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$$\Phi_{\varphi}(\vec{\nu}) = 0.0228 \; r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2} \right)^{-\frac{11}{6}}$$

Which, numerically written, and by considering wavefronts made of 'dim' pixels corresponding to 'L' meters, becomes:

(re-writing - ``de-dimensionalizing'' - the equation with $L_0 = L_0 \; L/L$ and $\nu = \nu \; L/L$…)

```
freq = findgen(dim)
dsp  = .0228*(L/r0)^(5/3.)*L^2*(freq^2+(L/L0)^2)^(-11./6)
```

And which (with the right frequency scale) can be plot with:

```
plot_oo, 1./L*findgen(dim), dsp, XR=[1/L/1.2,dim*1/L*1.2], /XS
```

=> make a function that computes PSD(L_0, r_0, dim, L) and plot it for different [r_0, L_0]… [with, for example: dim=1000L, L=100., r0=0.1, L0=100.,10.,1.]

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Example of a function that computes the sum of two parameters:

```
function sum_of_two_parameters, par1, par2
result=par1+par2
return, result
end
```

Compile and run the function (written, e.g., in a file sum2.pro):

```
idl > .r sum2
—> % Compiled module: SUM_OF_TWO_PARAMETERS
idl > res = sum_of_two_parameters(2,1)
idl > print, res
—> 3
```

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```
1  function dsp_theo, dim, L, r0, L0
2
3  freq = findgen(dim)
4  dsp  = .0228*(L/r0)^(5/3.)*L^2*(freq^2+(L/L0)^2)^(-11./6)
5
6  ;to be plotted afterwards with:
7  ;plot_oo, 1./L*findgen(dim), dsp, XR=[1/L/1.2,dim*1/L*1.2], /XS, $
8  ;         TIT='PSD(L0)', XTIT='spatial frequency [1/m]', YTIT='PSD'
9  ;oplot  , 1./L*findgen(dim), dsp, LINE=1
10 ;playing, e.g., with L0=100.,10.,1., or r0=.05, .1, .2
11
12 return, dsp
13 end
```


L0 = 100 m, 10 m, 1 m

r0 = 5 cm, 10 cm, 20 cm
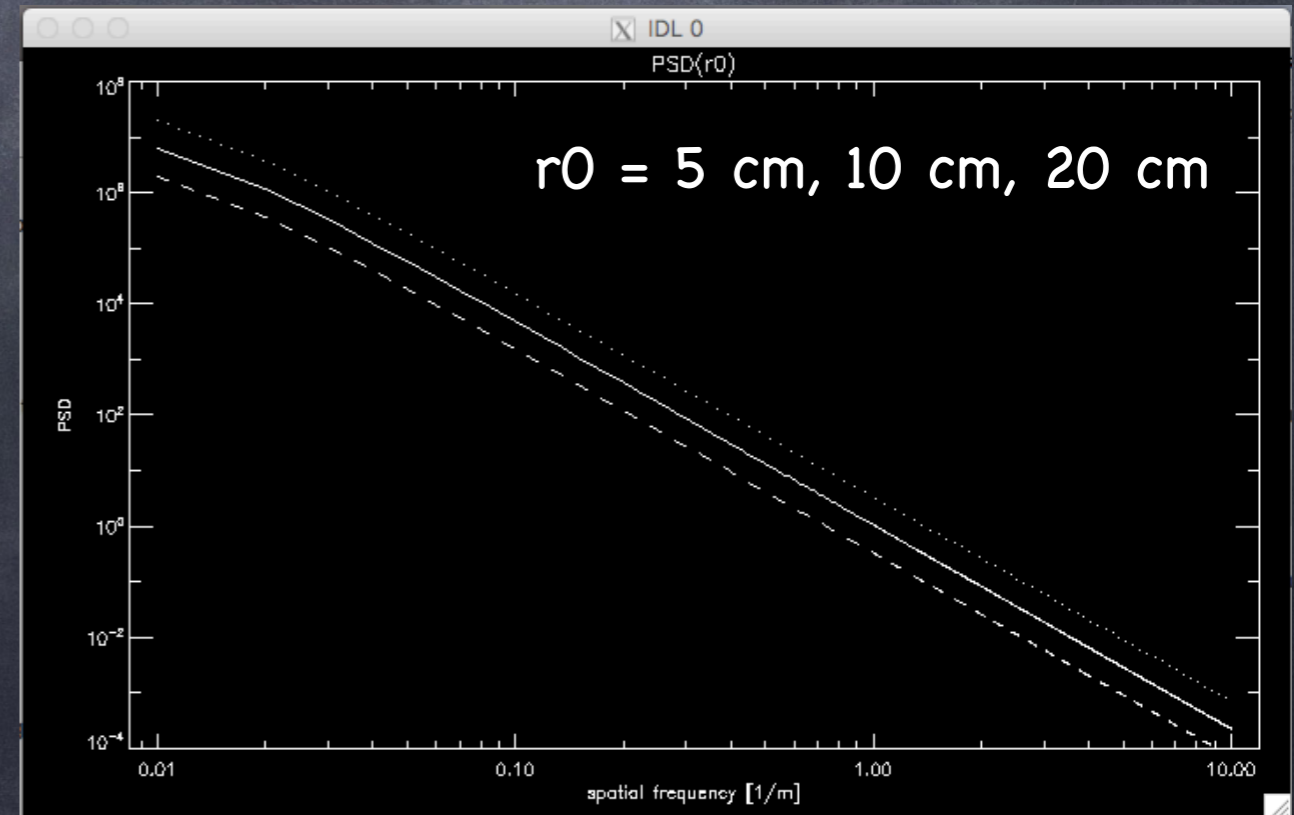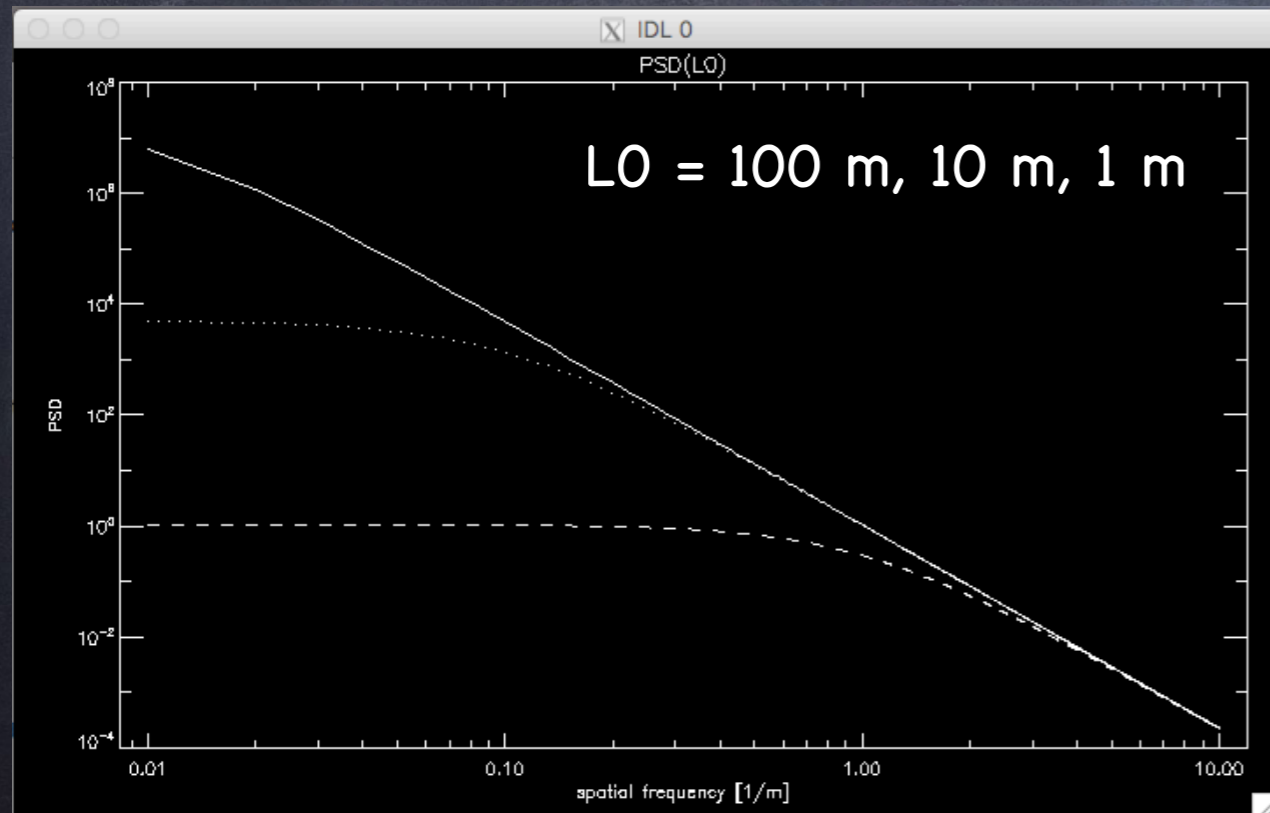
```
REPORT

- Preliminary measures
+ introduction
+ PSD|(r0, L0) plot
+ ccl on influence of r0 and L0
+ (more to come...)
```

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-> For next time: read Aime (Sec. 1 & Sec. 2) and Maire (Chap.1)...

Chapitre 1

Introduction

Jérôme Maire, PhD thesis (in French), chap.1

Long Telescopes may cause Objects to appear brighter and larger than short ones can do, but they cannot be so formed as to take away the confusion of the Rays which arises from the Tremors of the Atmosphere.
I. Newton, *1717 Optics, Sec. Ed., Book I, Part I, Prop.VIII*

Teaching astronomical speckle techniques

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Abstract

This paper gives an introduction to speckle techniques developed for high angular-resolution imagery in astronomy. The presentation is focussed on fundamental aspects of the techniques of Labeyrie and Weigelt. The formalism used is that of Fourier optics and statistical optics, and corresponds to graduate level. Several new approaches of known results are presented. An operator formalism is used to identify similar regions of the bispectrum. The relationship between the bispectrum and the phase closure technique is presented in an original geometrical way. Effects of photodetection are treated using simple Poisson statistics. Realistic simulations of astronomical speckle patterns illustrate the presentation.

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-> Perturbed wavefront generation

The well-known FFT method allows us to generate phase screens $\varphi(\vec{r})$, where $\vec{r}$ is the two-dimensional position within the phase screen, assuming usually either a Kolmogorov or a von Karman spectrum $\Phi_\varphi(\vec{\nu})$, where $\vec{\nu}$ is the two-dimensional spatial frequency, from which is computed the modulus of $\tilde{\varphi}(\vec{\nu})$, the Fourier transform of $\varphi(\vec{r})$. Assuming the near-field approximation and small phase perturbation [3], the von Karman/Kolmogorov spectrum is given by

$$\Phi_\varphi(\vec{\nu}) = 0.0229 r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2} \right)^{-\frac{11}{6}}, \qquad (1)$$

where r_0 is the Fried parameter and $\mathcal{L}_0$ is the wavefront outer scale (infinite for the Kolmogorov model). Within the framework of the classical FFT-based technique, a turbulent phase screen $\varphi_L(\vec{r})$ of physical length L is obtained by taking the inverse FFT of $\tilde{\varphi}_L(\vec{\nu})$, the modulus of which is obtained from Eq. (1) by applying the definition of the power spectrum, which is

$$\Phi_\varphi(\vec{\nu}) = \lim_{L \to \infty} \left(\frac{\langle |\tilde{\varphi}_L(\nu)|^2 \rangle}{L^2} \right)$$

$$\Rightarrow |\tilde{\varphi}_L(\nu)| \simeq L r_0^{-\frac{5}{6}} \sqrt{0.0228} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2} \right)^{-\frac{11}{12}}, \quad (2)$$

and which phase is random and uniformly distributed.

(From Carbillet & Riccardi, sec. 2: read it as well...)

(the same manipulation as before is applied here in order to obtain the numerical formulation here below.)

The obtained phase screen is thus numerically written

$$\varphi_L(i,j) = \sqrt{2}\sqrt{0.0228} \left(\frac{L}{r_0} \right)^{\frac{5}{6}} \left\{ \text{FFT}^{-1} \left[\left(k^2 + l^2 \right. \right. \right.$$
$$\left. \left. \left. + \left(\frac{L}{\mathcal{L}_0} \right)^2 \right)^{-\frac{11}{12}} \exp\{\imath \theta(k,l)\} \right] \right\}, \qquad (3)$$

where i and j are the indices in the direct space, k and l are the indices in the FFT space, {} stands for either *real part of* or *imaginary part of*, $\imath$ is the imaginary unit, and θ is the random uniformly distributed phase (between $-\pi$ and π). The factor $\sqrt{2}$ comes from the fact that here we use both the real and imaginary parts of the original complex generated FFT phase screens, which are independent one from the other [4]. This kind of phase screen suffers, however, from the lack of spatial frequencies lower than the inverse of the necessarily finite length L of the simulated array.