

Adaptive Optics (AO)

(and, en passant, also intro to image formation and atmospheric turbulence)

Marcel Carillet

[marcel.carillet@oca.eu]

lagrange.oca.eu/carillet/enseignement/M2-optiQ/

(Credit: Markus Feldt, Max Planck Institut für Astronomie — Heidelberg, 2025)

→ https://www2.mpia-hd.mpg.de/homes/feldt/post/02_adaptive_optics/

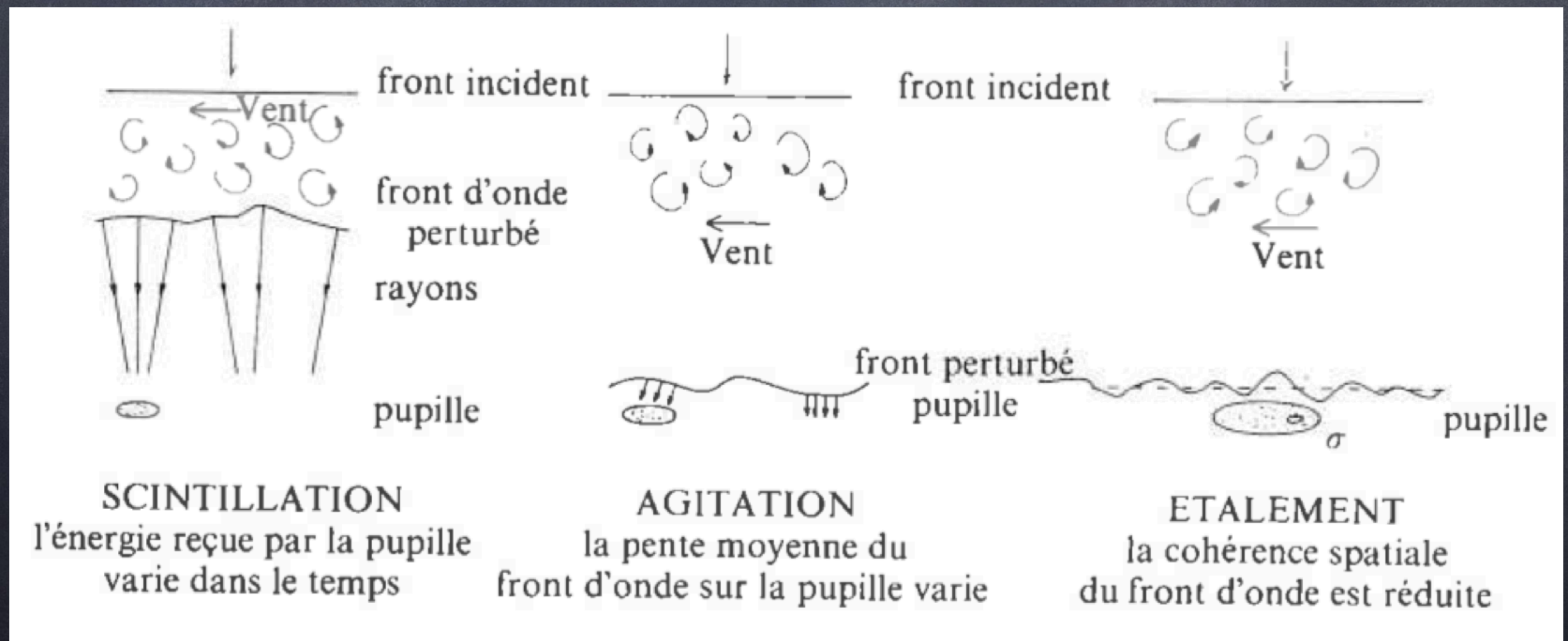
Menu

- High-angular resolution imaging in astronomy
- Atmospheric turbulence
- Numerical modelling of perturbed wavefronts
- Formation of resulting images (+detection noises)
- Adaptive optics (AO) error budget
- Post-AO point-spread function morphology
- Numerical modelling of an ideal AO system (but considering the anisoplanatic error)

Images & turbulence — 01

The image formed through turbulent atmosphere (optically speaking) is degraded:

- Scintillation (due to intensity fluctuation in the pupil).
- Agitation (due to angle-of-arrival variation).
- Spreading (due to a loss of spatial coherence).



Images & turbulence — 02

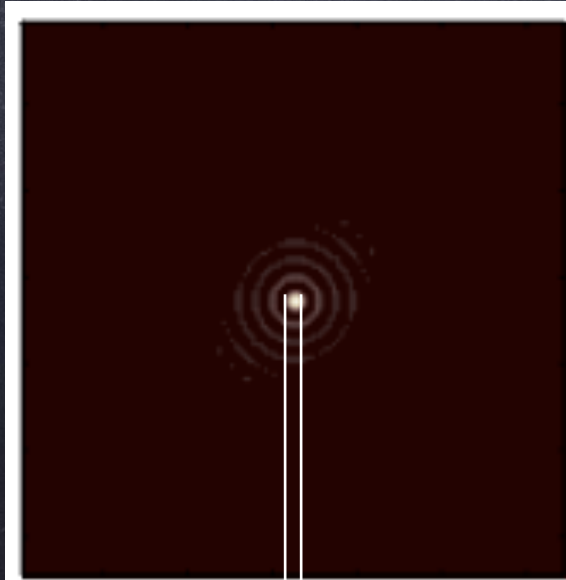
The object-image relation between the intensity $I(\alpha)$ in the image plane (i.e. the focal plane of the telescope) and the brightness $O(\alpha)$ of the object (in the sky) is a relation of convolution implying the point-spread function (PSF) $S(\alpha)$ of the whole ensemble telescope+atmosphere, with α the angular coordinates in the focal plane:

$$I(\vec{\alpha}) = O(\vec{\alpha}) * S(\vec{\alpha})$$

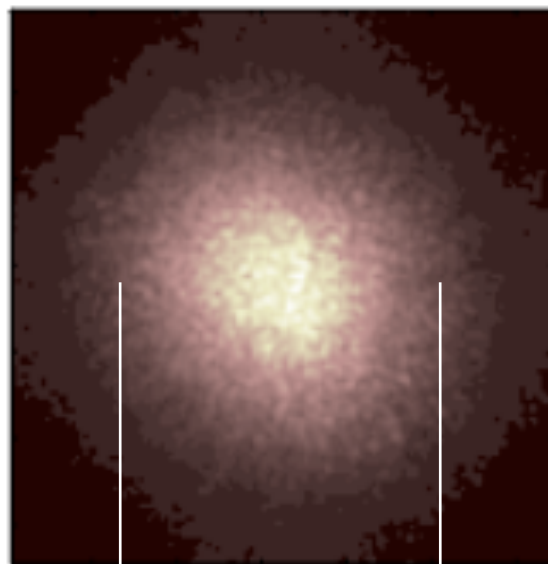
Images & turbulence — 03

$$I(\vec{\alpha}) = O(\vec{\alpha}) * S(\vec{\alpha})$$

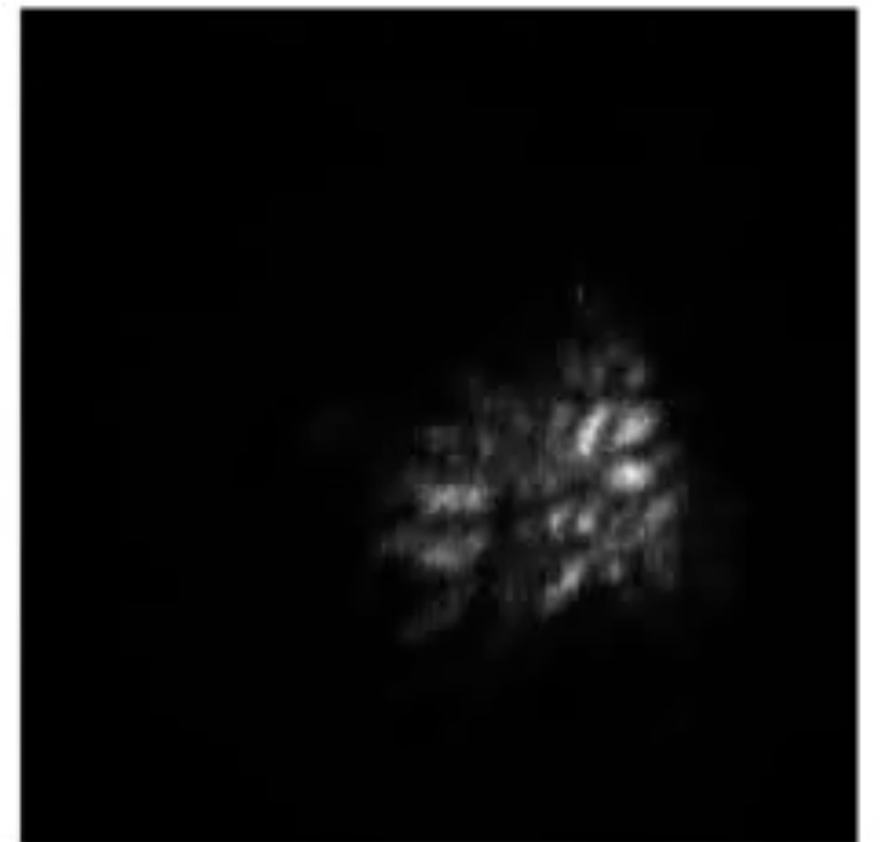
This relation is valid notably at the condition that the system is invariant by translation (everything happens within the isoplanatic domain)...



λ/D



λ/r_0

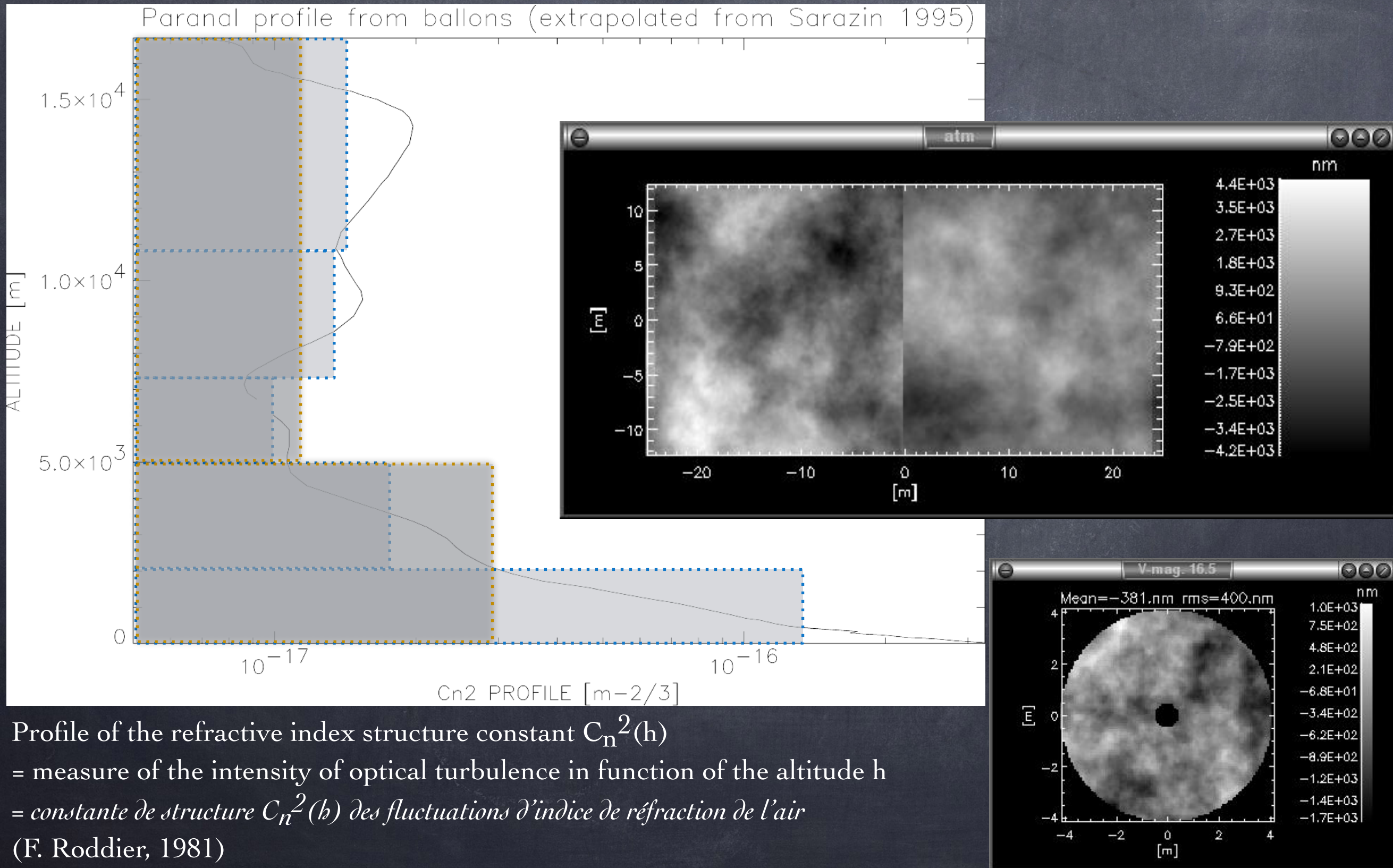


Images & turbulence — 04

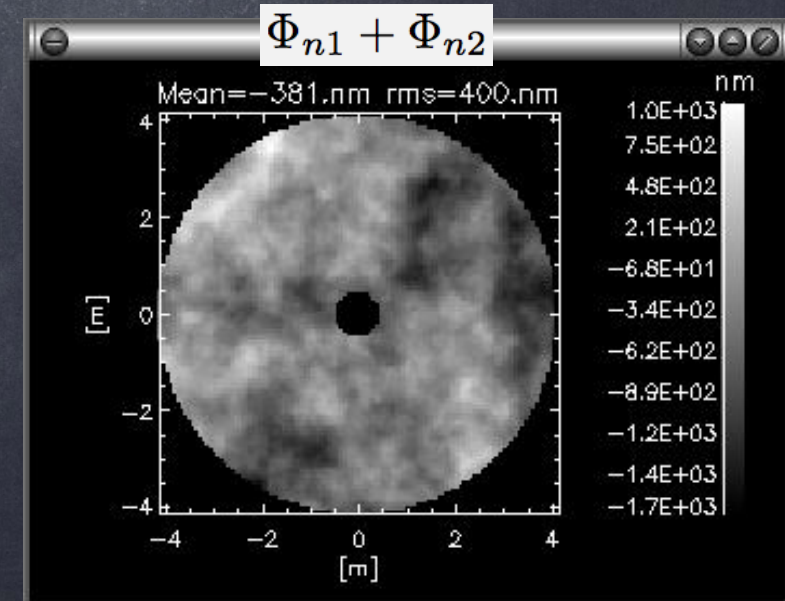
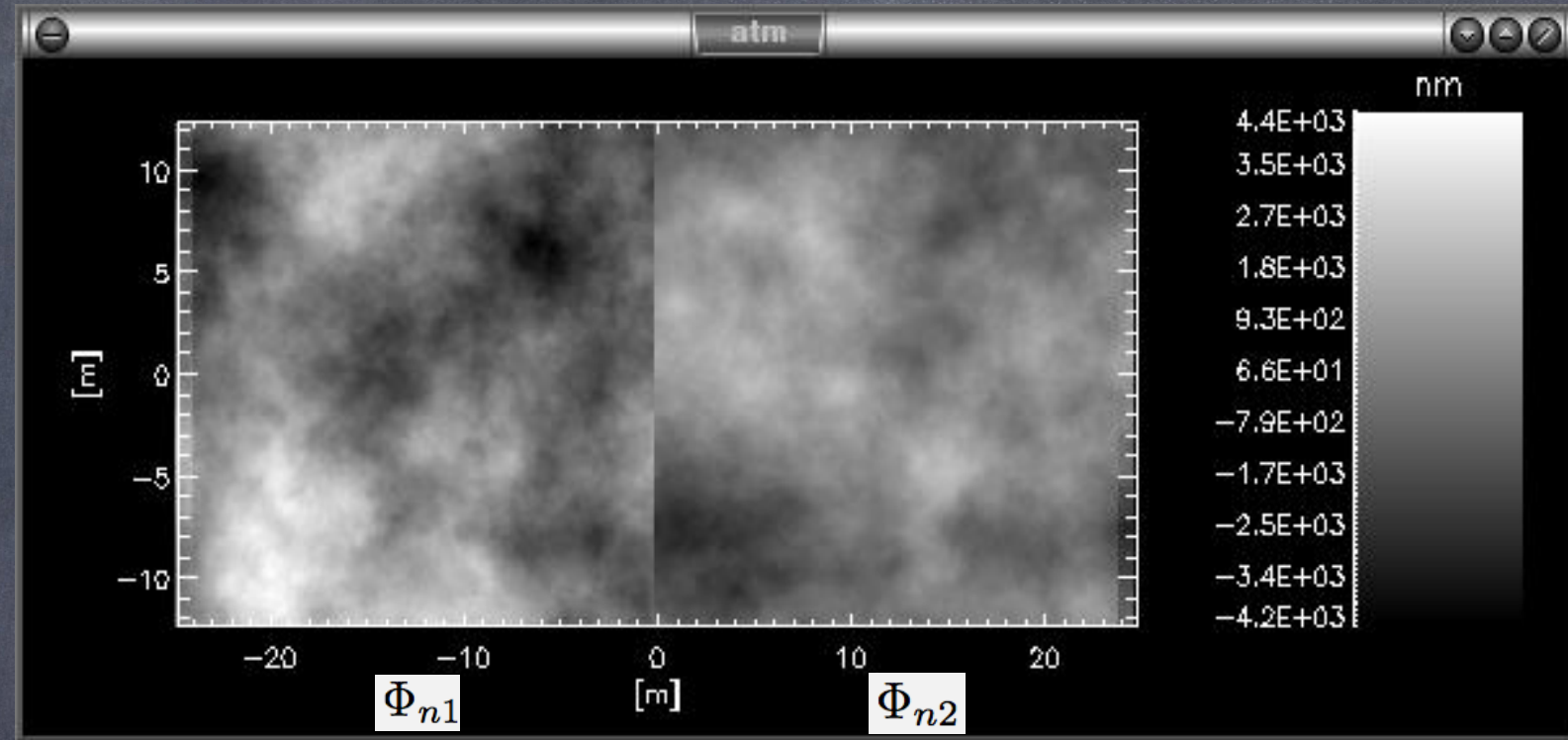
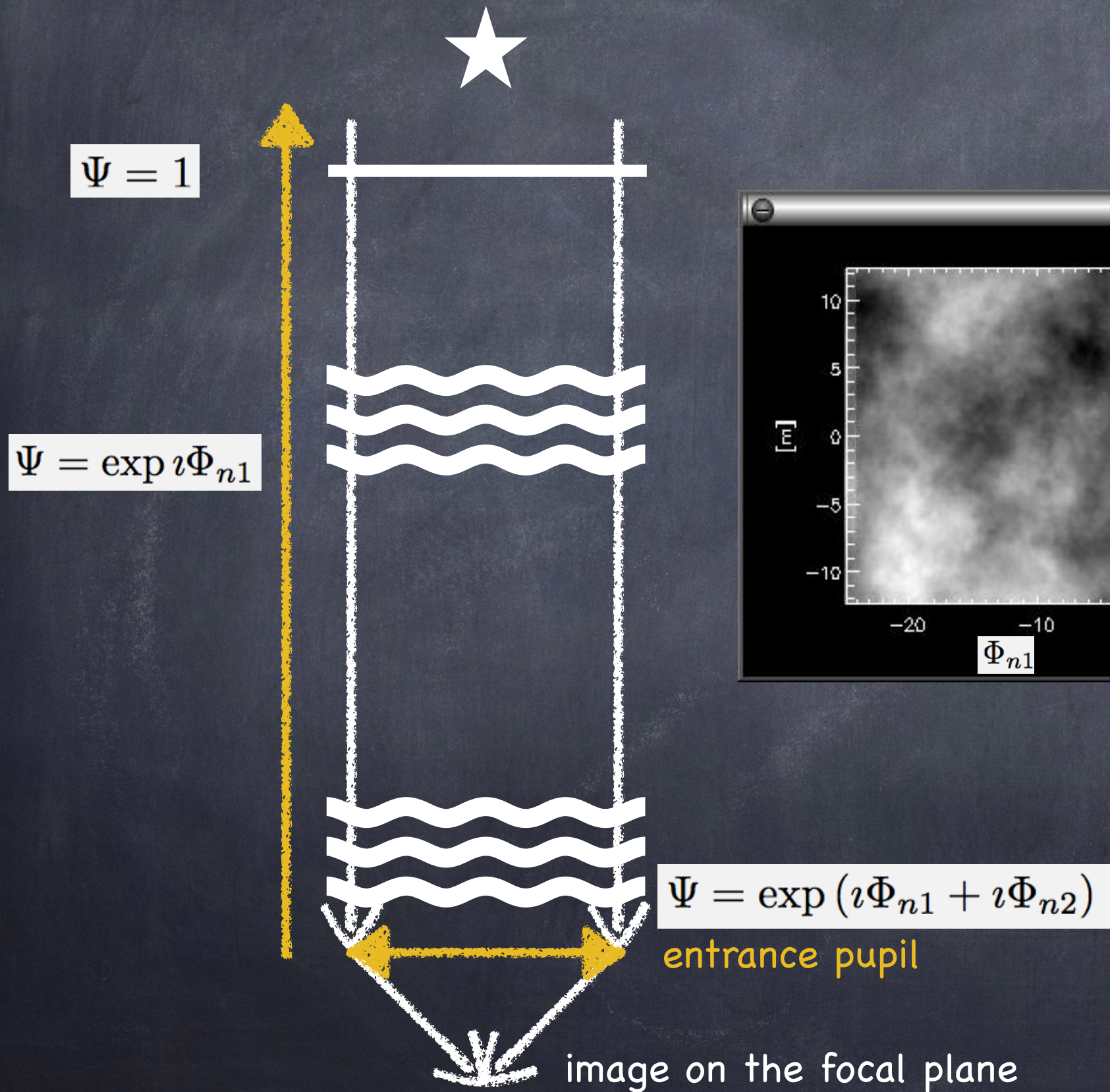
Some orders of magnitude concerning the turbulent atmosphere:

	$\lambda = 500 \text{ nm}$	$\lambda = 2.2 \mu\text{m}$
Fried parameter (r_0)	$\rightarrow 10 \text{ cm}$	60 cm
velocity of the turbulent layers (v)	$\rightarrow 10 \text{ m/s}$	id.
=> image FWHM ($\epsilon \approx \lambda/r_0$)	$\rightarrow 1''$	$\sim 1''$
=> evolution time ($\tau_0 \propto r_0/v$)	$\rightarrow 3 \text{ ms}$	18 ms

Images & turbulence — 05



Images & turbulence — 06



Images & turbulence — 07

entrance pupil



image on the focal plane



Images & turbulence — 08

The wavefront is, modulo $\lambda/2\pi$, proportional to the phase $\Phi(r)$ of the wave $\Psi(r)$ which has went through the turbulent atmosphere before reaching the telescope:

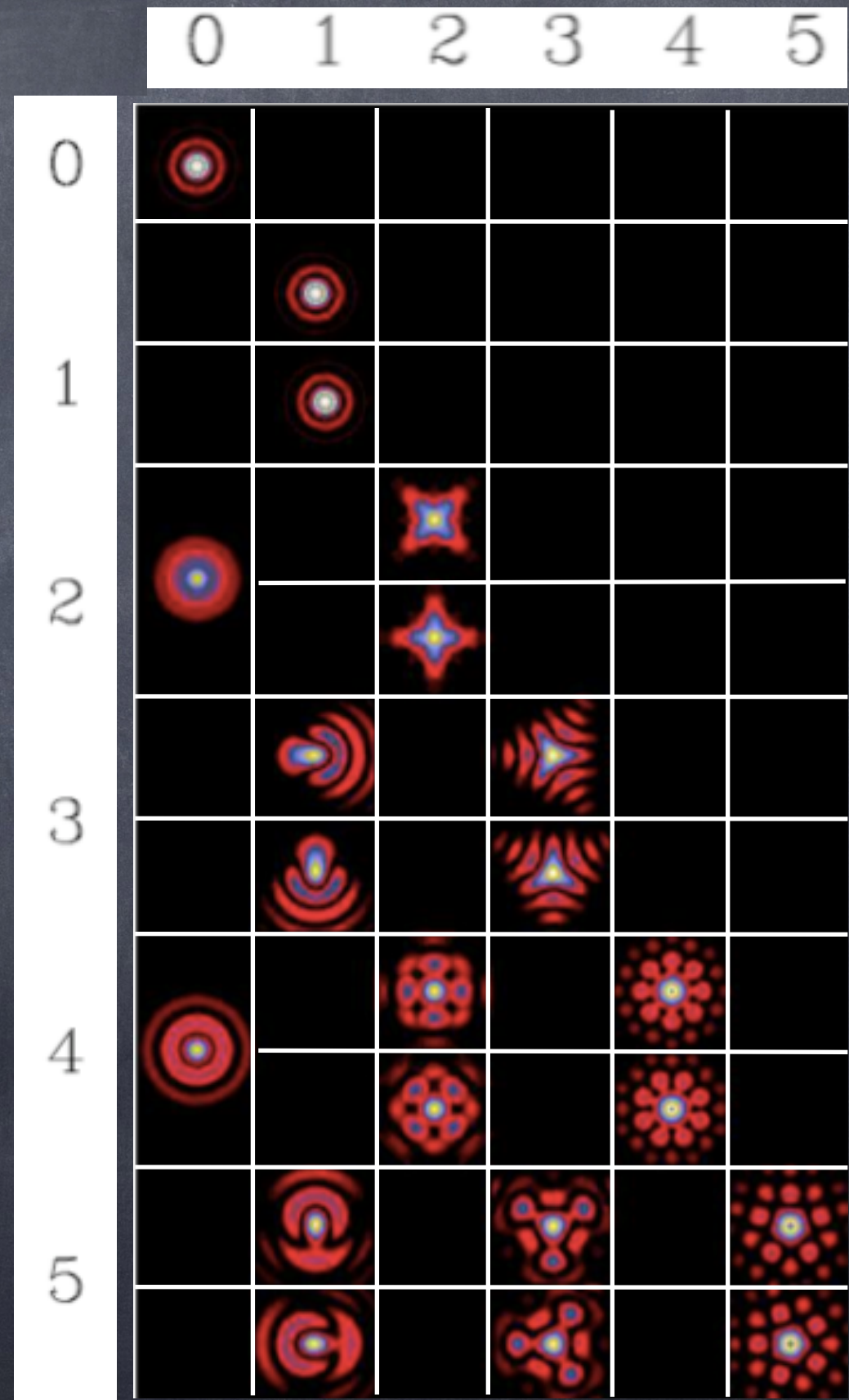
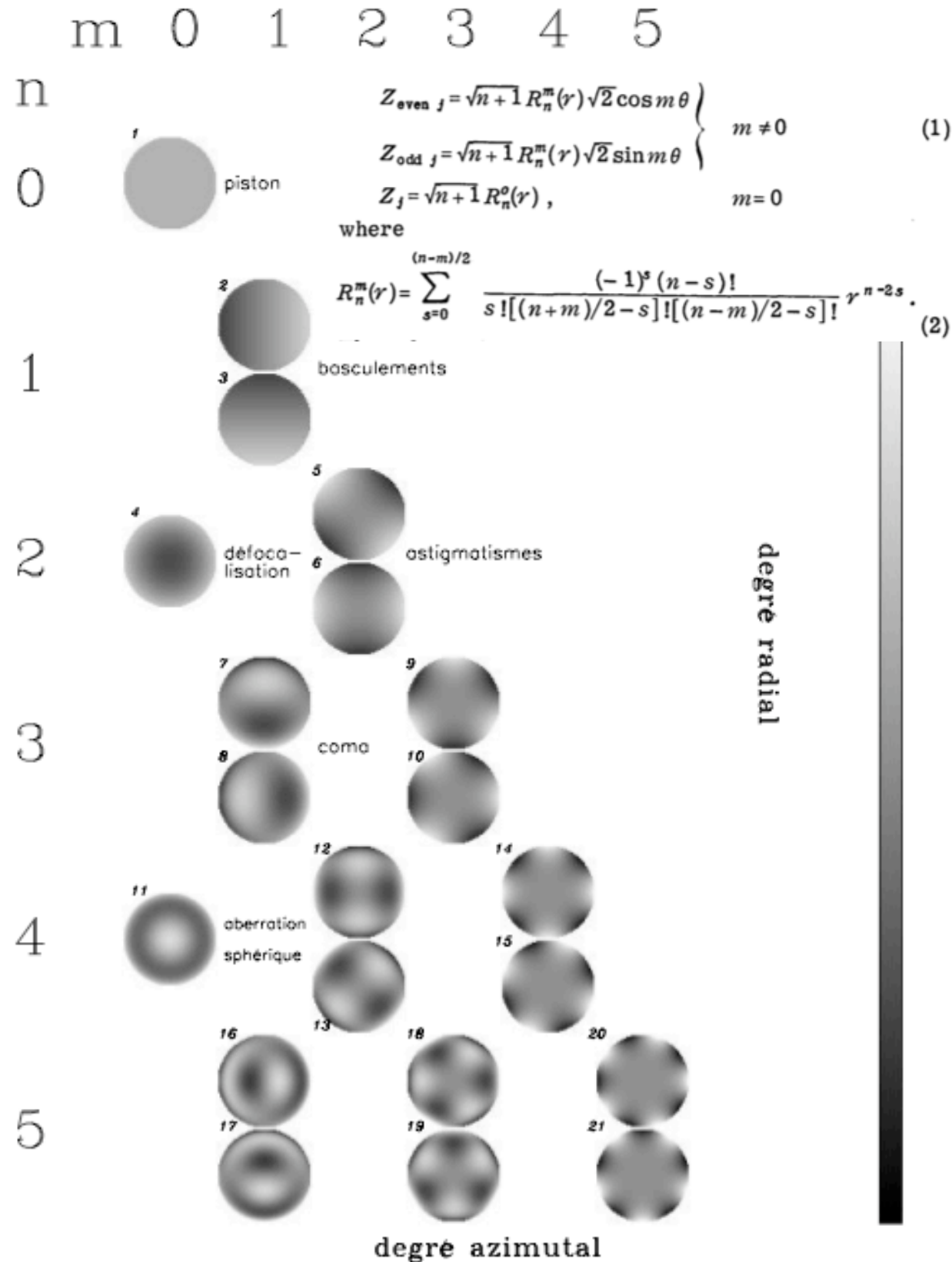
$$\Psi(\vec{r}) = A(\vec{r}) \exp\{i\Phi(\vec{r})\}$$

Note that this phase can be decomposed following a base of polynomials, for example Zernike ones:

$$\Phi(\vec{r}) = \sum_i a_i Z_i(\vec{r})$$

Images & turbulence — 09

polynômes de Zernike



entrance pupil

Images & turbulence — 10

turbulence intensity [$m^{1/3}$]

$$r_0 = 0.185 \lambda^{\frac{6}{5}} \cos(\gamma)^{\frac{3}{5}} \left[\int_0^\infty C_n^2(z) dz \right]^{-\frac{3}{5}}$$

dimension of r_0 ? r_0 in band H knowing that $r_0@500nm=10cm$?

$$\tau_0 = 0.36 \frac{r_0}{\bar{v}}$$

$$\epsilon_0 = 0.98 \frac{\lambda}{r_0}$$

$$\theta_0 = 0.314 \frac{r_0}{\bar{h}}$$

$$\bar{v} = \left(\frac{\int C_n^2(h) v(h)^{\frac{5}{3}} dh}{\int C_n^2(h) dh} \right)^{\frac{3}{5}}$$

$$N_s \simeq 0.34 \left(\frac{D}{r_0} \right)^2$$

$$\bar{h} = \left(\frac{\int C_n^2(h) h^{\frac{5}{3}} dh}{\int C_n^2(h) dh} \right)^{\frac{3}{5}}$$

Ref.: D.L.Fried (1955)
& F.Roddier (1981)

Number of speckles for $r_0=10cm$ and $D=1m$?
(also in H band)

Images & turbulence – 11

r_0 in band H knowing r_0 at 500nm ?...

$$r_0 = 0.185 \lambda^{\frac{6}{5}} \cos(\gamma)^{\frac{3}{5}} \left[\int_0^\infty C_n^2(z) dz \right]^{-\frac{3}{5}}$$

$$r_0^{H=1.65 \mu\text{m}} = r_0^{500 \text{ nm}} \left(\frac{1.65}{0.5} \right)^{\frac{6}{5}} \simeq 0.42$$

Number of speckles for $r_0=10\text{cm}$ and $D=1\text{m}$?...

$$N_S^{500 \text{ nm}} \simeq 0.34 \left(\frac{1.0}{0.1} \right)^2 \simeq 34$$

$$N_S^H \simeq 0.34 \left(\frac{1.0}{0.42} \right)^2 \simeq 2$$

Images & turbulence — 12

$$\Phi_{\varphi}(\vec{\nu}) = 0.0228 \, r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2} \right)^{-\frac{11}{6}}$$

Power Spectral Density (PSD) of the phase, function of the spatial frequency

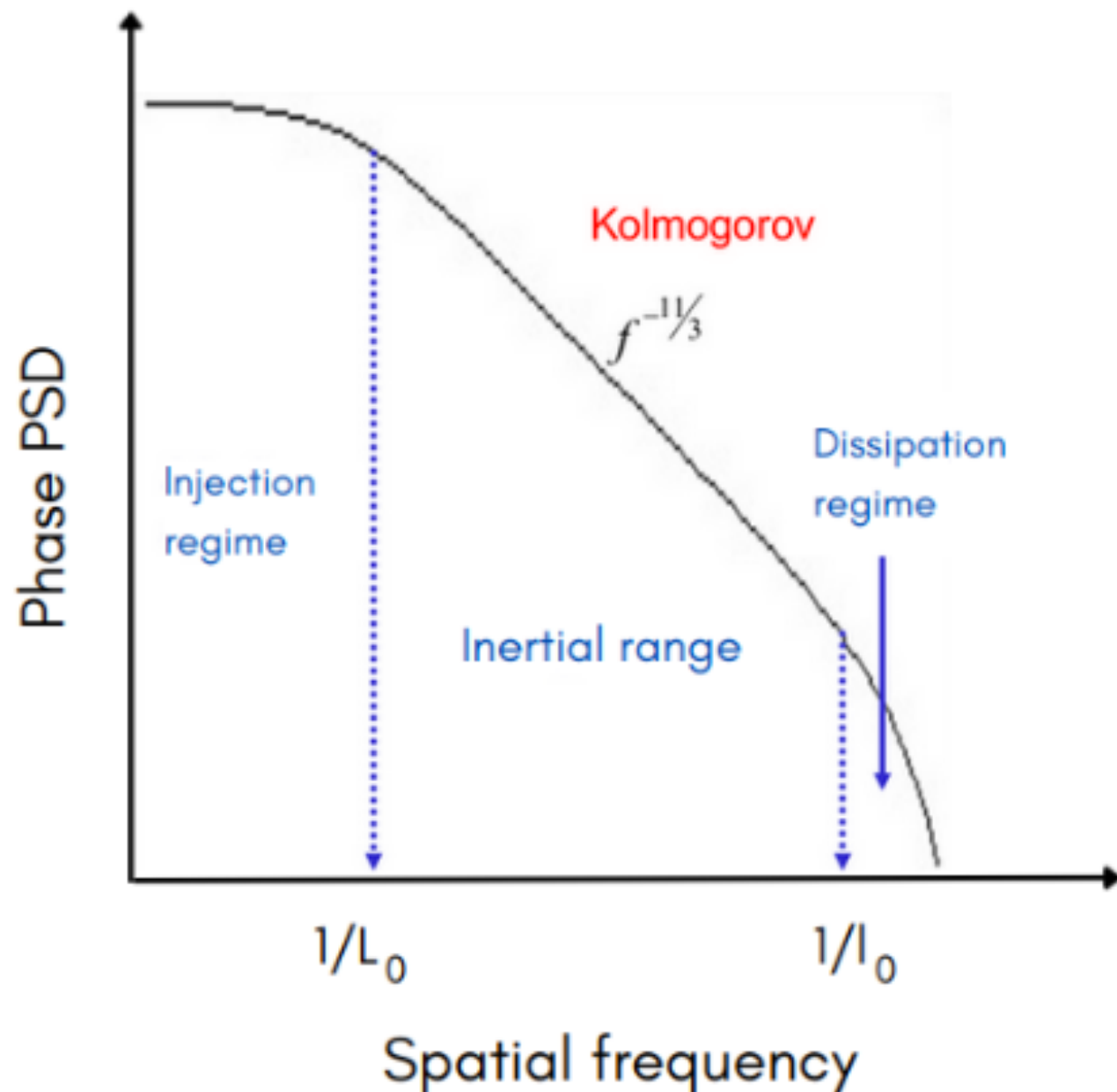
Kolmogorov/von Kármán model

- Kolmogorov : outerscale of turbulence $\mathcal{L}_0$ is infinite.
- One can refine the model by considering also ℓ_0 .
- $\exists$ other models with a finite $\mathcal{L}_0$ (and a non-zero ℓ_0).

Energy cascade: wind shear $\Rightarrow$ turbulent energy injected into the system via a large eddy ($\mathcal{L}_0$) which splits into smaller and smaller eddies (ℓ_0), and is finally viscously dissipated. Interval $[\ell_0, \mathcal{L}_0] =$ inertial range.

Images & turbulence — 13

$$\Phi_{\varphi}(\vec{\nu}) = 0.0228 \, r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2} \right)^{-\frac{11}{6}}$$

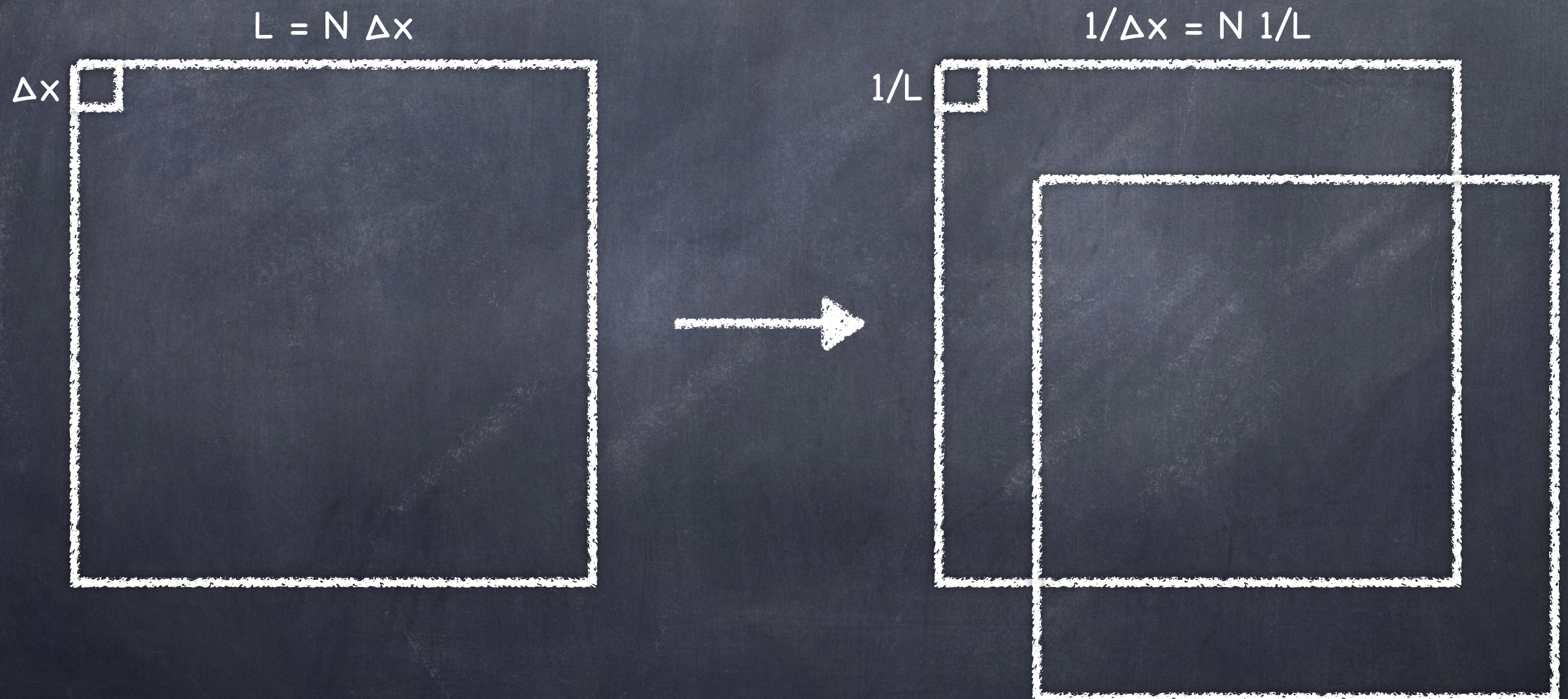


Energy cascade:

wind shear $\Rightarrow$ turbulent energy injected into the system via a large eddy ($\mathcal{L}_0$) which splits into smaller and smaller eddies (ℓ_0), and is finally viscously dissipated.

Interval $[\ell_0, \mathcal{L}_0]$ = inertial range.

(A reminder of discrete Fourier transform (DFT)...)



Images & turbulence — 14

$$\Phi_{\varphi}(\vec{\nu}) = 0.0228 \, r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2} \right)^{-\frac{11}{6}}$$

Which, numerically written, and by considering wavefronts made of 'dim' pixels corresponding to 'L' meters, becomes:
(re-writing - "de-dimensionalizing" - the equation with $L_0 = L_0 \, L/L$ and $\nu = \nu \, L/L \dots$)

```
freq = findgen(dim)
dsp   = .0228*(L/r0)^(5/3.)*L^2*(freq^2+(L/L0)^2)^(-11./6)
```

And which (with the right frequency scale) can be plot with:

```
plot_oo, 1./L*findgen(dim), dsp, XR=[1/L/1.2,dim*1/L*1.2], /XS
```

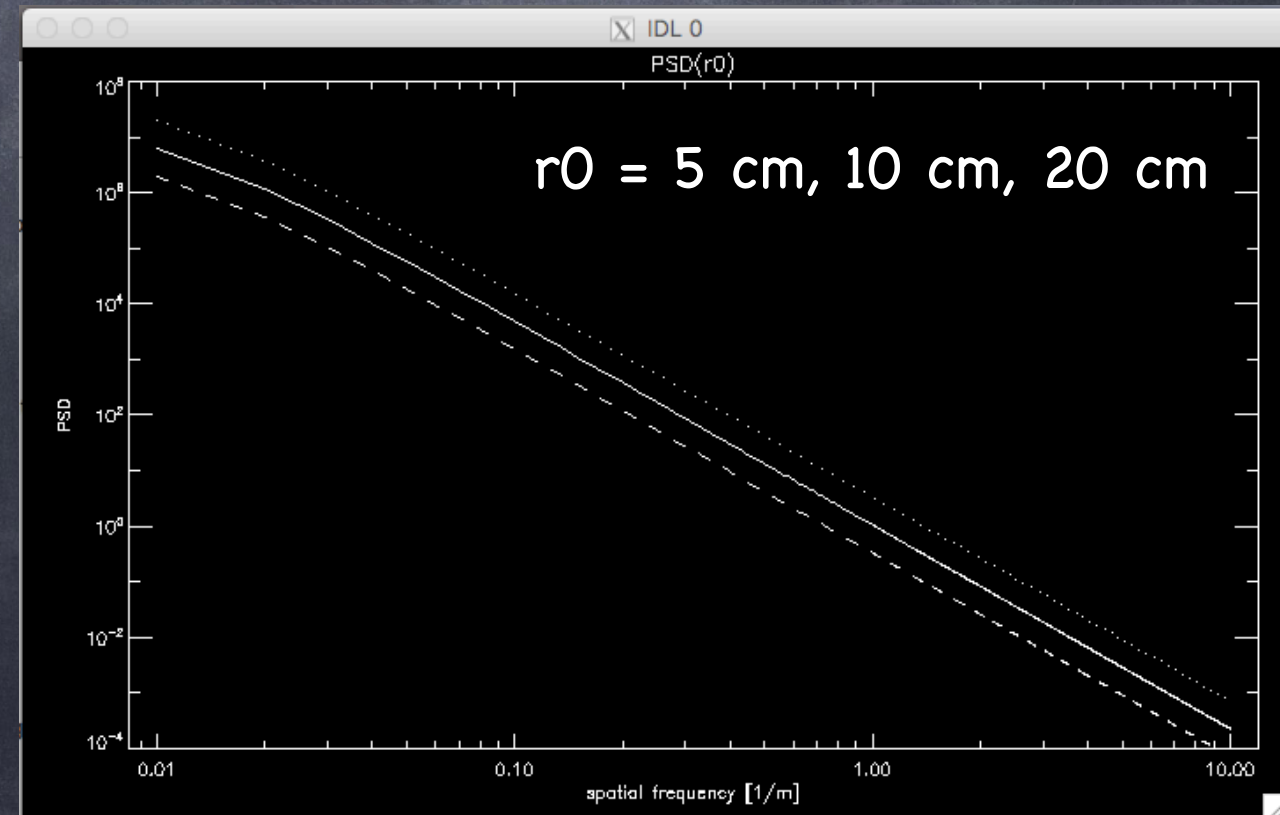
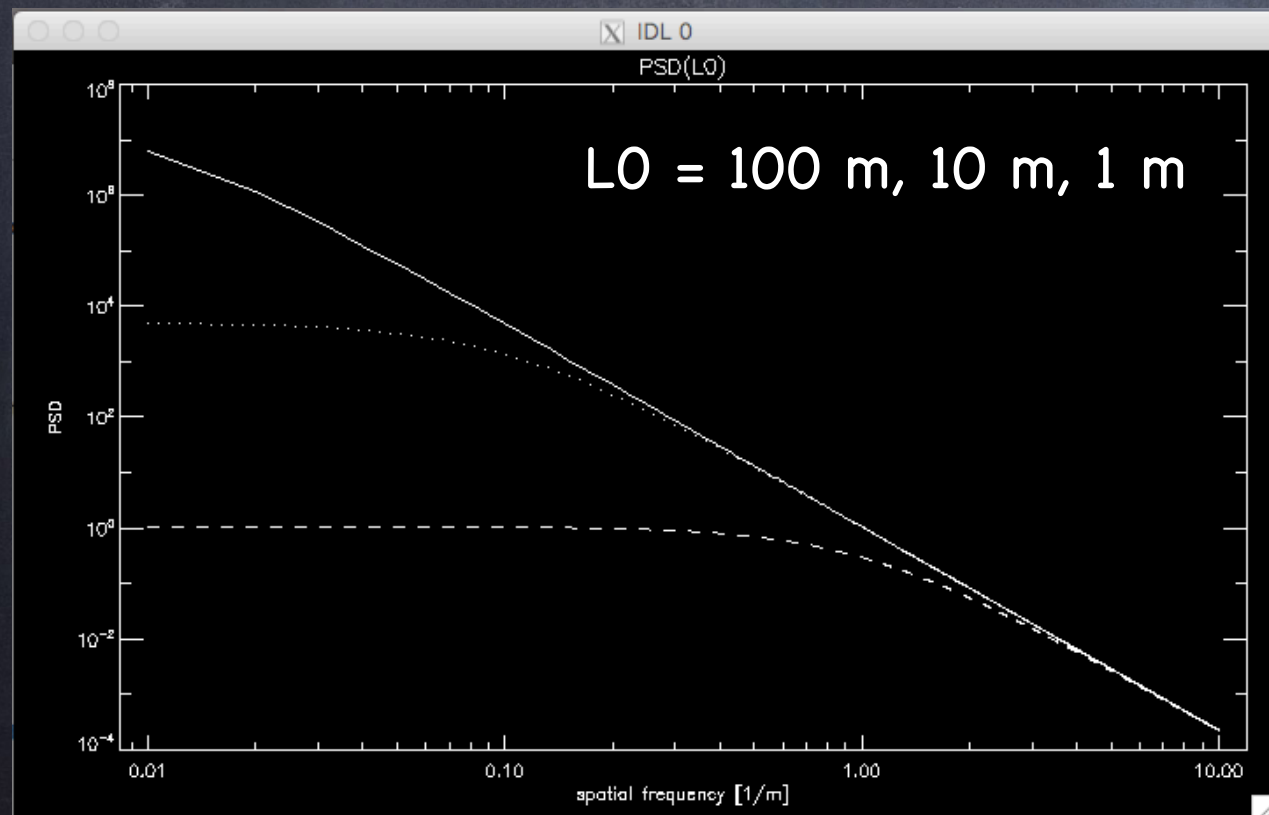
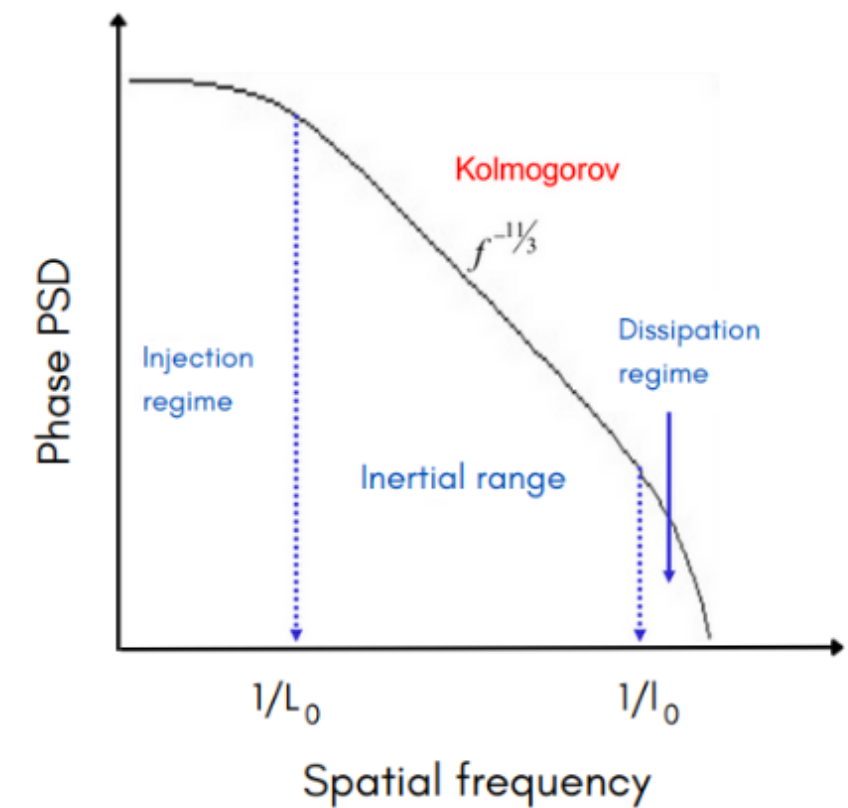
=> make a function that computes $\text{PSD}(L_0, r_0, \text{dim}, L)$ and plot it for different $[r_0, L_0] \dots$ [with, for example: $\text{dim}=1000, L=100., r_0=0.1, L_0=100., 10., 1.]$

Images & turbulence — 15

```

1 function dsp_theo, dim, L, r0, L0
2 ;
3 ; dim = array linear dimension [px]
4 ; L = array physical length [m]
5 ; r0 = phase screen Fried parameter [m]
6 ; L0 = phase screen outerscale [m]
7 ; use: dsp=dsp_theo(dim,L,r0,L0)
8 ; to be plotted afterwards with:
9 ; plot_oo, 1./L*findgen(dim), dsp, XR=[1/L/1.2,dim*1/L*1.2], /XS, $
10 ; YR=[0.1, 1E7], TIT='PSD(L0)', XTIT='freq. [1/m]', YTIT='PSD'
11 ; oplot , 1./L*findgen(dim), dsp, LINE=1
12 ; playing, e.g., with L0=100.,10.,1., or r0=.05, .1, .2
13 ;
14 freq = findgen(dim)
15 dsp = .0228*(L/r0)^(5/3.)*L^2*(freq^2+(L/L0)^2)^(-11./6)
16
17 return, dsp
18 end

```



Images & turbulence — 16

→ For next time: read Aime (Sec. 1 & Sec. 2) and Maire (Chap.1)...

Chapitre 1 Introduction

Jérôme Maire, PhD
thesis (in French), chap.1

INSTITUTE OF PHYSICS PUBLISHING

EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **22** (2001) 169–184

www.iop.org/Journals/ej PII: S0143-0807(01)14580-0

Long Telescopes may cause Objects to appear brighter and larger than short ones can do, but they cannot be so formed as to take away the confusion of the Rays which arises from the Tremors of the Atmosphere.

I. Newton, 1717 *Optics*, Sec. Ed., Book I, Part I, Prop. VIII

Teaching astronomical speckle techniques

Claude Aime

UMR 6525 Astrophysique, Faculté des Sciences de l'Université de Nice Sophia-Antipolis,
Parc Valrose-06108, Nice Cedex 2, France

Received 7 June 2000, in final form 6 December 2000

Abstract

This paper gives an introduction to speckle techniques developed for high angular-resolution imagery in astronomy. The presentation is focussed on fundamental aspects of the techniques of Labeyrie and Weigelt. The formalism used is that of Fourier optics and statistical optics, and corresponds to graduate level. Several new approaches of known results are presented. An operator formalism is used to identify similar regions of the bispectrum. The relationship between the bispectrum and the phase closure technique is presented in an original geometrical way. Effects of photodetection are treated using simple Poisson statistics. Realistic simulations of astronomical speckle patterns illustrate the presentation.

also have a look here [A. Tokovinin
tutorial on atm'c turbulence]:

[https://www.ctio.noirlab.edu/
~atokovin/tutorial/part1/turb.html](https://www.ctio.noirlab.edu/~atokovin/tutorial/part1/turb.html)