On the Difference between Seeing and Image Quality: When the Turbulence Outer Scale Enters the Game

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We attempt to clarify the frequent confusion between seeing and image quality for large telescopes. The full width at half maximum of a stellar image is commonly considered to be equal to the atmospheric seeing. However the outer scale of the turbulence, which corresponds to a reduction in the low frequency content of the phase perturbation spectrum, plays a significant role in the improvement of image quality at the focus of a telescope. The image quality is therefore different (and in some cases by a large factor) from the atmospheric seeing that can be measured by dedicated seeing monitors, such as a differential image motion monitor.

Seeing and *image quality* are two quantities that are frequently confused in the field of astronomical instrumentation. The first is an inherent property of the atmospheric turbulence, which is independent of the telescope that is observing through the atmosphere. The second, defined as the full width at half maximum (FWHM) of long-exposure stellar images, is a property of the images obtained in the focal plane of an instrument mounted on a telescope observing through the atmosphere. Without considering instrumental aberrations, one remaining property of the turbulence that affects the image quality is the outer scale: the size of the largest turbulent eddies present in the atmosphere. It has been observed that the image quality in a large telescope is always lower than the seeing, owing to the finite outer scale of the turbulence, as opposed to the commonly used Kolmogorov theory that considers an infinite outer scale.

In this article we discuss the dependence of atmospheric long-exposure resolution on the outer scale of the turbulence over the practically interesting range of telescope diameters and wavelengths. We show that this dependence is efficiently predicated by a simple approximate formula introduced in the literature in 2002. The practical consequences for operation of large telescopes are discussed and an application to on-sky data is presented.

Background and definitions

In practice the resolution of ground-based telescopes is limited by the atmospheric turbulence, called “seeing”. It is traditionally characterised by the Fried parameter (r_0) – the diameter of a telescope such that its diffraction-limited resolution equals the seeing resolution. The well-known Kolmogorov turbulence model describes the shape of the atmospheric long-exposure point spread function (PSF), and many other phenomena, by this single parameter r_0. This model predicts the dependence[1] of the PSF FWHM (denoted ε_0) on wavelength (λ) and inversely on the Fried parameter, r_0, where r_0 depends on wavelength (to the power –1/5) and airmass (to the power 3/5). In the following, we assume that r_0 and ε_0 refer to observations at the zenith. In addition, by adopting a standard wavelength of 500 nm, we can refer to ε_0 in place of r_0 for defining the strength of the turbulence, and this single parameter is nowadays usually called seeing. The equivalence between FWHM of a long-exposure image and seeing is indeed only valid in the Kolmogorov model, in which the energy is injected into the atmosphere at infinite scales, and is gradually transferred to smaller and smaller scales (a cascade process) until air viscosity dissipates it on scales of few mm (the inner scale l_0).

In reality, the physics of turbulence implies that the spatial power spectral density of phase distortions deviates from the pure power law at low frequencies, i.e. the energy is not injected into the atmosphere at infinite scales, but rather at finite scales. The popular von Kàrmàn turbulence model introduces an additional parameter, the outer scale L_0, referring to a cut-off in the turbulence spectrum at low frequencies. The Kolmogorov model corresponds then to the particular case of L_0 of infinity.

A finite L_0 reduces the variance of the low order modes of the turbulence, and in particular decreases the image motion (the tip-tilt). The result is a decrease of the FWHM of the PSF. In the von Kàrmàn model, r_0 describes the high frequency asymptotic behaviour of the spectrum where L_0 has no effect, and thus r_0 loses its sense of an equivalent wavefront coherence diameter. The differential image motion monitors (DIMM; Sarazin & Roddier, 1990) are devices that are commonly used to measure the seeing at astronomical sites. The DIMM delivers an estimate of r_0 based on measuring wavefront distortions at scales of ~ 0.1 m, where L_0 has no effect. By contrast, the absolute image motion and long-exposure PSFs are affected by large-scale distortions and depend on L_0. In this context the Kolmogorov expression for ε_0[1] is therefore no longer valid.

Proving the von Kàrmàn model experimentally would be a difficult and eventually futile goal as large-scale wavefront perturbations are anything but stationary. However, the increasing number of estimation campaigns worldwide over the past few years has firmly established that the turbulence phase spectrum does deviate from a power law (i.e. it does not match the Kolmogorov model at low frequencies), and the additional L_0 parameter provides a useful first-order description of this behaviour.

The purpose of this article is precisely to discuss the modifications of the Kolmogorov expression for ε_0 implied by the presence of a finite outer scale L_0, and to further establish the difference between seeing and FWHM of a stellar image. A first order approximation of the FWHM of the atmospheric PSFs (ε_{vK}) under von Kàrmàn turbulence was proposed by Tokovinin (2002), with a dependence (see note[2]) on ε_0 scaled by an expression involving the ratio of r_0 and L_0. The FWHM of long-exposure PSFs, ε_{vK}, is no longer equivalent to the seeing, but is a function of the seeing (ε_0), r_0 and L_0. We recall that while r_0 depends on the wavelength, L_0 does not. In the following, we discuss the validity of using ε_{vK} rather than ε_0, by presenting several results from extensive numerical simulations. At this point it is important to note that the FWHM ε_{vK} is independent of the telescope diameter.

Our investigations focus on telescope diameters ranging from 0.1 m to 42 m, with wavelength domain ranges from the *U*-band to *M*-band, while the seeing ranges from 0.1 to 1.8 arcseconds. Several L_0 cases were considered from 10 m to an infinite value. Our long-exposure PSFs are generated by Fourier transforms of 1000 atmospheric turbulence phase screen realisations adopting the von Kàrmàn model. The phase screens consist of 8192 × 8192 arrays to handle both atmospheric statistics and aliasing effects, and the same set is used for all telescope diameters. Several investigations were carried out on the phase screens to ascertain their properties (r_0, L_0). For the sake of simplicity, we do not discuss the details of these investigations or the analytical treatment leading to the expression[2] for ε_{vK}. For more information one should refer to Martinez et al. (2010).

Outer scale and telescope diameter

Figure 1 aims at defining the general trend of atmospheric FWHM in large telescopes in the presence of the outer scale of the turbulence. Several cases of L_0 are presented: 10 m, 22 m (Paranal median value), 50 m and 65 m. We compare the numerical FWHM (extracted from the simulated PSFs) to the analytical expectation, ε_{vK}, and the seeing, ε_0. From Figure 1 it is straightforward to see that the FWHM is lower than the seeing ε_0 in all cases. In addition the FWHM nicely fulfilled the analytical approximation for ε_{vK} for all telescope diameters except small ones, where our treatment of the diffraction is too crude (the results are affected by coarse pupil sampling).

The validity of the expression[2] for ε_{vK} is hereby confirmed in an $L_0/r_0 > 80$ domain. As the outer scale L_0 gets smaller, the difference between FWHM and seeing increases. In all cases, the difference is significant and cannot be neglected. In addition, the effect of the outer scale is observable for all telescope diameters, and not only for large telescopes where diameters correspond to a significant fraction of L_0 (i.e., the common assumption that ε_0 is valid in a domain where telescope diameters are smaller than L_0 does not hold). We found the expression[2] for ε_{vK} to be valid at least for $L_0/D \leq 500$. Very small telescope diameters do asymptotically converge to ε_0, but for smaller diameters than usually considered and for very large outer scale L_0 values (e.g., D = 0.1 m and L_0 = 100 m, D = 0.2 m and L_0 > 400 m).

Wavelength and seeing dependence

Here we consider an 8-metre telescope, a fixed outer scale L_0 = 22 m, and 0.83 arcsecond seeing at 0.5 μm, while the imaging wavelength is varying from the *U*-band to the *M*-band (0.365–4.67 μm). The results are presented in Figure 2 (left), where a stronger dependence of the FWHM on wavelength compared to that expected by ε_0 is noticeable, and again an agreement with the expression for ε_{vK} is demonstrated. Considering the same value of L_0 and telescope diameter, but fixed wavelength (0.5 μm), we analysed the seeing dependency of the FWHM; the results are shown in Figure 2 (right). For

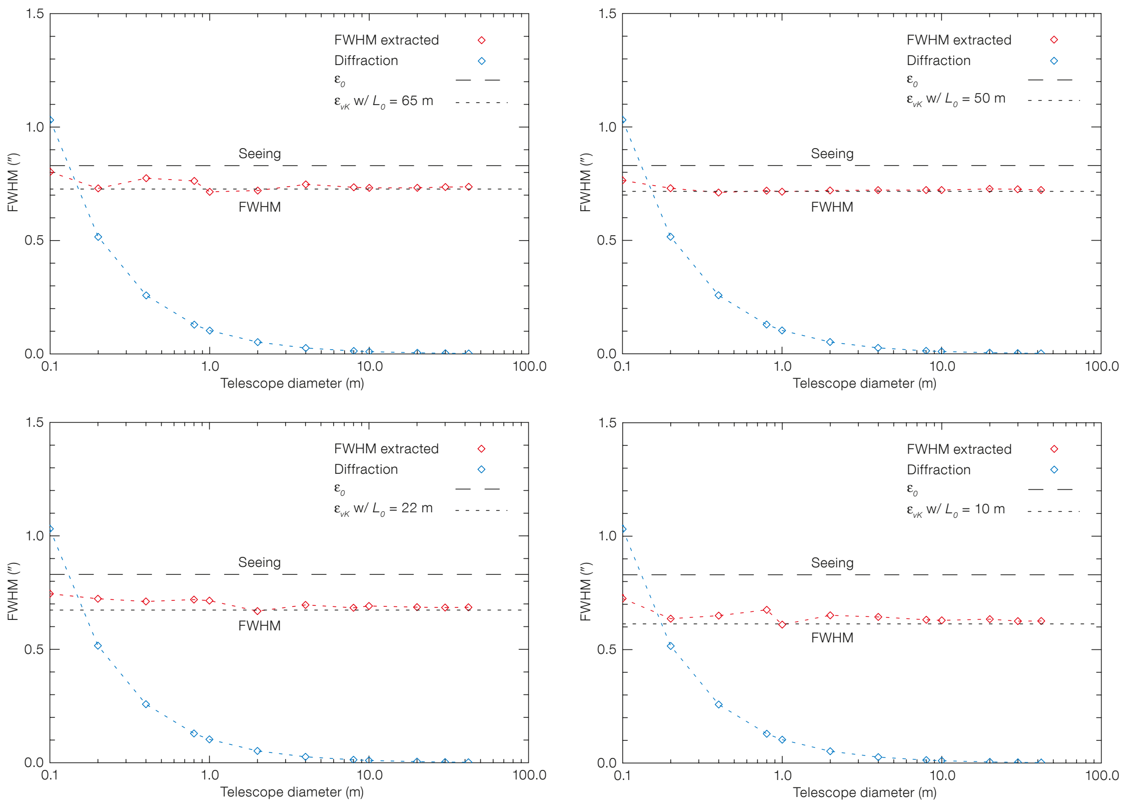

Figure 1. The atmospheric FWHM of simulated long-exposure PSFs versus telescope diameter for several turbulence outer scale L_0 values (10, 22, 50, and 65 m, for ε_0 = 0.83 arcseconds at λ = 0.5 μm). The diffraction FWHM has been quadratically removed from the extracted FWHM.

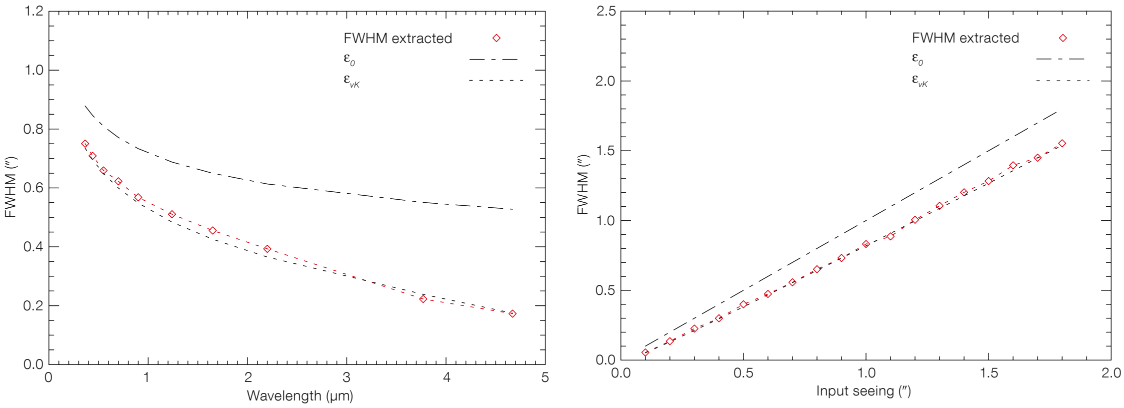

Figure 2. Dependence of the FWHM on wavelength (left, fixed ε_0 = 0.83 arcseconds) and seeing (right, fixed λ = 0.5 µm). Other parameters are L_0 = 22 m, D = 8 m (typical for the VLT).

all seeing values, the extracted FWHM clearly follows ε_{vK} and not ε_0. The agreement with the expression[2] for ε_{vK} is therefore demonstrated for both wavelength (L_0/r_0 > 10) and seeing dependence (L_0/r_0 > 20). **The FWHM of long-exposure PSFs is not the seeing.**

Discussion of the case of Paranal

We discuss here the particular case of the VLT site at Paranal assuming standard seeing conditions (0.83 arcsecond at 0.5 µm), and for several outer scale L_0 values including the Paranal median value (22 m) and its corresponding 1σ values (13 and 37 m). Figure 3 quantifies the ratio of the seeing ε_0 to the FWHM ε_{vK} for different wavelengths. The difference is substantial and can exceed a factor of two in the infrared (IR). For instance, the FWHM ε_{vK} is lower than ε_0 by 19 % in the visible, and it is even more dramatic in the near-IR, where it is lower by 29.7 % (*H*-band) and 36.3 % (*K*-band). Figure 3 also strongly emphasises the importance of obtaining reliable estimation of L_0 at a telescope site, thus requiring simultaneous measurements of ε_0 and L_0.

An intensive multi-instrument campaign of L_0, surface layer, and seeing characterisation was carried out at Paranal in 2007 and has been recently presented in Dali Ali et al. (2010). This study has, for the first time, provided the profile of the outer scale L_0 (*h*) (where *h* stands for the altitude) at Paranal, enabling the whole profile of the atmospheric turbulence to be separated into the respective contributions from the free, ground and surface layers. In this extensive study, the authors found outer scale L_0 values varying from a few metres (~ 10 m) in the ground layer to a maximum value of ~ 35 m appearing in the boundary layer (at 1 km). In addition, by comparing PSFs at visible and mid-IR wavelengths simultaneously, it is possible to extract the two parameters, ε_0 and L_0, assuming that the telescope's contribution to the image degradation can be neglected (Tokovinin et al., 2007).

On-sky data application

To relate the previous results to real situations, we have evaluated several stellar FHWMs from an image of Omega Centauri recorded with the IR-camera of MAD (the ESO Multi-conjugate Adaptive Optics Demonstrator, formerly installed at the VLT UT3). The image was obtained on 29 March 2007 in open loop (i.e. no AO correction is applied) with a 65-second integration time at a wavelength of 2.166 µm (bandwidth of 0.04 µm). We use this example as it provides a well-sampled image of a large field of view (57 arcseconds × 57 arcseconds).

The image is presented in Figure 4. The FWHM has been evaluated in the elongation-free direction of the stars, and derived using a 10th order polynomial fit to the radial profiles (a telescope PSF is a convolution of the atmosphere blur with diffraction, aberrations, guiding errors, etc. … and none of these factors is described by a Gaussian). A mean FWHM value of 0.51 arcseconds has been measured. By converting this FWHM into a seeing value ε_{vK} assuming the Paranal outer scale median value of 22 m, and with a proper scaling for wavelength (0.5 µm) and airmass (1.1), we found that the seeing during the acquisition of the image was equal to 1.01 arcseconds.

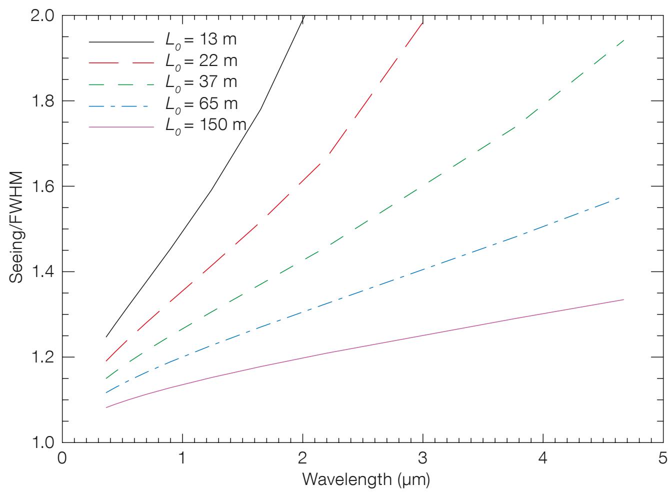

Figure 3. Ratio of seeing ε_0 to FWHM (ε_{vK}) as a function of the wavelength for several values of L_0. The atmospheric seeing is set to the Paranal standard value of 0.83 arcsecond at 0.5 µm.

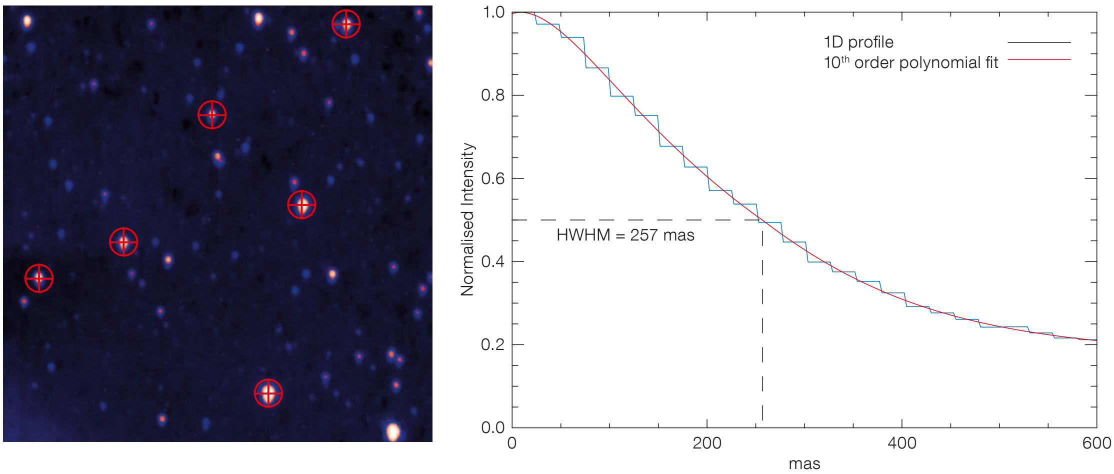

Figure 4. Left: VLT MAD image of Omega Centauri recorded in March 2007. The stars indicated were used for FWHM evaluation. Right: Normalised intensity profile of one star and the result of the FWHM evaluation (0.514 arcseconds) from a fit on the image minor axis.

DIMM seeing measurements were indeed evolving between 0.94 and 1.04 arcseconds (see Table 1) during this exposure. Not considering the outer scale presence (i.e. adopting ε_0), would have led to a value of 0.69 arcsecond seeing (wavelength and airmass corrected), more than 0.3 arcseconds different from the measured value ... quite a difference!

Although this test nicely supports the expression[2] for ε_{vK}, it relies on the chosen value of L_0; as a matter of fact we cannot guarantee that 22 m is a correct guess, nor did we consider potential internal telescope defects (a PSF is broadened by non-atmospheric factors as well). Indeed, optical aberrations and the outer scale of the turbulence act in opposite directions, and they can partially compensate for each other. Besides, the active optics also plays a role that is similar to the effect of the outer scale L_0. In analogy with the finite outer scale impact, partially corrected wavefronts, resulting e.g., from tip-tilt compensation (fast guiding) or low order adaptive optics (AO) correction, lead to a small effective L_0. All three effects — turbulence outer scale, partial AO correction and tip-tilt correction — reduce the low frequency content of the phase perturbation spectrum, but the gain in resolution over Kolmogorov turbulence is not cumulative. Therefore the search for an agreement with DIMM measurements should always be carefully considered, and would require statistical investigations, to prevent for example the surface layer (thin and time-varying turbulence layer occurring over the mountain) to confuse the situation (see Sarazin et al., 2008).

On the other hand, by considering the DIMM seeing measurements during the acquisition of the image (0.94–1.04 arcseconds), one could retrieve the outer scale L_0 value occurring at that time and the measured FWHM. We thus obtained a value of L_0 confined between 25 and 45 m, which appears to be realistic considering the Paranal median value of 22 m, and measurements obtained in the 2007 campaign at Paranal and presented in Dali Ali et al. (2010). Floyd et al. (2010) derived the value of the outer scale of the turbulence at the Magellan telescopes likewise, and they found an outer scale L_0 of 25 m.

Conclusion

This study has confirmed several aspects of the difference between seeing and image quality at an optical telescope:
– the FWHM of long-exposure stellar images obtained at a telescope is not the seeing;
– the outer scale of the atmospheric turbulence plays a significant role in the relationship between the seeing and the FWHM of an image. The effect of the outer scale is apparent for all telescope diameters. The expression ε_{vK} proposed by Tokovinin (2002) accurately predicts the dependence of atmospheric long-exposure resolution on the outer scale.

By not considering the presence of the outer scale of the turbulence, one is currently: (a) overestimating the image size expected for a large telescope, i.e. our telescopes could perform better than we predict; (b) underestimating the seeing if deduced from the FWHM of a long-exposure PSF, i.e. the seeing is actually poorer than we predict.

FWHM (arcsecond)	0.51
Seeing (arcsecond) (ε_0)	0.69
Seeing (arcsecond) (ε_{vK})	1.01
DIMM seeing (arcsecond)	0.94–1.04

Table 1. Conversion of the FWHM obtained on the MAD image of Omega Centauri (1st row) into seeing values (assuming L_0 = 22 m) using the expression[1] for ε_0 (2nd row), the expression[2] for ε_{vK} (3rd row) and compared to DIMM seeing (4th row). All values are for a wavelength of 0.5 μm and are corrected for airmass.

Reference

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Notes

1 The full expression for the FWHM of the Kolmorgorov PSF is $\varepsilon_0 = 0.976\ \lambda/r_0$.
2 The full expression for the FWHM of the van Kàrmàn PSF is $\varepsilon_{vK} \approx \varepsilon_0 \sqrt{(1 - 2.183\ (r_0/L_0)^{0.356})}$.