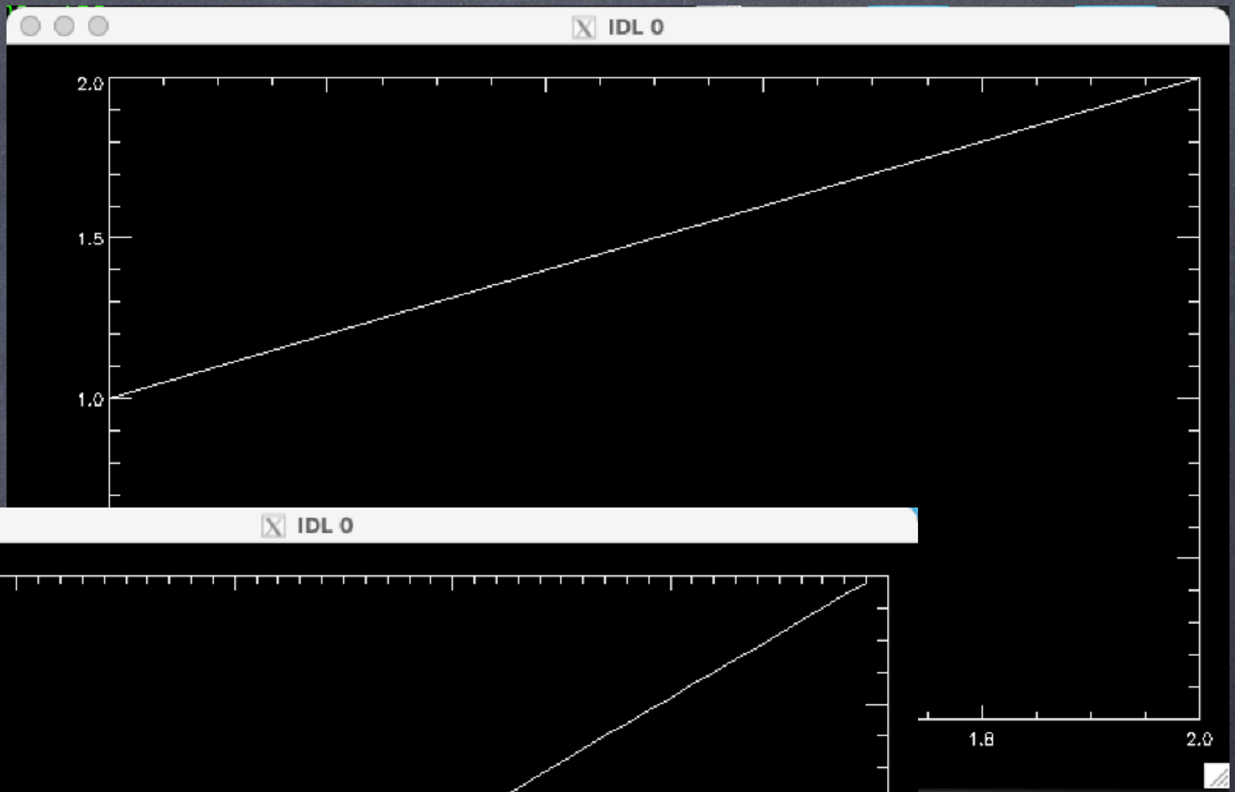
Menu

- High-angular resolution imaging in astronomy
- Atmospheric turbulence (reminder)
- Numerical modelling of perturbed wavefronts
- Formation of resulting images (+detection noises)
- *(Introduction to speckle interferometry)*
- Introduction to adaptive optics (AO)
- AO error budget
- Post-AO point-spread function morphology
- Anisoplanatic error study (ideal AO system)

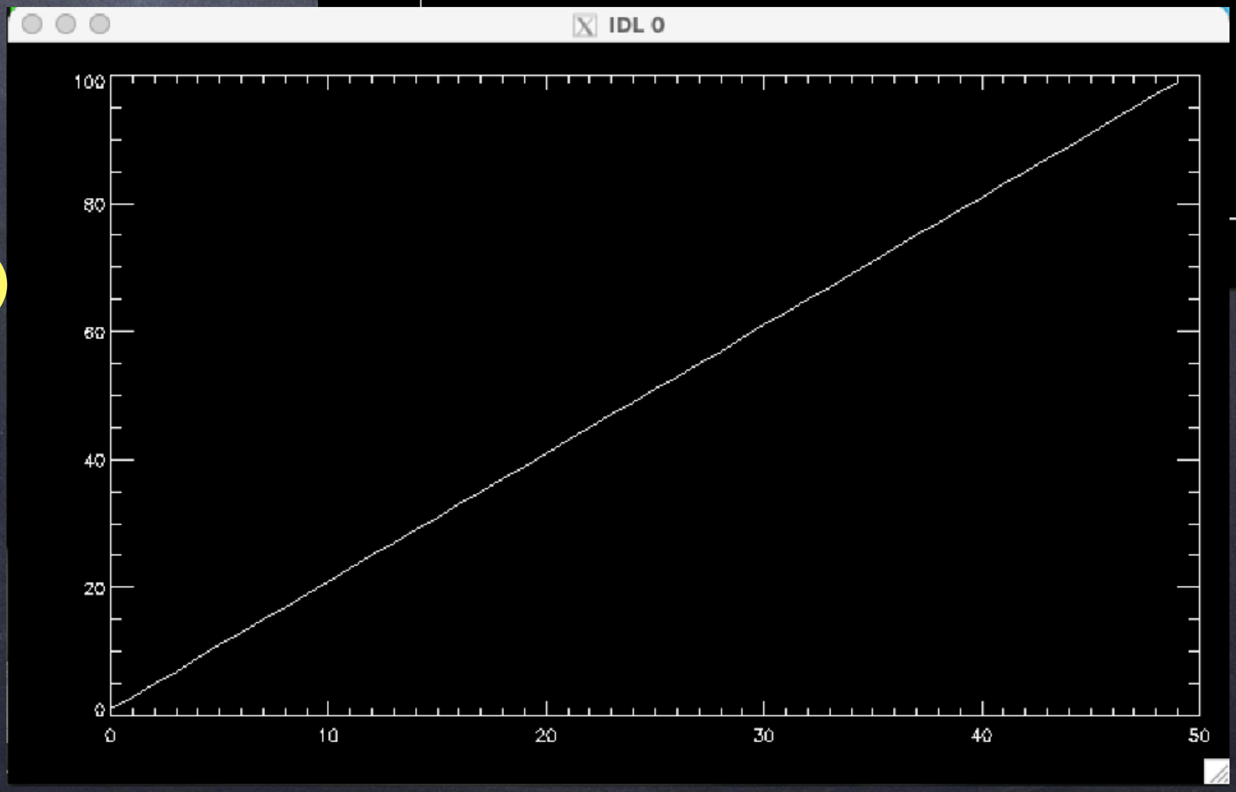
Preliminarily

- launch IDL (or IDLDE=IDL+interface), with Tunnelblick or the like (VPN) running if necessary (wifi).



- test it:
  IDL> print, ‘hello’
  IDL> plot, [1,2], [1,2]

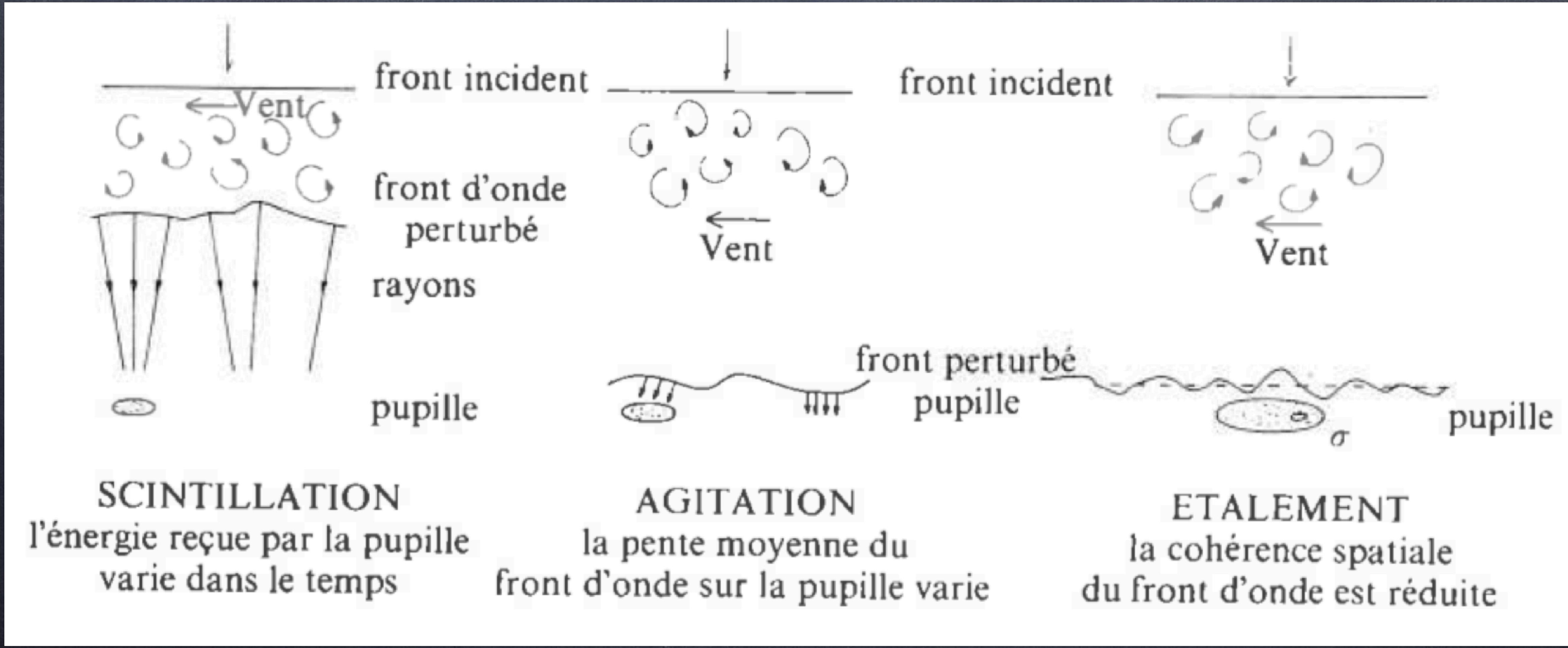
- test it more:
  IDL> xx=findgen(50)
  IDL> yy=2*xx+1
  IDL> plot, xx, yy



Images & turbulence - 1

The image formed through turbulent atmosphere (optically speaking) is degraded:

- Scintillation (due to intensity fluctuation in the pupil).
- Agitation (due to angle-of-arrival variation).
- Spreading (due to a loss of spatial coherence).



(Illustration from Pierre Léna, Astrophysique — Méthodes physiques de l'observation, CNRS Éd. (2me éd.), p.177)

Images & turbulence - 2

The object-image relation between the intensity $I(\alpha)$ in the image plane (i.e. the focal plane of the telescope) and the brightness $O(\alpha)$ of the object (in the sky) is a relation of convolution implying the point-spread function (PSF) $S(\alpha)$ of the whole ensemble telescope+atmosphere, with α the angular coordinates in the focal plane:

$$I(\vec{\alpha}) = O(\vec{\alpha}) * S(\vec{\alpha})$$

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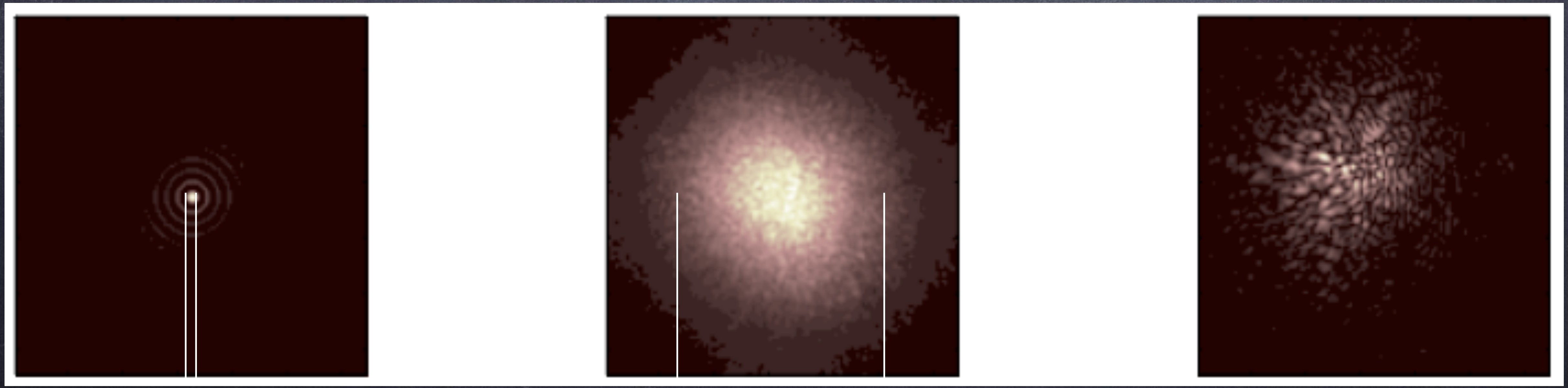
$$I(\vec{\alpha}) = O(\vec{\alpha}) * S(\vec{\alpha})$$

This relation is valid notably at the condition that the system is invariant by translation (everything happens within the isoplanatic domain)...



λ/D λ/r_0

Images & turbulence - 4

Some orders of magnitude concerning the turbulent atmosphere:

	λ = 500 nm	λ = 2.2 μm
Fried parameter (r_0)	→ 10 cm	60 cm
velocity of the turbulent layers (v)	→ 10 m/s	id.
=> image FWHM ($\epsilon \approx \lambda / r_0$)	→ 1"	~1"
=> evolution time ($\tau_0 \propto r_0 / v$)	→ 3 ms	18 ms

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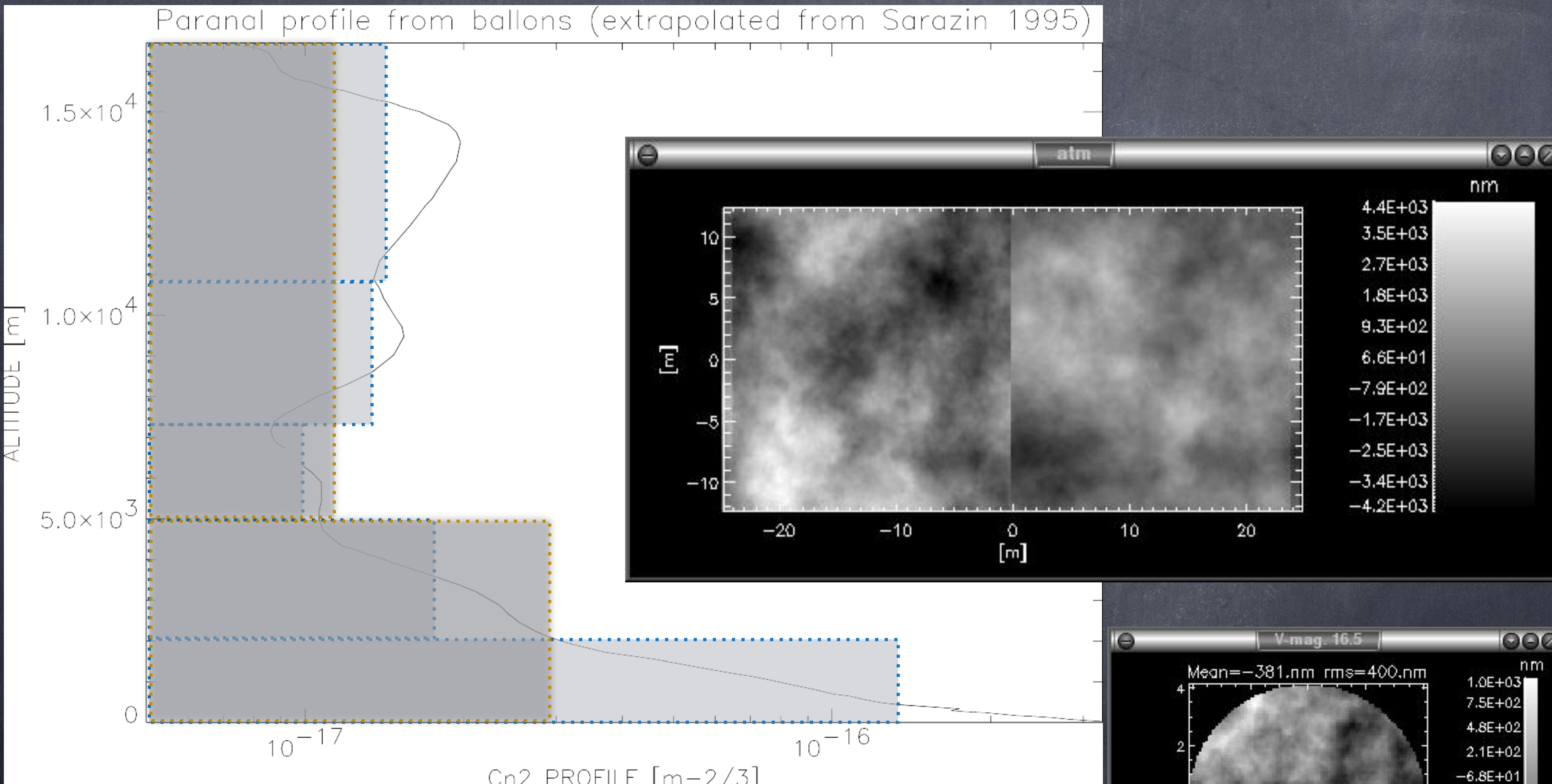

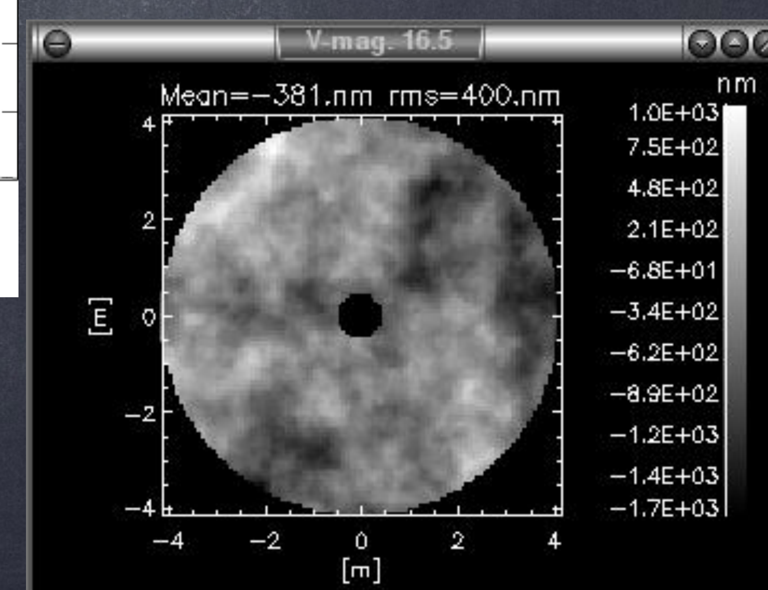

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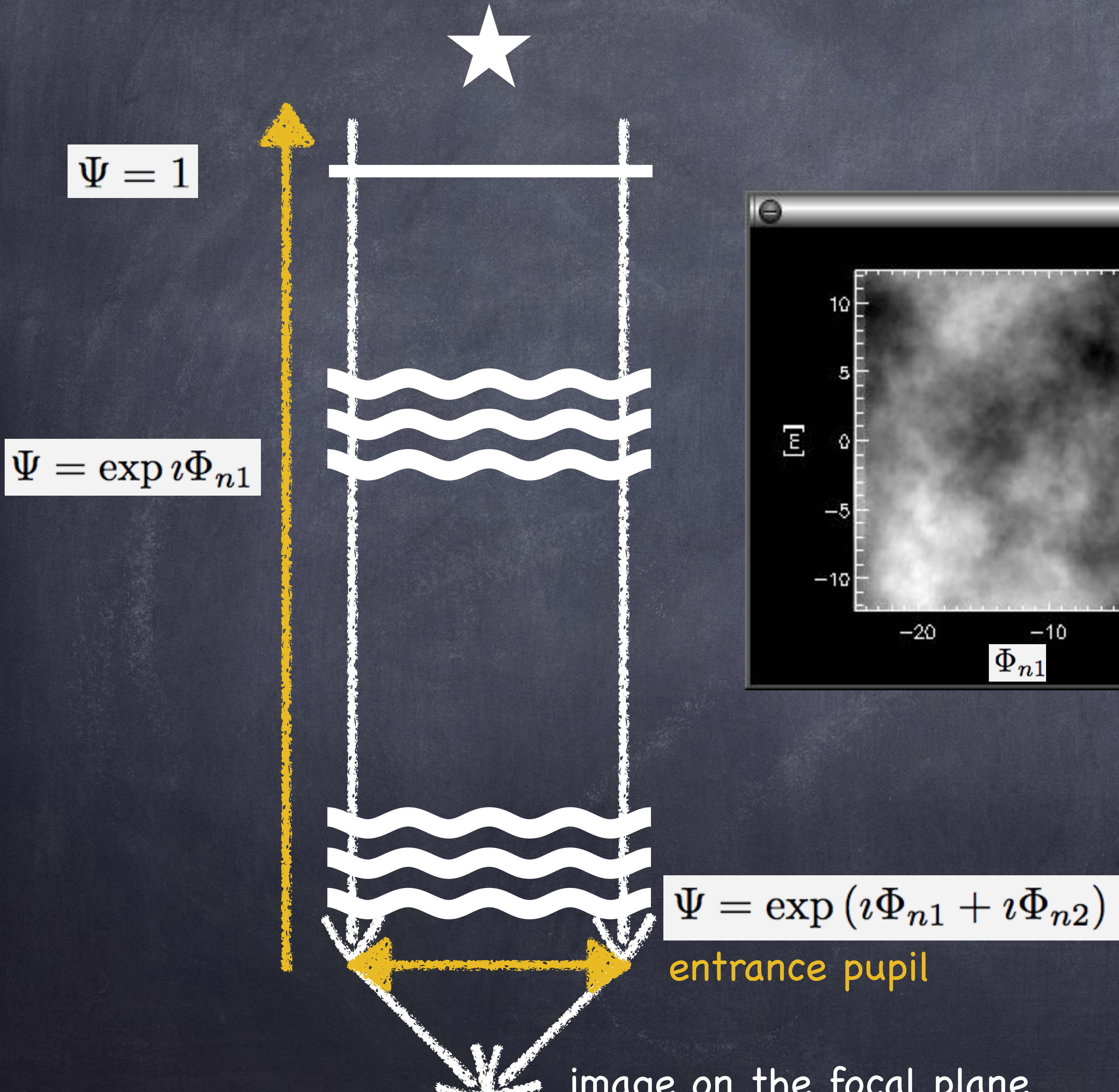

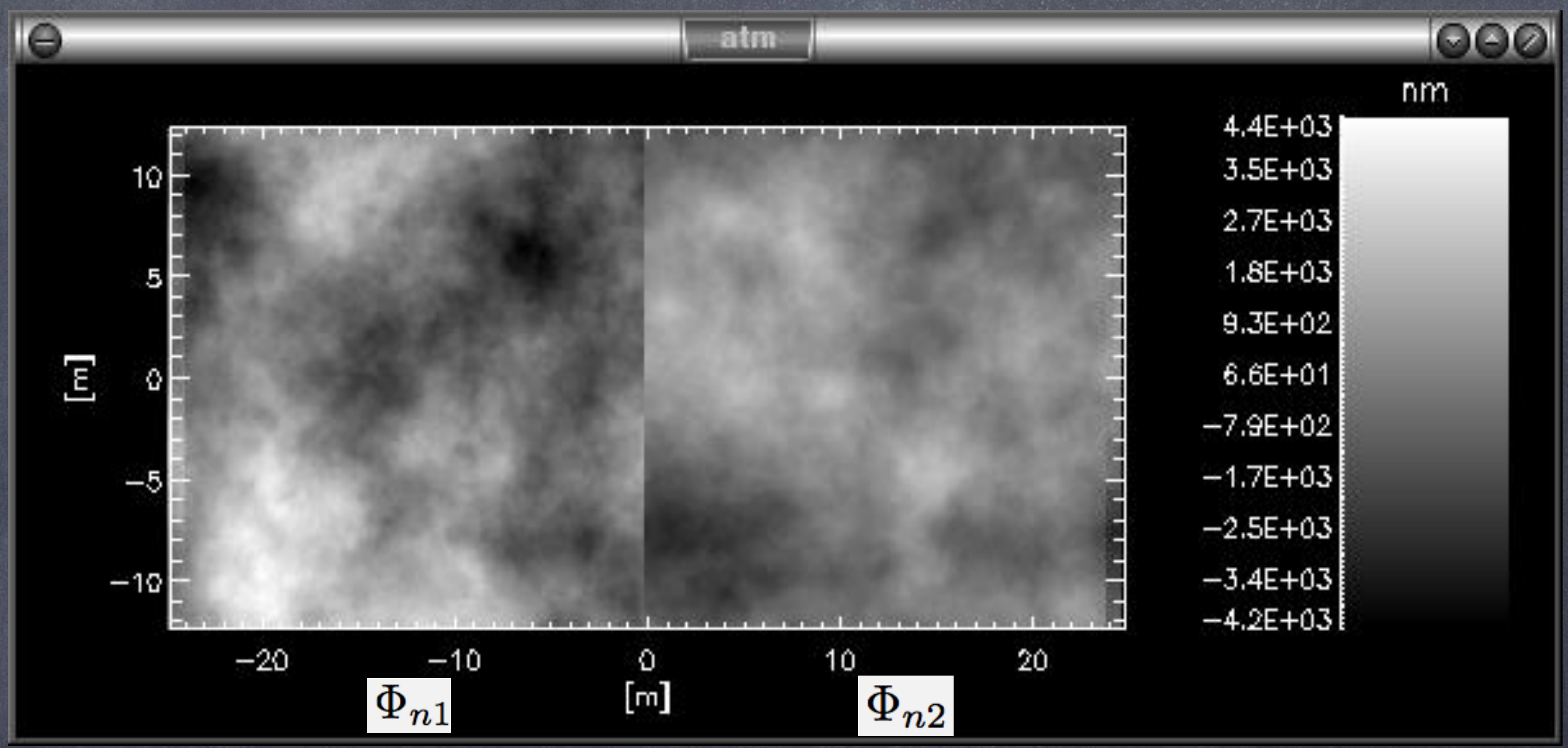

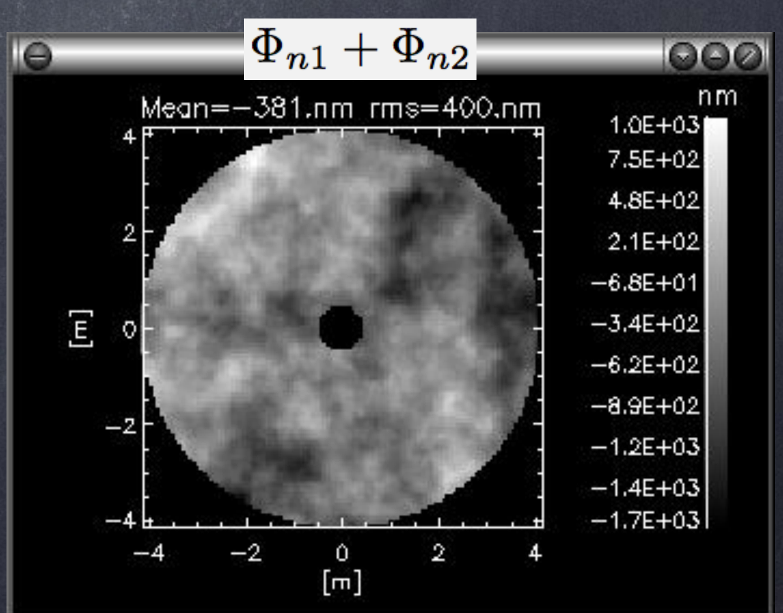

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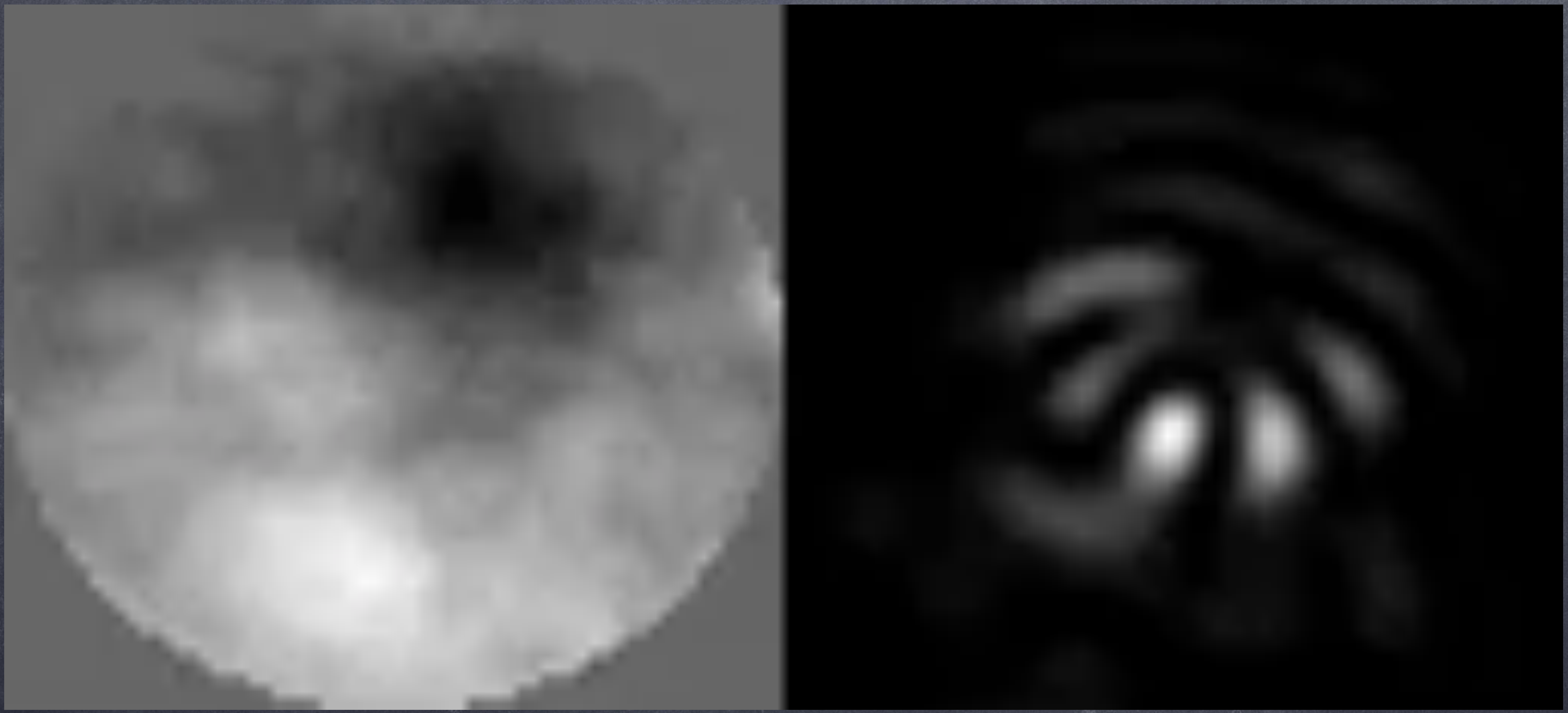
entrance pupil

image on the focal plane



remembering eq. 2.17 from the course of Éric Aristidi:

$$I(x,y) = \frac{1}{\lambda^2 F^2} \left| \hat{f}_0 \left(\frac{x}{\lambda F}, \frac{y}{\lambda F} \right) \right|^2$$

directly coming from (eq. 2.16):

$$\boxed{f_F(x,y) = \frac{e^{ikF}}{i\lambda F} \, e^{\frac{i\pi\rho^2}{\lambda F}} \, \hat{f}_0 \left(\frac{x}{\lambda F}, \frac{y}{\lambda F} \right)}$$

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The wavefront is, modulo $\lambda/2\pi$, proportional to the phase $\Phi(r)$ of the wave $\Psi(r)$ which has went through the turbulent atmosphere before reaching the telescope:

$$\Psi(\vec{r}) = A(\vec{r}) \exp\{\imath \Phi(\vec{r})\}$$

Note that this phase can be decomposed following a base of polynomials, for example Zernike ones:

$$\Phi(\vec{r}) = \sum_i a_i \, Z_i(\vec{r})$$

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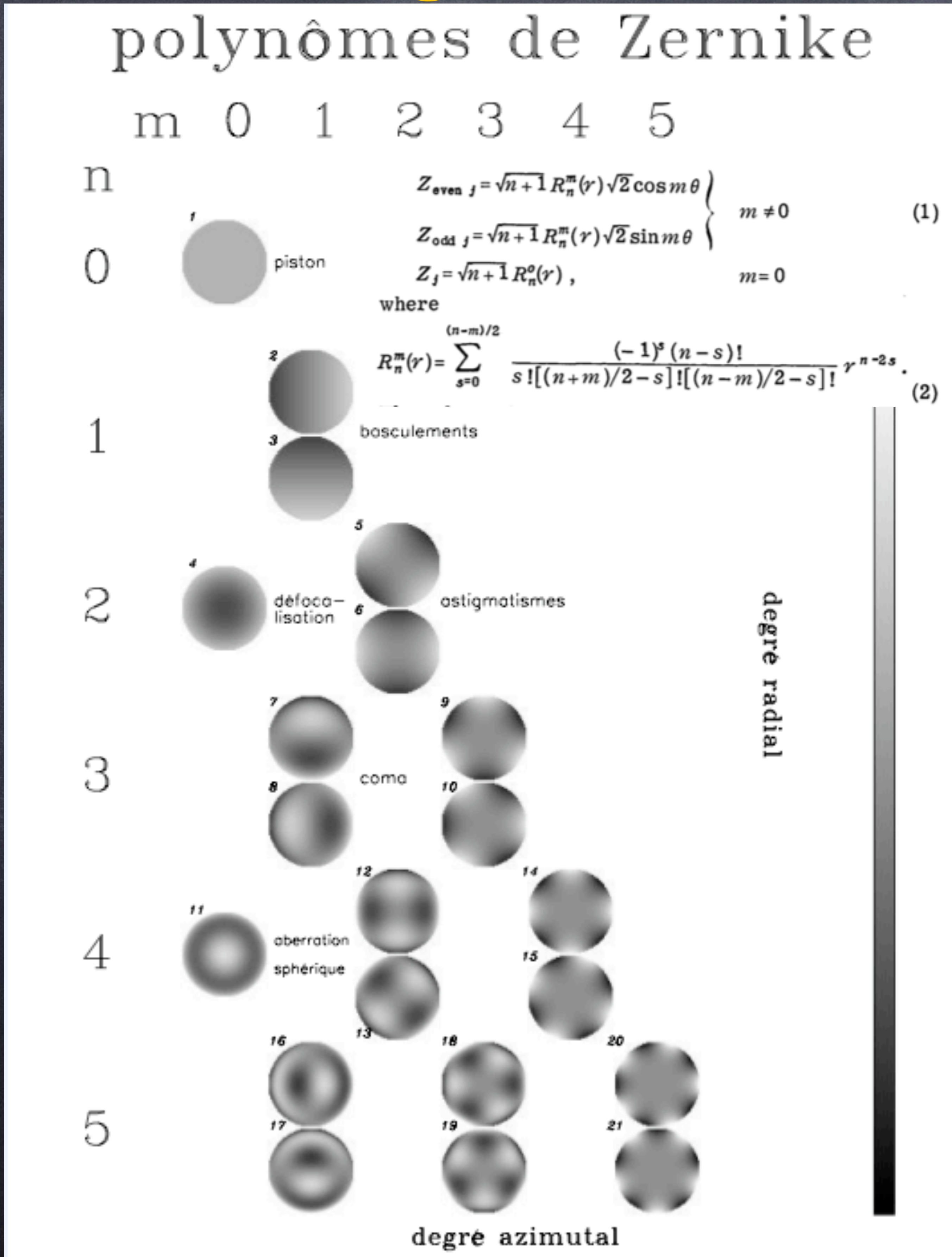
entrance pupil



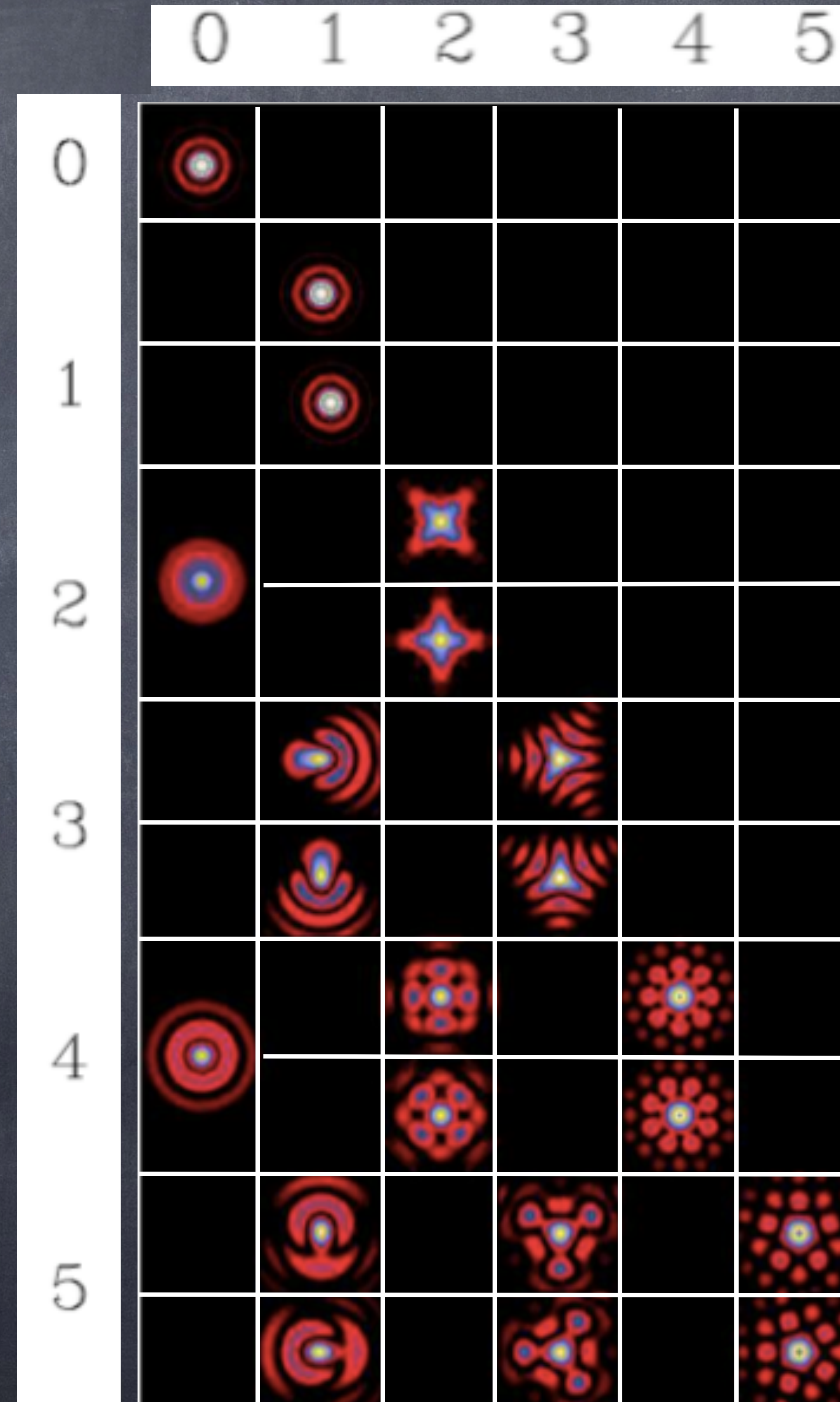


image on the focal plane

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turbulence intensity [$m^{1/3}$]

$$r_0 = 0.185\ \lambda^{\frac{6}{5}}\ \cos(\gamma)^{\frac{3}{5}}\ \left[\int_0^\infty C_n^2(z)\ dz\right]^{-\frac{3}{5}}$$

dimension of r_0 ? value in band H knowing r_0 at 500nm (10cm) ?...

$$\tau = 0.36\ \frac{r_0}{v}$$

$$\bar{v} = \left(\frac{\int C_n^2(h) v(h)^{\frac{5}{3}} dh}{\int C_n^2(h) dh}\right)^{\frac{3}{5}}$$

$$\epsilon = 0.98\ \frac{\lambda}{r_0}$$

$$\theta_0 = 0.314\ \frac{r_0}{\bar{h}}$$

$$\bar{h} = \left(\frac{\int C_n^2(h) h^{\frac{5}{3}} dh}{\int C_n^2(h) dh}\right)^{\frac{3}{5}}$$

$$N_s \simeq 0.34 \left(\frac{D}{r_0}\right)^2$$

Number of speckles for r_0=10cm and D=1m ?...

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r_0 in band H knowing r_0 at 500nm ?...

$$r_0 = 0.185\ \lambda^{\frac{6}{5}}\ \cos(\gamma)^{\frac{3}{5}}\ \left[\int_0^\infty C_n^2(z)\ dz\right]^{-\frac{3}{5}}$$

$$r_0^{\mathrm{H}=1.65\,\mu\mathrm{m}} = r_0^{500\,\mathrm{nm}} \left(\frac{1.65}{0.5}\right)^{\frac{6}{5}} \simeq 0.42$$

Number of speckles for r_0=10cm and D=1m ?...

$$N_S^{500\,\mathrm{nm}} \simeq 0.34\ \left(\frac{1.0}{0.1}\right)^2 \simeq 34$$

$$N_S^{\mathrm{H}} \simeq 0.34\ \left(\frac{1.0}{0.42}\right)^2 \simeq 2$$

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$$\Phi_\varphi(\vec{\nu}) = 0.0228\ r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{6}}$$

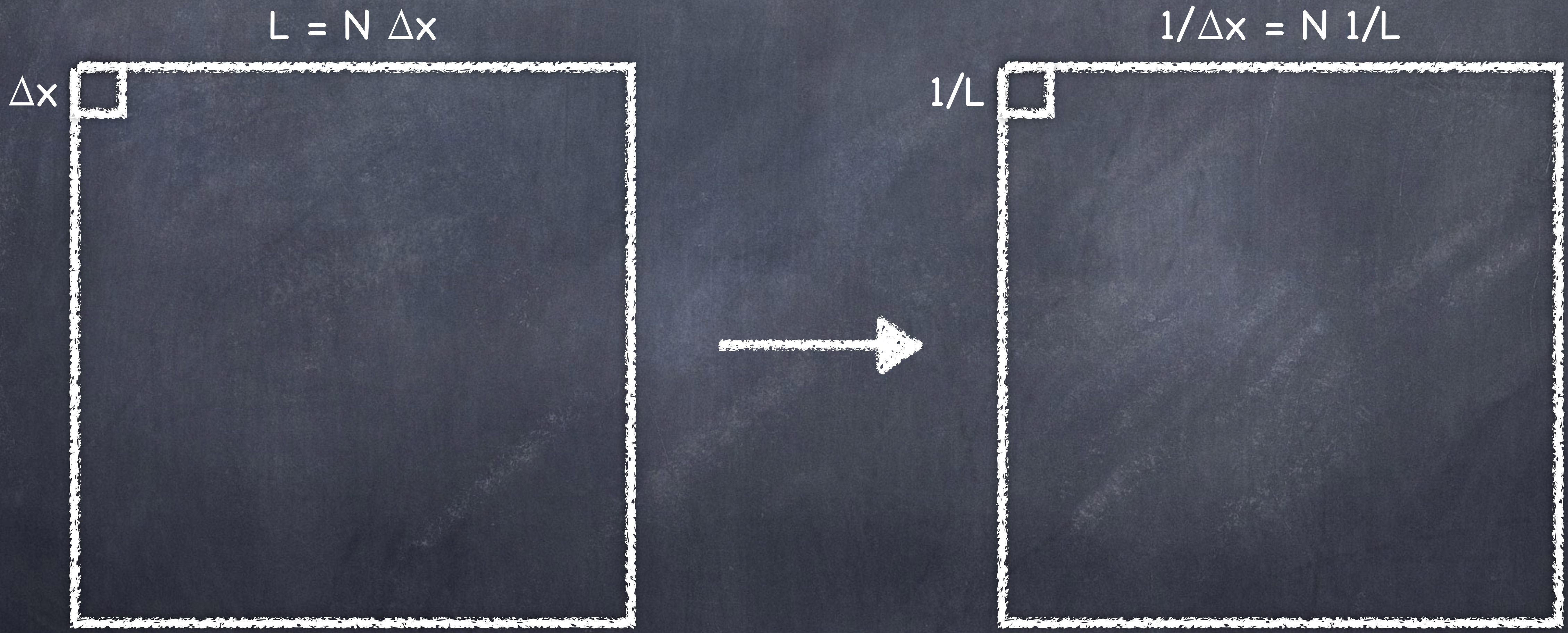
Power Spectral Density (PSD) of the phase, function of the spatial frequency

Kolmogorov/von Kármán model

- Kolmogorov : outerscale of turbulence $\mathcal{L}_0$ is infinite.
- One can refine the model by considering also ℓ_0.
- ∃ other models with a finite $\mathcal{L}_0$ (and a non-zero ℓ_0).

Energy cascade: wind shear => turbulent energy injected into the system via a large eddy ($\mathcal{L}_0$) which splits into smaller and smaller eddies (ℓ_0), and is finally viscously dissipated. Interval [ℓ_0, $\mathcal{L}_0$] = inertial range.

(A reminder of discrete Fourier transform (DFT)…)



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$$\Phi_\varphi(\vec{\nu}) = 0.0228\ r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{6}}$$

Which, numerically written, and by considering wavefronts made of 'dim' pixels corresponding to 'L' meters, becomes:
(re-writing - "de-dimensionalizing" - the equation with L_0=L_0 L/L and ν=ν L/L…)

```
freq = findgen(dim)
dsp  = .0228*(L/r0)^(5/3.)*L^2*(freq^2+(L/L0)^2)^(-11./6)
```

And which (with the right frequency scale) can be plot with:

```
plot_oo, 1./L*findgen(dim), dsp, XR=[1/L/1.2,dim*1/L*1.2], /XS
```

=> make a function that computes PSD(L_0, r_0, dim, L) and plot it for different [r_0 , L_0]… [with, for example: dim=1000L, L=100., r0=0.1, L0=100.,10.,1.]