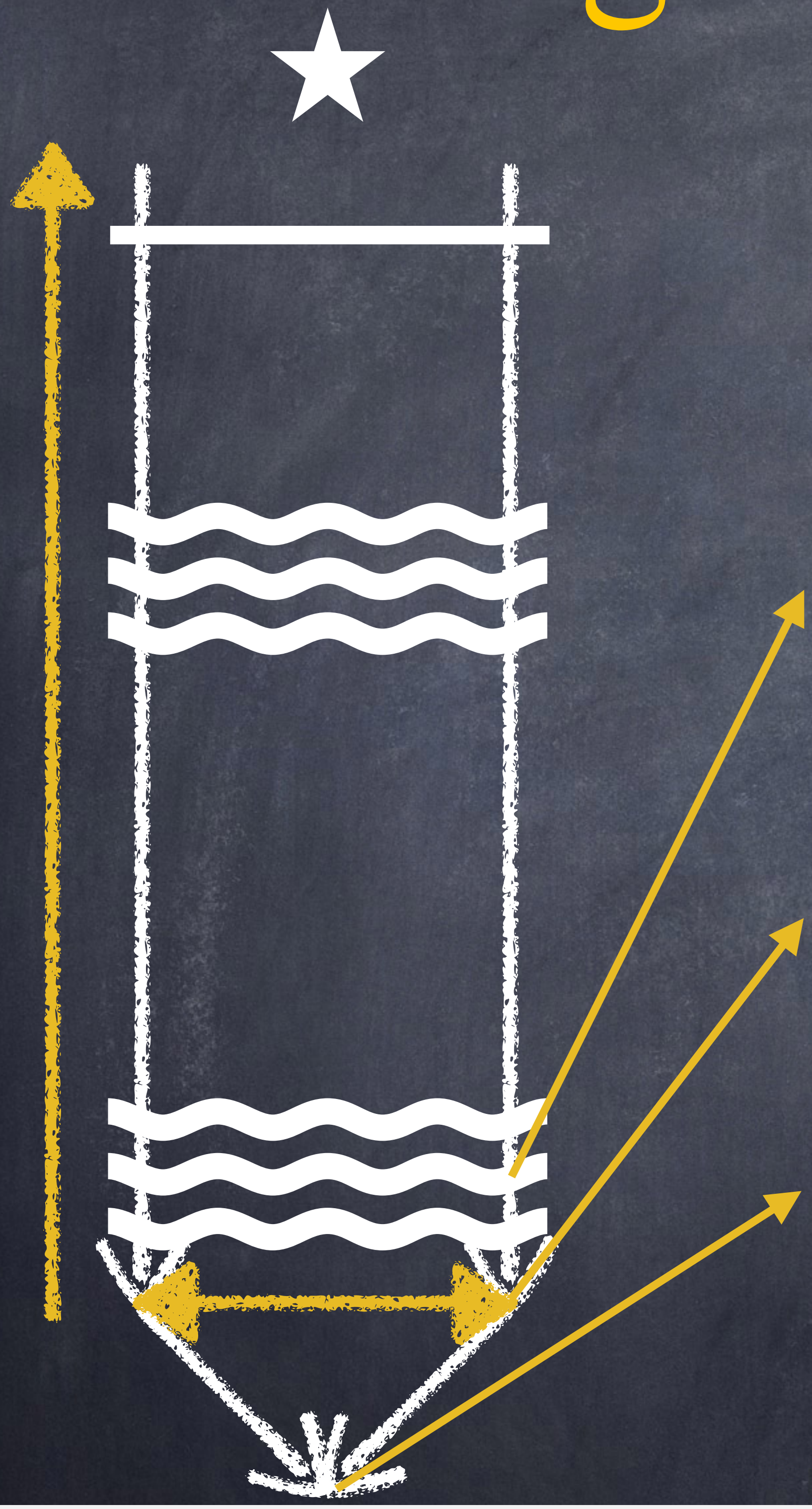
Images & turbulence — 20



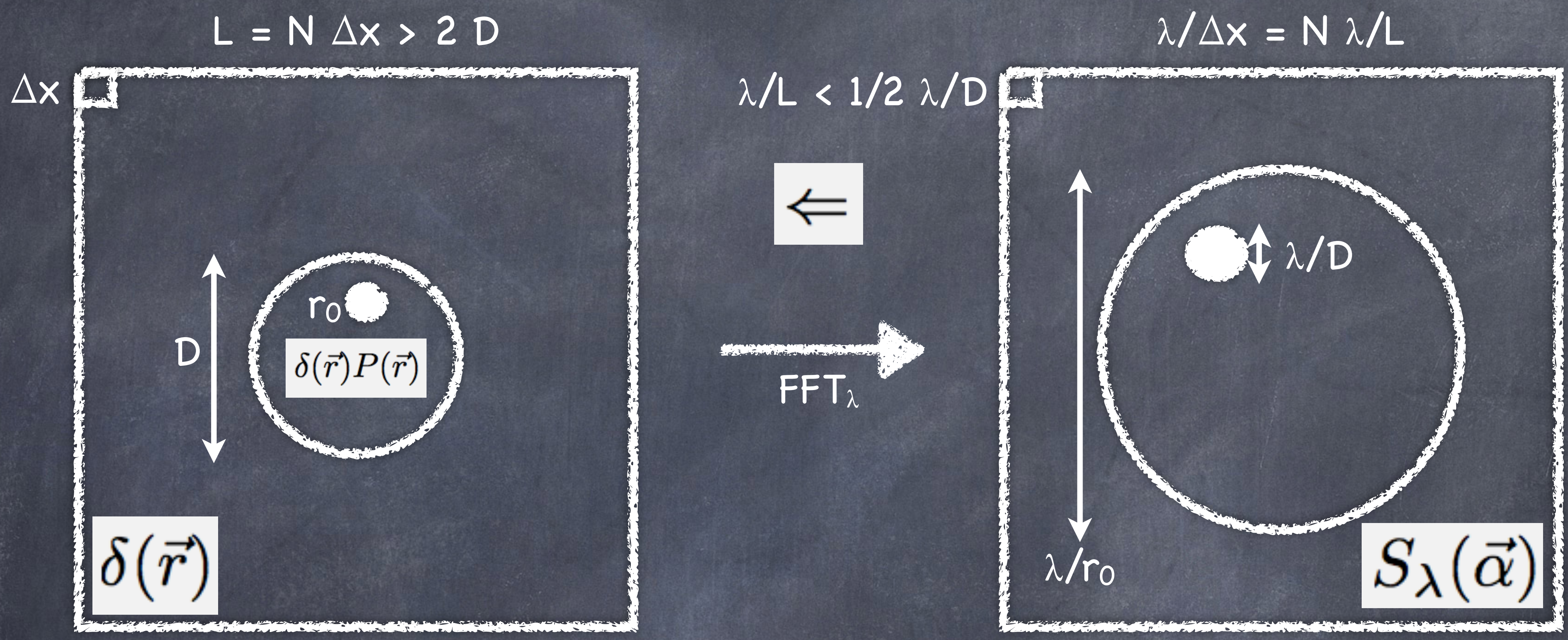
$$\Psi(\vec{r}) = A \ \exp\left(\imath \Phi(\vec{r})\right)$$

$$P(\vec{r}) \Rightarrow A \ P(\vec{r}) \ \exp\left(\imath \Phi(\vec{r}) P(\vec{r})\right)$$

$$S_\lambda(\vec{\alpha}) \propto \| FT\{A \ P(\vec{r}) \ \exp\left(\imath \Phi(\vec{r}) P(\vec{r})\right)\}\|^2$$

$$A = 1 \text{ and } \Phi(\vec{r}) = \frac{2\pi}{\lambda}\delta(\vec{r}) \Rightarrow S_\lambda(\vec{\alpha}) \propto \| FT\{P(\vec{r}) \ \exp\left(\imath \frac{2\pi}{\lambda}\delta(\vec{r}) P(\vec{r})\right)\}\|^2$$

Images & turbulence — 21



Shannon (=Nyquist) criterium

=> the image pixel λ/L must be at most half the resolution element (resel!) λ/D (in other words : one must have AT LEAST 2 image pixels per λ/D)

=> the simulated wavefronts must be at least twice the telescope diameter (L>2D)

In addition

- λ/r_0 should be smaller than λ/Δx (=> N large enough)

Images & turbulence — 22

```
function wfimg2, diam, obs, lambda_psf, filewf, filepsf

;+
; example of use:
;    diam      = 64L           ; [px] telescope pupil dimension
;    obs       = 0. [0-1]      ; (linear) obscuration ratio
;    lambda_psf= 500E-9        ; [m] PSF wavelength
;    filewf    = 'cube.sav'    ; cube of wf filename
;    filepsf   = 'cube_psf.sav'; cube of PSFs filename
;    print, wfimg2(diam,obs,lambda_psf,filewf,filepsf)
;    -> compute the cube of PSFs, save it, and tell how it went
;
; sub-routines needed: make_PSF.pro, wfgeneration.pro, makepup.pro
;
; Marcel Carbillet [marcel.carbillet@unice.fr], Lagrange (UniCA, OCA, CNRS)
; written: Feb. 2018, last modified: March 11th 2024.
;-

; preliminaries
restore, filewf               ; restores variable 'cube' containing nn wf
dim= (size(cube))(1)          ; linear size of wf
nn = (size(cube))(3)          ; nb of wf
cube_psf=fltarr(dim,dim,nn) ; initialize cube of PSFs

; compute and save PSFs
pup = makepup(dim,diam,obs) ; compute entrance pupil
for i=0, nn-1L do cube_psf[*,*,i] = make_PSF(pup,cube[*,*,i],lambda_psf)
                              ; compute the PSF corresponding to each wf
save, cube_psf, FI=filepsf    ; save cube of PSFs to disk

; return back
return, 'Cube of PSFs '+filepsf+' saved on disk...'
end
```

<u>image formation:</u>

1- cube of instantaneous PSFs (500nm & H-band)

```
function make_PSF, pup, wf, lambda

;+
; PSF computation from a wavefront
;
; pup    = input pupil,
; wf     = input wavefront [float],
; lambda = wavelength at which PSF is computed.
; PSF = make_PSF(pup, wf, lambda)
; -> compute the PSF corresponding to wf and pup, at wavelength lambda
;
; Marcel Carbillet [marcel.carbillet@unice.fr],
; UMR 7293 Lagrange (UNS/CNRS/OCA), Feb. 2013.
; Last modification: March 11th 2024
;-

; preliminary
dim = (size(wf))[1]

; compute PSF
psf = (abs(fft(pup*exp(complex(0,1)*2*!PI/lambda*wf*pup))))^2
; NB: (abs(fft(pup*exp(complex(0,1)*2*!PI/lambda*wf))))^2 would suffice
psf = shift(psf, dim/2, dim/2)

; return back
return, psf
end
```

```
IDL> .r wfimg2
% Compiled module: WFIMG2.
IDL> print, wfimg2(64L, 0., 500E-9, 'wf_r0=10cm_L0=10m.sav', 'PSF_r0=10cm_L0=10m_lambda=500nm.sav')
Cube of PSFs PSF_r0=10cm_L0=10m_lambda=500nm.sav saved on disk...
```

Images & turbulence — 23

```
IDL> restore, 'PSF_r0=10cm_L0=10m_lambda=500nm.sav'
IDL> help
% At $MAIN$
CUBE_PSF        FLOAT     = Array[128, 128, 100]
I               INT       =      100
Compiled Procedures:
    $MAIN$

Compiled Functions:
  COMPUTE_RMS DIST        MAKEPUP      MAKE_PSF     WFCUBE2      WFGENERATION            WFIMG2

IDL> window, XS=512, YS=512, /FREE
IDL> for i=0,99 do tvscl, rebin(cube_PSF[*,*,i], 512, 512, /SAMPLE)
```

```
IDL> longexp=total(cube_PSF,3)
IDL> tvscl, rebin(longexp, 512, 512, /SAMPLE)^.1
```

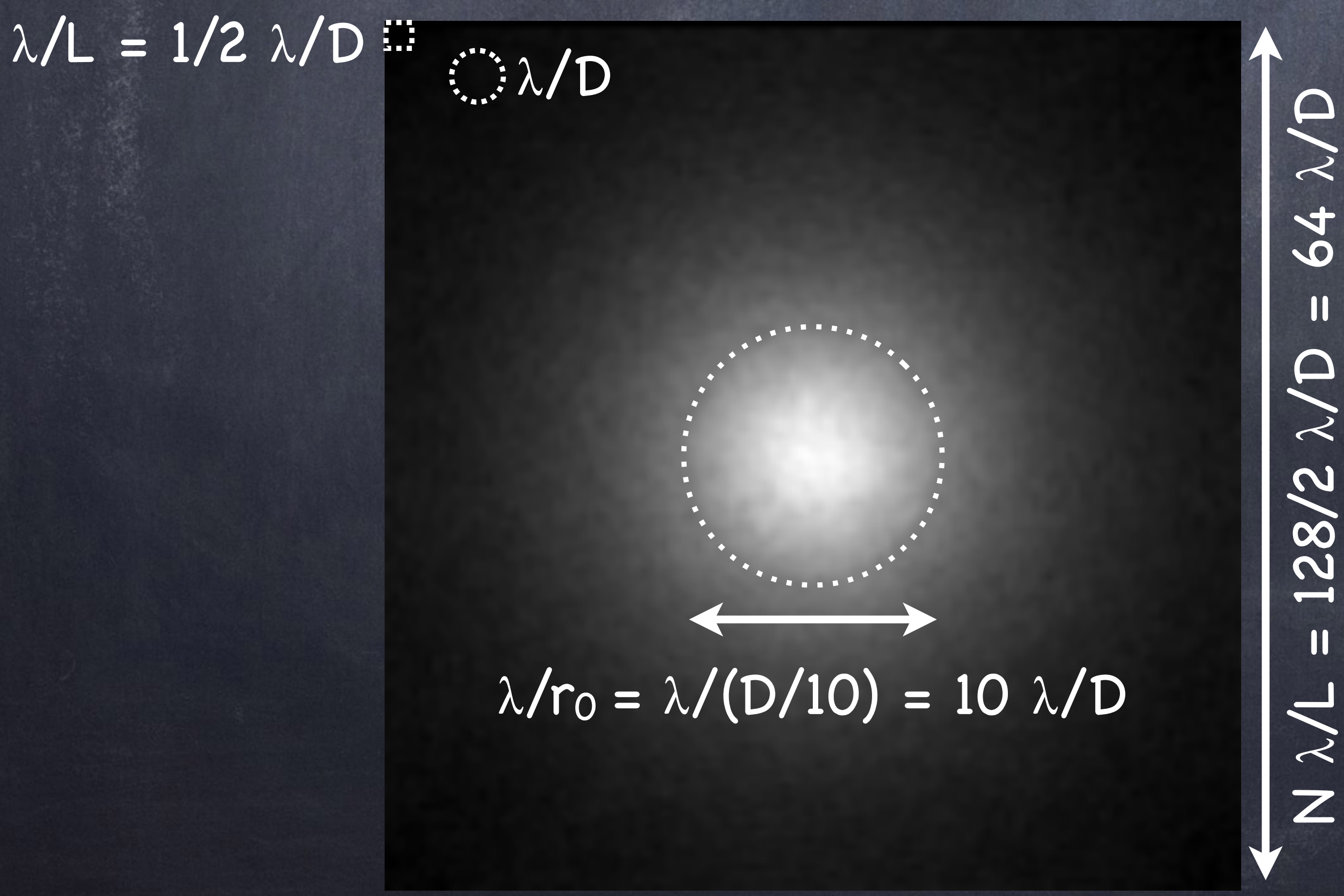
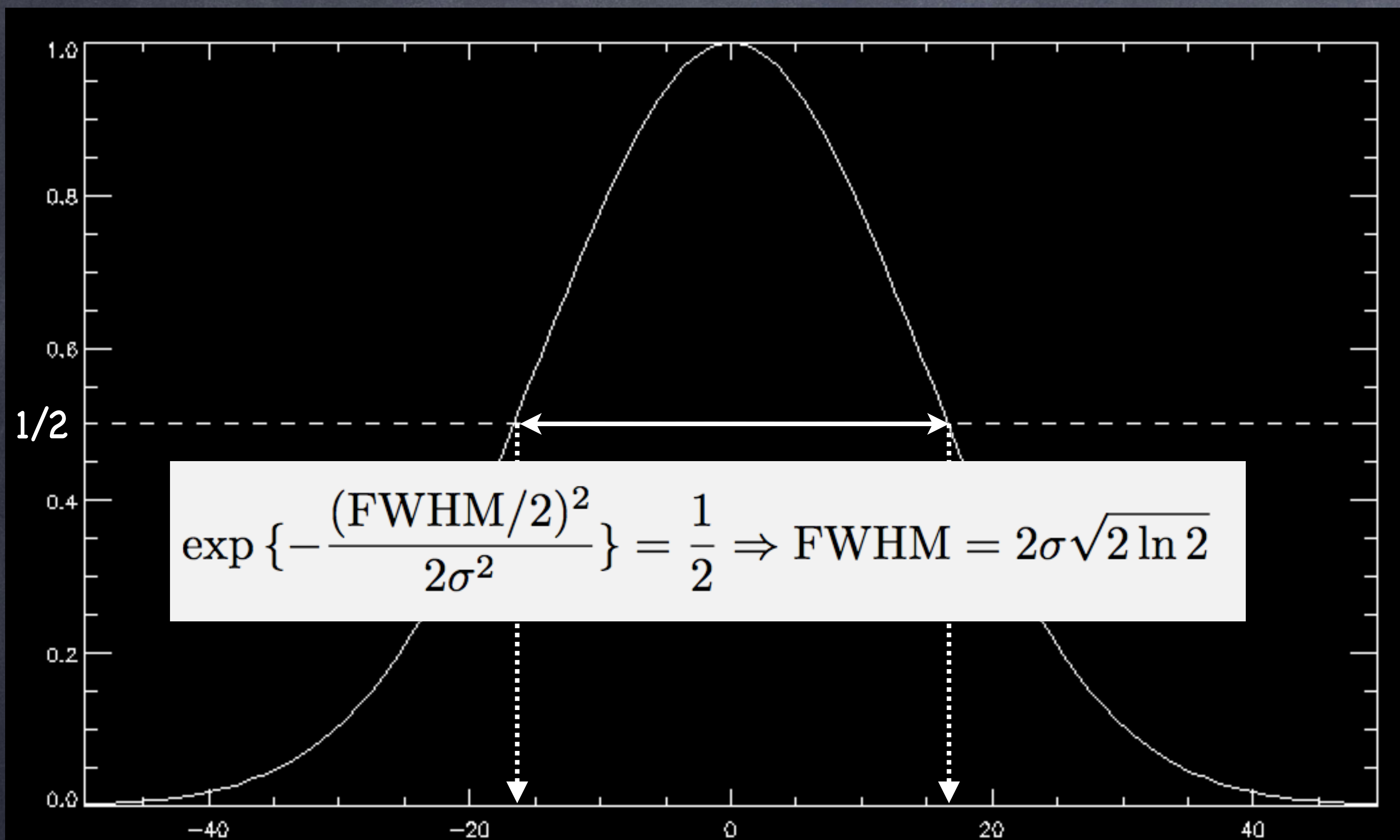


image formation:

1- cube of instantaneous PSFs (500nm & band H)

2- long-exposure PSF

Images & turbulence — 24



$$\exp\{-\frac{(\mathrm{FWHM}/2)^2}{2\sigma^2}\} = \frac{1}{2} \Rightarrow \mathrm{FWHM} = 2\sigma\sqrt{2\ln 2}$$

<u>image formation:</u>

1- cube of instantaneous PSFs (500nm & H-band)

2- long-exposure PSFs

3- fit with gaussian and compare FWHM vs. λ/r_0 (seeing), also in function of the outerscale L_0.

-> Also read Martinez...

```
IDL> res=gauss2dfit(longexp,a)
% Program caused arithmetic error: Floating underflow
IDL> print, 2*((a[2]+a[3])/2)*sqrt(2*alog(2))
      15.9637
IDL>
```

In this example, the FWHM is ≈16px and, since we have here: 1px=(λ/D)/2, we have hence: FWHM≈8 (λ/D)
[i.e. 8*0.1″≈0″8 here (@500nm)]

Images & turbulence — 25

On the Difference between Seeing and Image Quality: When the Turbulence Outer Scale Enters the Game

Patrice Martinez[1]
Johann Kolb[1]
Marc Sarazin[1]
Andrei Tokovinin[2]

[1] ESO
[2] Cerro-Tololo Inter American Observatory, Chile

We attempt to clarify the frequent confusion between seeing and image quality for large telescopes. The full width at half maximum of a stellar image is commonly considered to be equal to the atmospheric seeing. However the outer scale of the turbulence, which corresponds to a reduction in the low frequency content of the phase perturbation spectrum, plays a significant role in the improvement of image quality at the focus of a telescope. The image quality is therefore different (and in some cases by a large factor) from the atmospheric seeing that can be measured by dedicated seeing monitors, such as a differential image motion monitor.

of telescope diameters and wavelengths. We show that this dependence is efficiently predicated by a simple approximate formula introduced in the literature in 2002. The practical consequences for operation of large telescopes are discussed and an application to on-sky data is presented.

Background and definitions

In practice the resolution of ground-based telescopes is limited by the atmospheric turbulence, called "seeing". It is traditionally characterised by the Fried parameter (r_0) – the diameter of a telescope such that its diffraction-limited resolution equals the seeing resolution. The well-known Kolmogorov turbulence model describes the shape of the atmospheric long-exposure point spread function (PSF), and many other phenomena, by this single parameter r_0. This model predicts the dependence[1] of the PSF FWHM (denoted ε_0) on wavelength (λ) and inversely on the Fried parameter, r_0, where r_0 depends on wavelength (to

A finite L_0 reduces the variance of the low order modes of the turbulence, and in particular decreases the image motion (the tip-tilt). The result is a decrease of the FWHM of the PSF. In the von Kàrmàn model, r_0 describes the high frequency asymptotic behaviour of the spectrum where L_0 has no effect, and thus r_0 loses its sense of an equivalent wavefront coherence diameter. The differential image motion monitors (DIMM; Sarazin & Roddier, 1990) are devices that are commonly used to measure the seeing at astronomical sites. The DIMM delivers an estimate of r_0 based on measuring wavefront distortions at scales of ~ 0.1 m, where L_0 has no effect. By contrast, the absolute image motion and long-exposure PSFs are affected by large-scale distortions and depend on L_0. In this context the Kolmogorov expression for ε_0[1] is therefore no longer valid.

Proving the von Kàrmàn model experimentally would be a difficult and eventually futile goal as large-scale wavefront perturbations are anything but stationary. However, the increasing number of esti-

```
REPORT

- Preliminary measures
+ introduction
+ PSD(r0, L0) plot
+ => ccl on the influence of r0 and L0
+ rms(r0, L0) plot or table
+ => ccl on the influence of r0 and L0
+ image formation and FWHM(r0 or lambda, possibly L0)
+ => ccl on the influence of r0 or lambda (and poss. L0)
+ => comparison with the 'seeing' lambda/r0
+ (more to come...)
```

Images & turbulence — 26

-> Detection noises:

• At first: *photon noise* (or *shot noise*), poissonian, actually a transformation of the image.

$$p(n) = \frac{N^n e^{-N}}{n!}, \text{with} : N = \frac{L\Delta t}{h\nu}, L = \text{luminosity}, \Delta t = \text{time exp.}$$

p(n) = probability to detect n photons when N are expected

For large N: ~gaussian…

$$p(n) \simeq \exp\left(-\frac{(n-N)^2}{2N}\right)$$

Images & turbulence — 27

-> Detector noises:

• At first: *photon noise* (or *shot noise*), poissonian, actually a transformation of the image.

• At last: *read-out noise* (*RON*), gaussian with zero mean and rms σ_e [e-/px], additive noise.

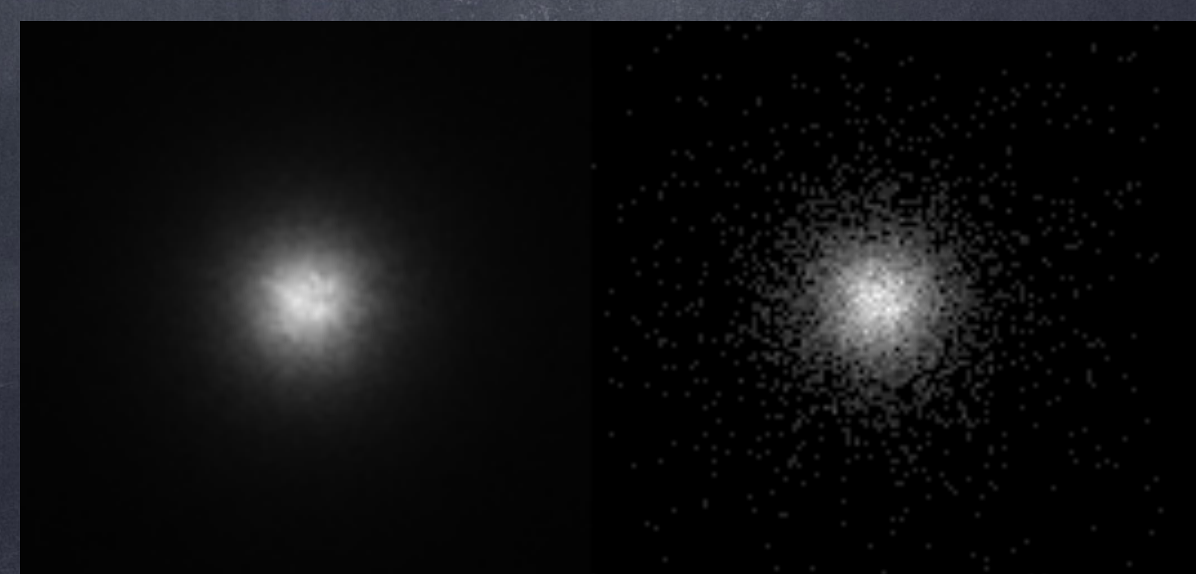
• In between: *dark current noise*, *amplification noise* & *exotic dark current noise* in the case of EMCCDs, noise due to the *calibration* of the *flat field*, *'salt & pepper' noise* ('hot' and 'cold' pixels), etc.

Images & turbulence — 28

```
;; Photon noise (Poisson)
if keyword_set(PHOT_NOISE) then begin
   idx=where((image GT 0.) AND (image LT 1E8),c)
                                                ; For values higher than 1E8, should one
   if (c NE 0) then for i=0l,c-1l do $          ; really has to worry about photon noise ?
      noisy_image[idx[i]]=randomn(seed_pn,POISSON=image[idx[i]],/DOUBLE)
endif
```

<u>image formation with noise:</u>

1- ‘add’ photon noise on one short-exp. PSF (in function of N…),
2- long-exp. PSF (100N photons!),
3- ‘add’ photon noise on the long-exp. PSF,
4- compare long-exp. & short-exp. noisy images (and ‘clean’ images),
5- compare also with the sum of the (100) short-exp. noisy images…

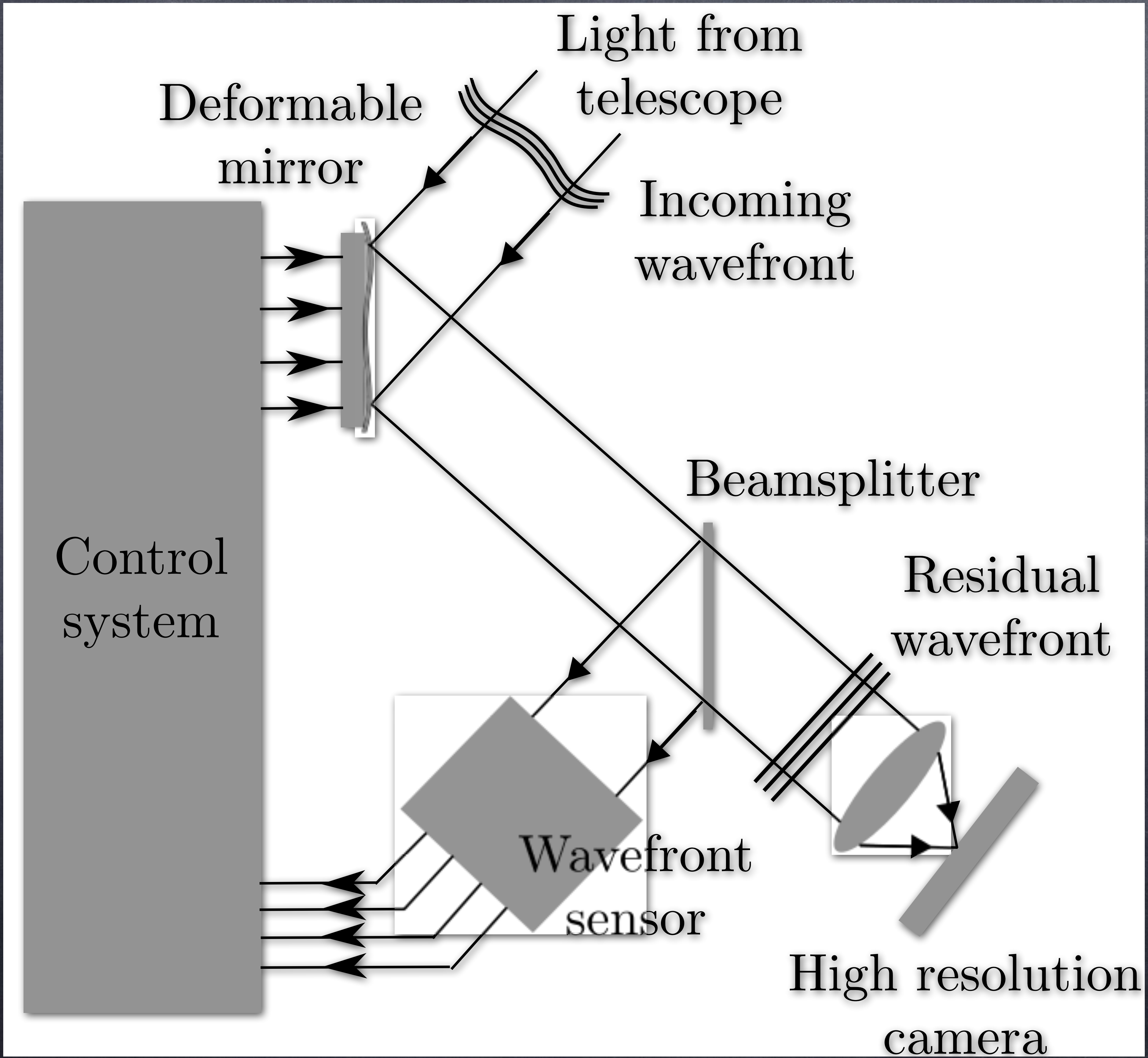


```
---
REPORT
---

- Preliminary measures
+ introduction/context
+ PSD(r0, L0)
+ => influence of r0 and L0
+ rms(r0, L0)
+ => influence of r0 and L0
+ FWHM(r0 or lambda=>r0, L0)
+ => influence of r0 and L0
+ => comparison with the "seeing" lambda/r0
+ noisy images
```

Adaptive optics — 01

Adaptive optics — 02

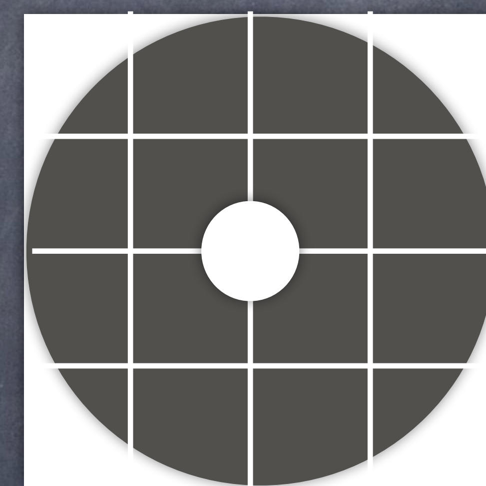


Adaptive optics — 03

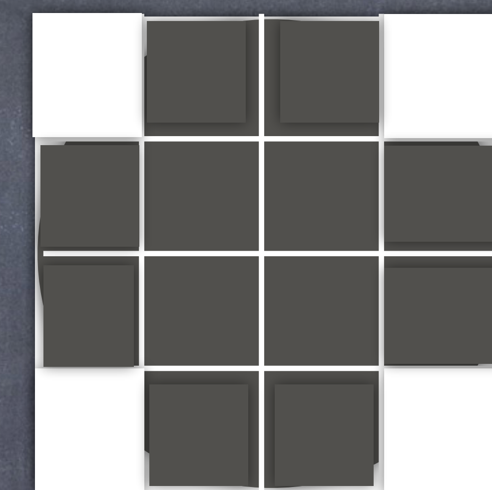
Fried configuration

Here is an example of an AO system based on a 4x4 lenslet array (i.e. a 4x4 SH WFS) and a 5x5 actuators array (i.e. a 5x5 DM)…

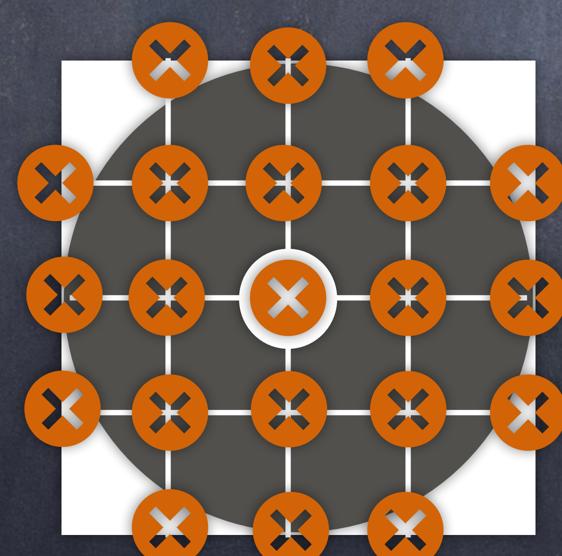
Entrance pupil of the telescope



Pupil projected onto the 4x4 lenslet array of the wavefront sensor (a 4x4 Shack-Hartman)



12 valid sub-apertures of the Shack-Hartmann (onto the 4x4 array)

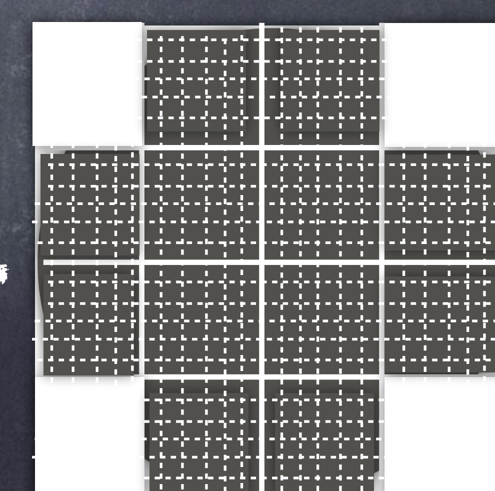


Pupil projected on the deformable mirror with its 21 actuators (on the 5x5 corresponding array)

commands to the DM

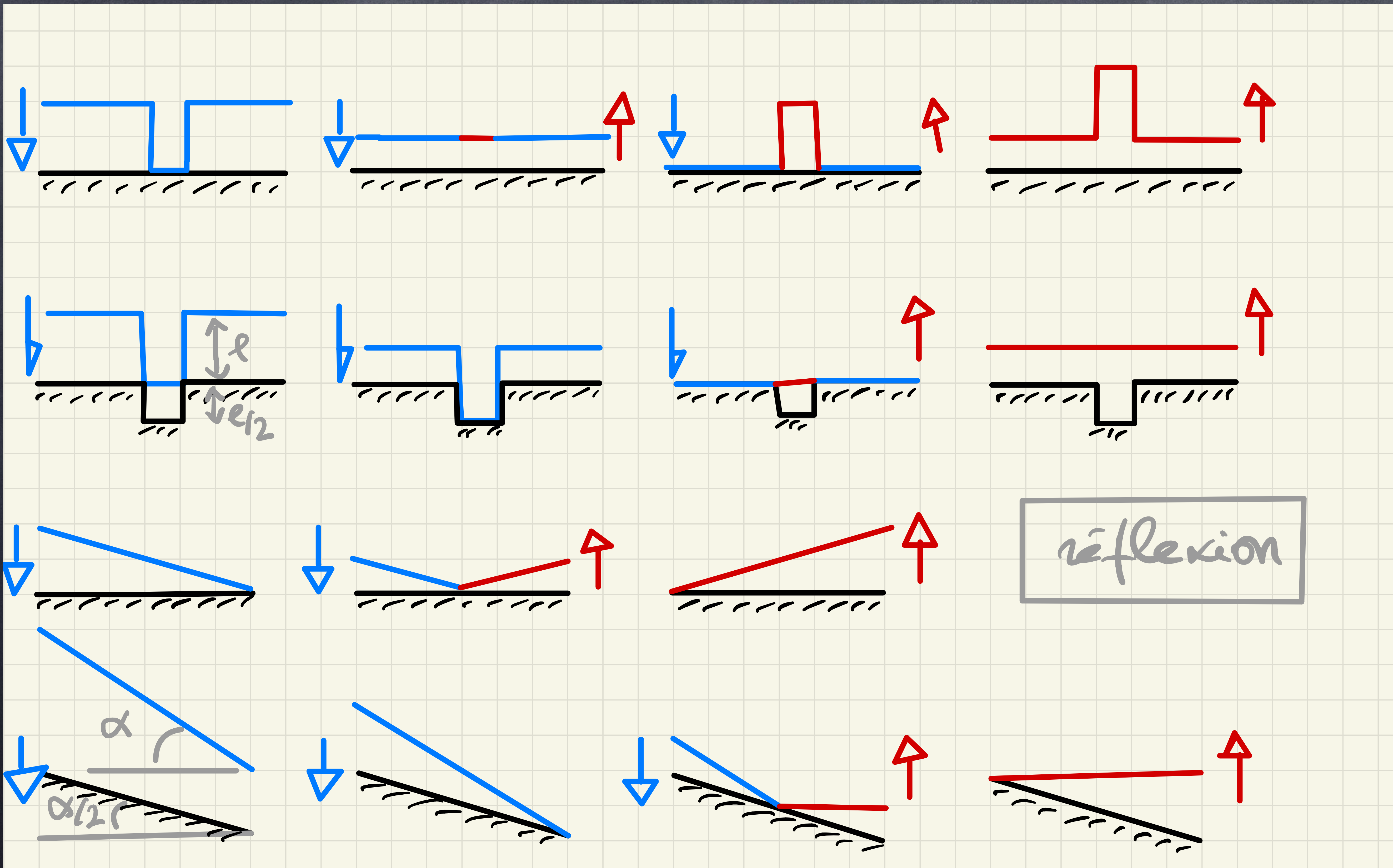
measures from WFS

reconstruction of the wavefront, control of the command



12 sub-parts of the detector placed in the focal plane of the SH lenslet array, with 6x6 pixels each

Adaptive optics — 04



Adaptive optics — 05

Some orders of magnitude concerning AO systems:

	@500nm	@2.2μm
spatial sampling (WFS analysis elements size)		
→ $d \approx r_0$	**≈ 10 cm**	**≈ 60 cm**
number of WFS analysis elements (≈ number of DM actuators)		
→ $N \propto (D/d)^2$, with D=10m	**≈ 7500**	**≈ 200**
temporal sampling		
→ $f \propto 10\ v/r_0$	**≈ 1 kHz**	**≈ 0.2 kHz**

Adaptive optics — 06

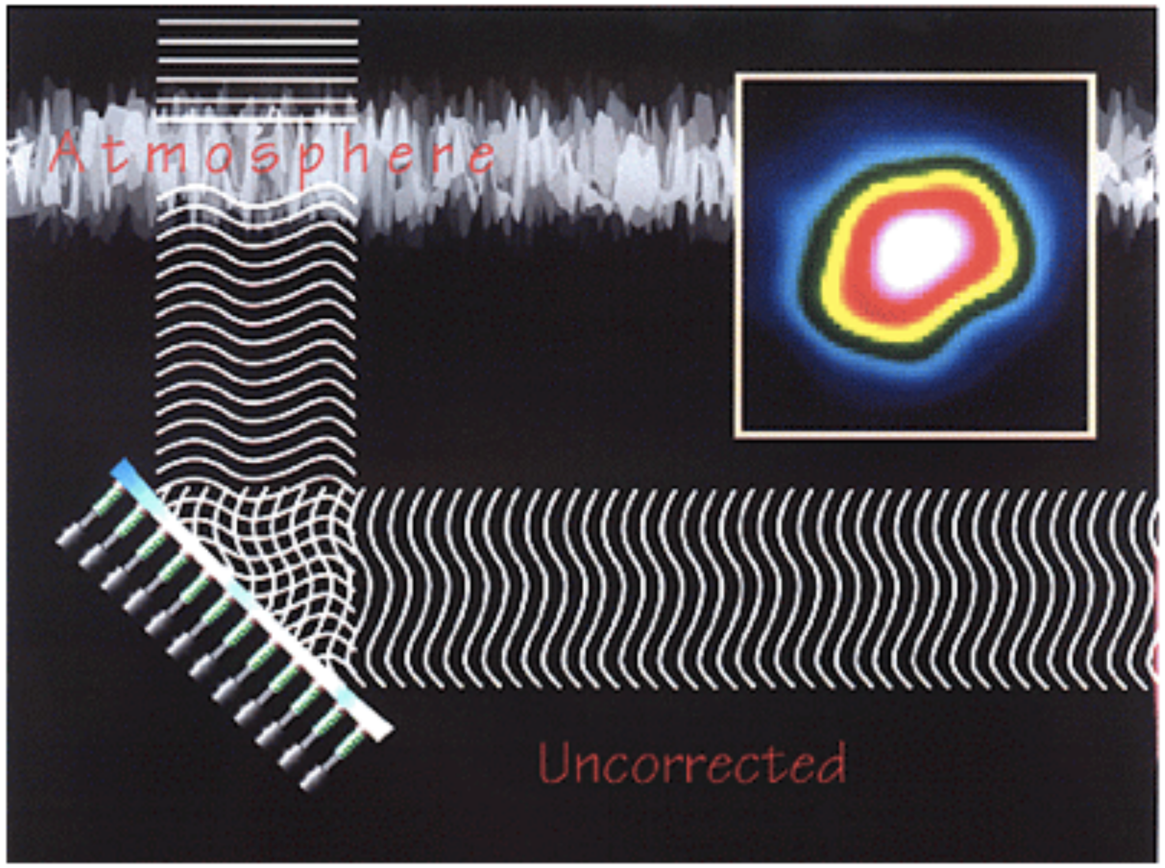
Introduction to Adaptive Optics

Credits: ESO and Jennifer Lotz

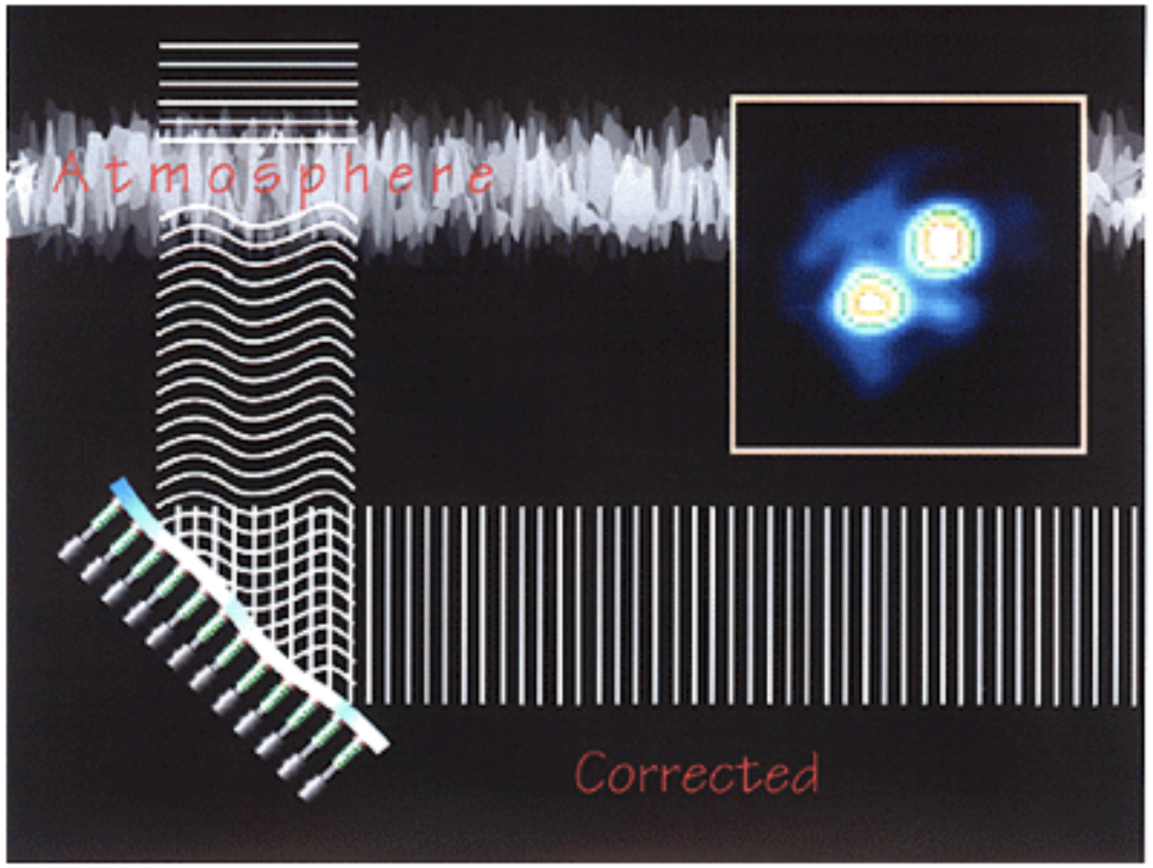
As astronomers attempt to understand the limits of the physical universe, they must look deep into the night sky with a sharp eye. Unfortunately, looking into the night sky is like looking up from the bottom of a swimming pool. Turbulence in the upper atmosphere causes spatial and temporal anomalies in atmosphere's refractive index and any planar wavefront of light passing through this turbulence will experience phase distortions by the time it reaches a ground-based telescope. These phase distortions blur the images obtained by the telescope and result in resolution an order of magnitude worse than the theoretical capabilities of the telescope. The power of ground-based telescopes to observe and resolve distant faint astronomical objects is limited by the effects of the atmosphere on the light coming from these objects.

The desire to avoid the image degradation due to the atmosphere was one of the main motivations behind the MPIA ALFA Project.

In recent years, astronomers have developed the technique of adaptive optics to actively sense and correct wavefront distortions at the telescope during observations. A telescope with adaptive optics measures the wavefront distortions with a wavefront sensor and then applies phase corrections with a deformable mirror on a time scale comparable to the temporal variations of the atmosphere's index of refraction. Adaptive optics dramatically improves image resolution as shown in the AO principle drawings below.



Blurred, uncorrected image (without Adaptive Optics)

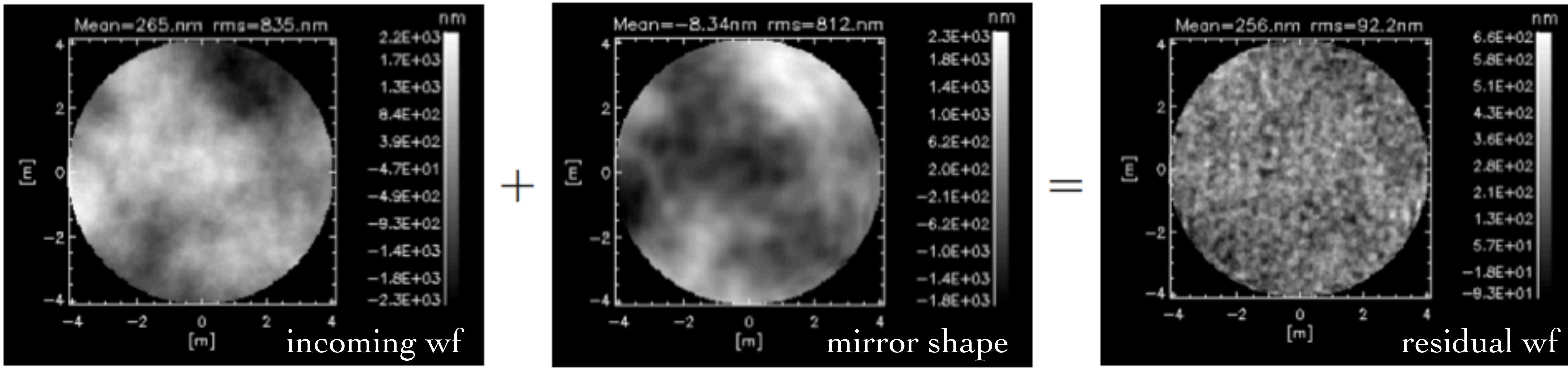


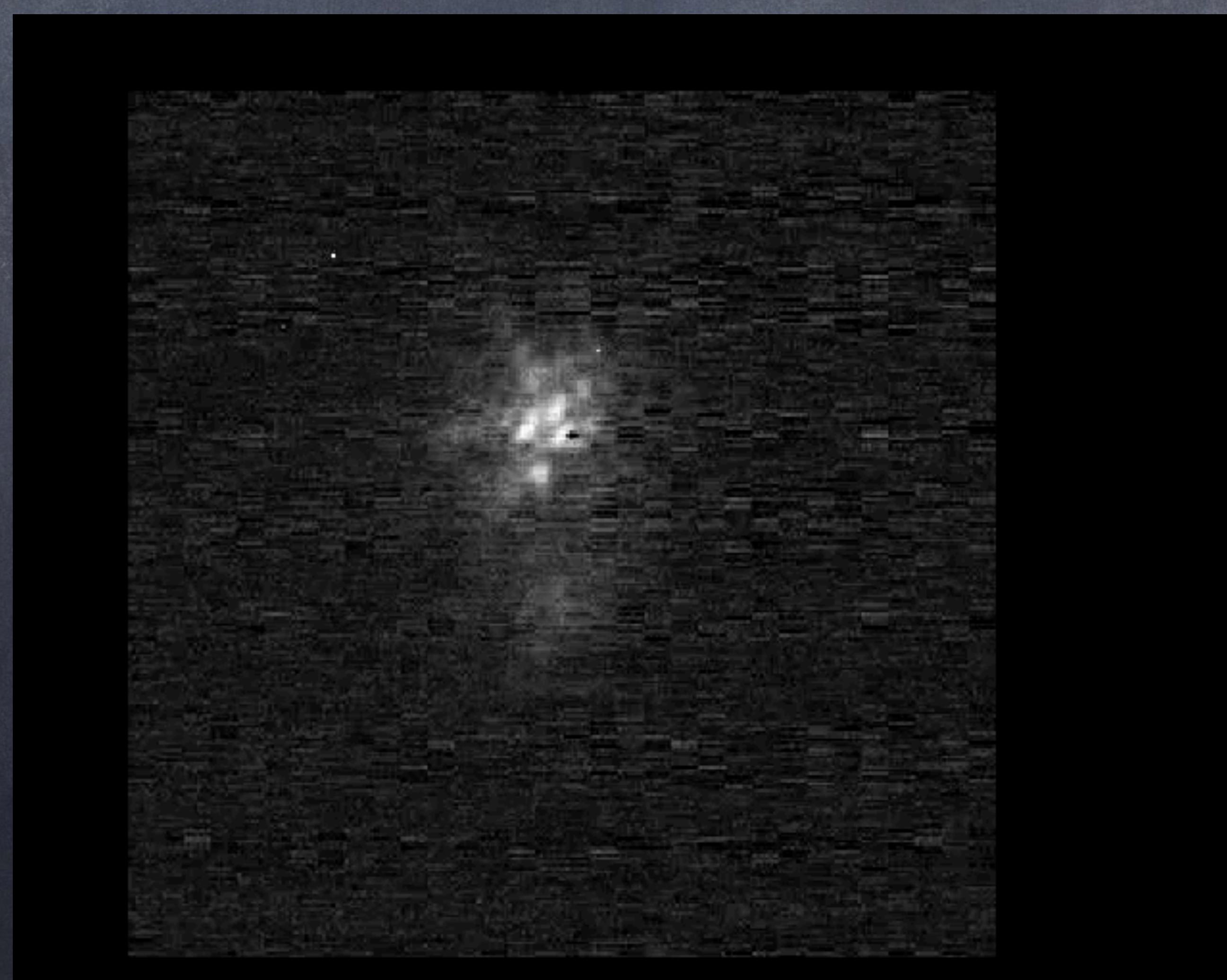
With Adaptive Optics corrected image

For more information see Adaptive Optics Tutorial in german or english by Stefan Hippler and Andrei Tokovinin.

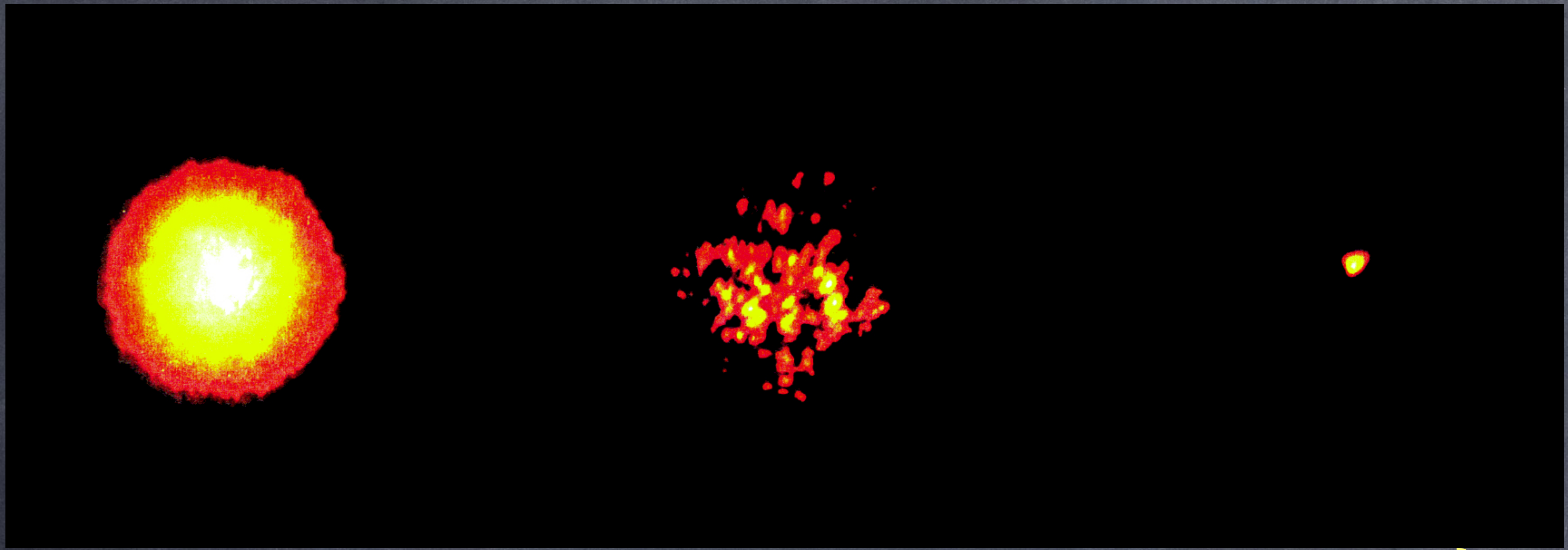
last update: 3 April 2007
editor of this page: Stefan Hippler

Adaptive optics — 07

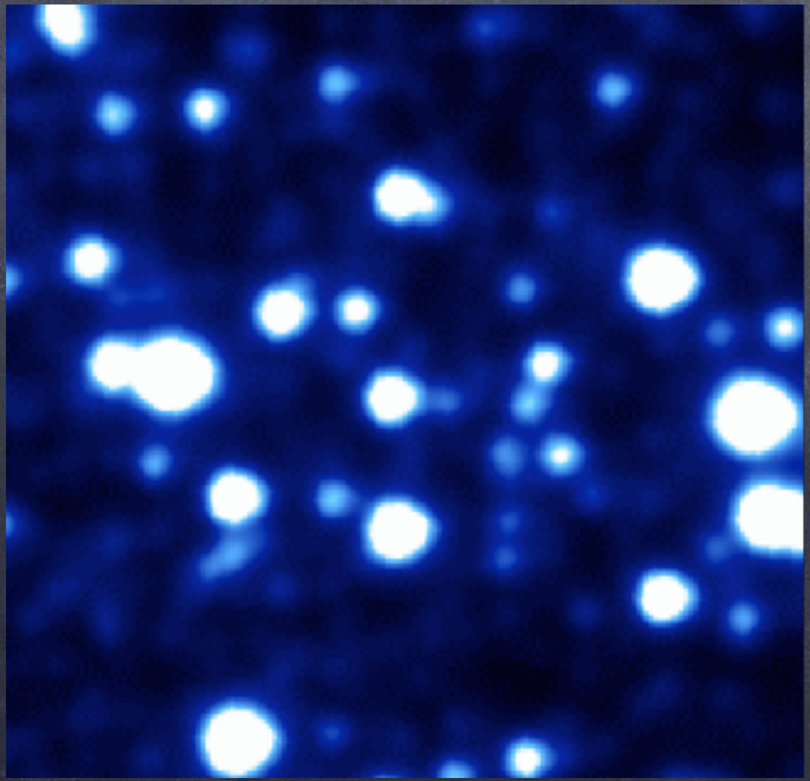


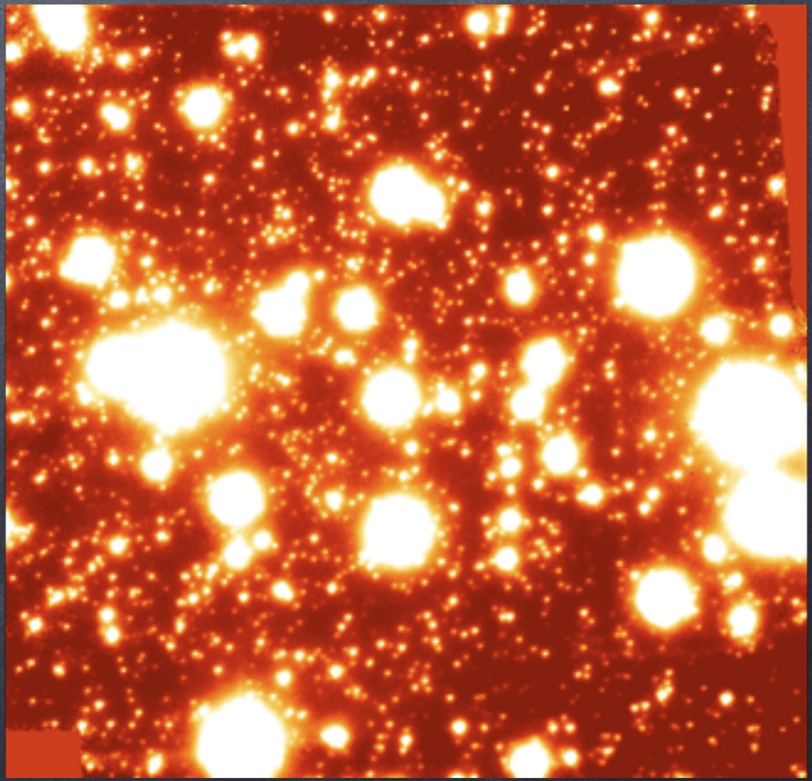


Adaptive optics — 08



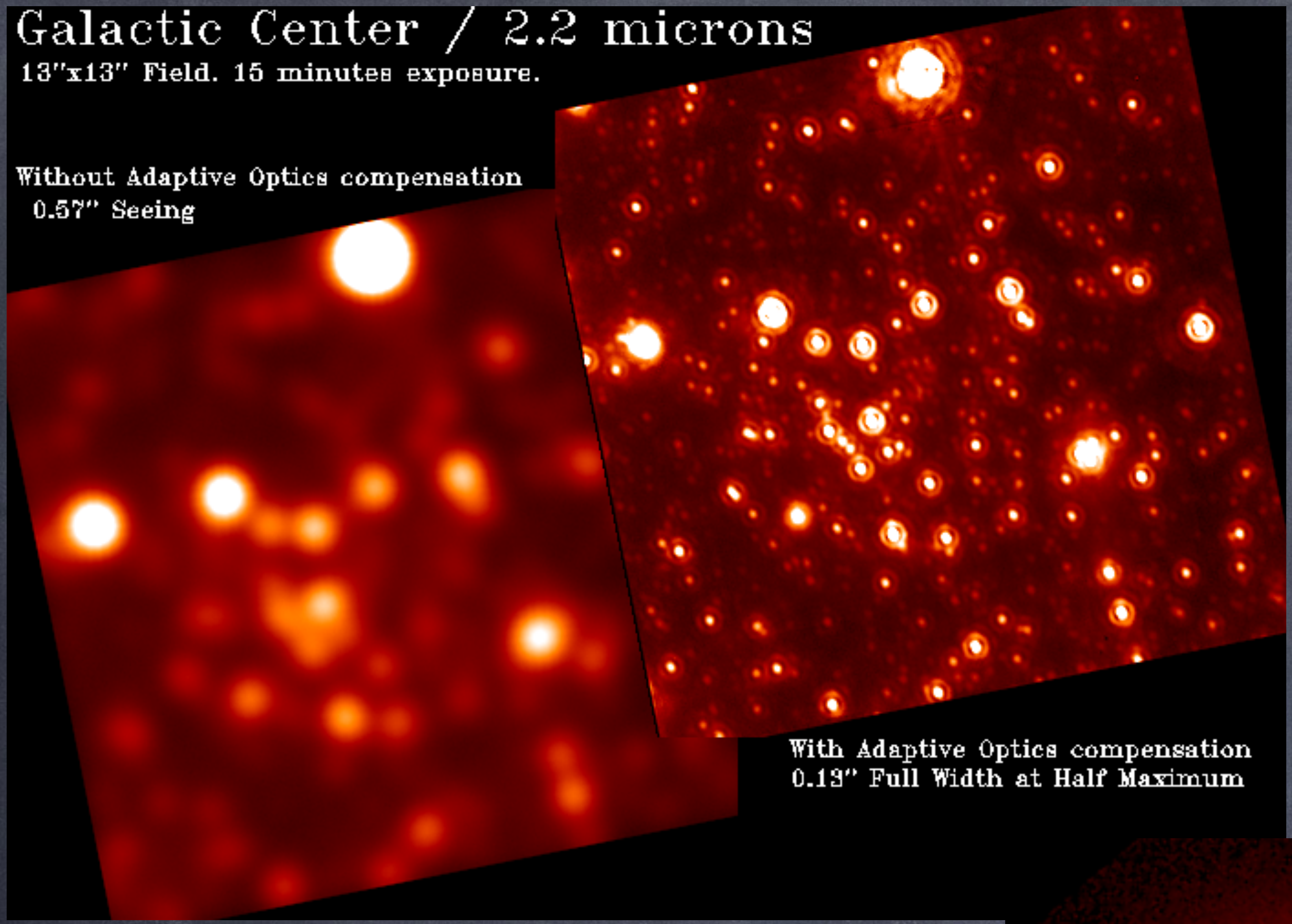
(Lick Observatory, 1-m telescope, left: FWHM≈1", right: FWHM≈λ/D)



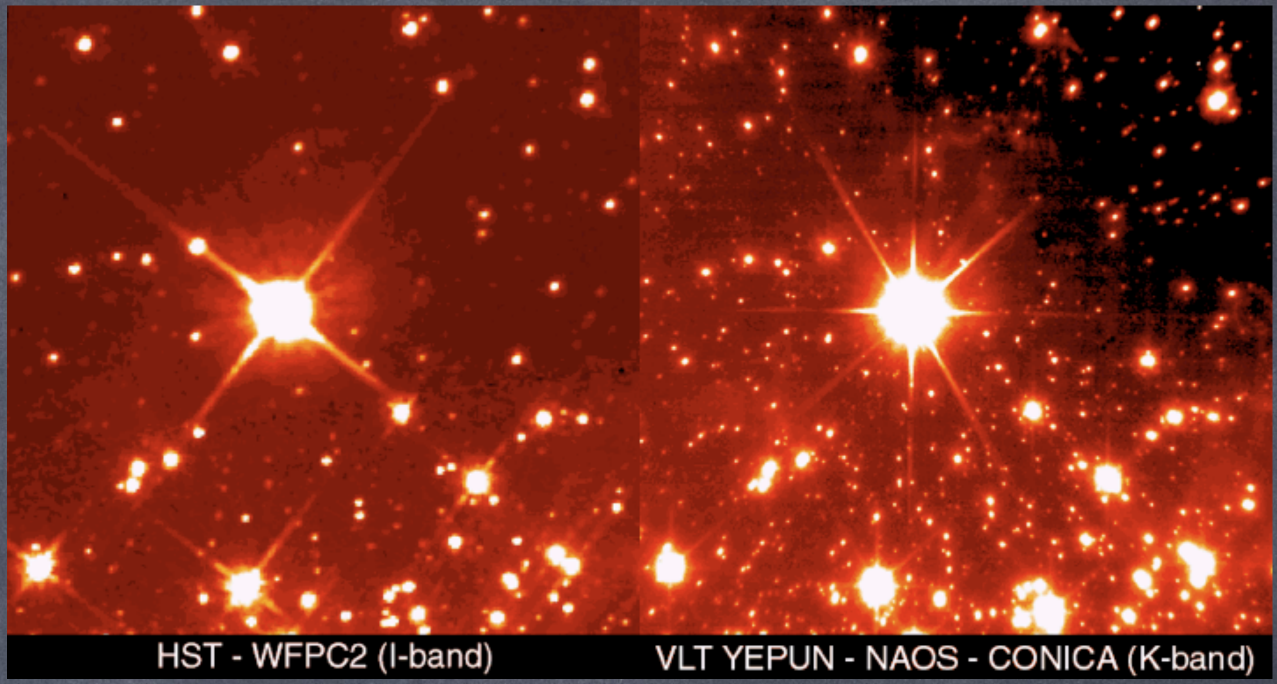


(Gemini Observatory, Hokupa'a+Quirc, left: FWHM≈0"85, right: FWHM≈0"09)

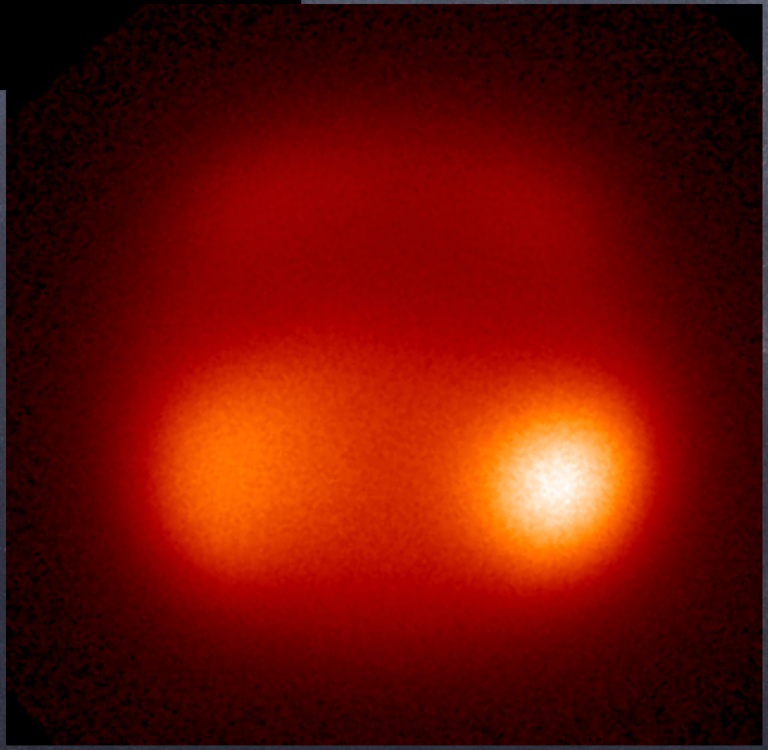
Adaptive optics — 09

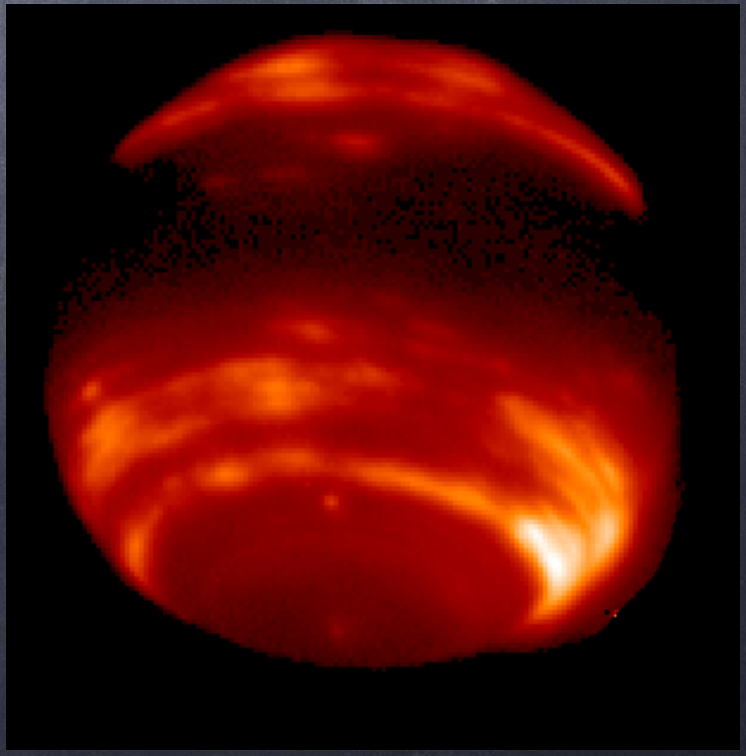


(CFHT, long-exp. images (15'))



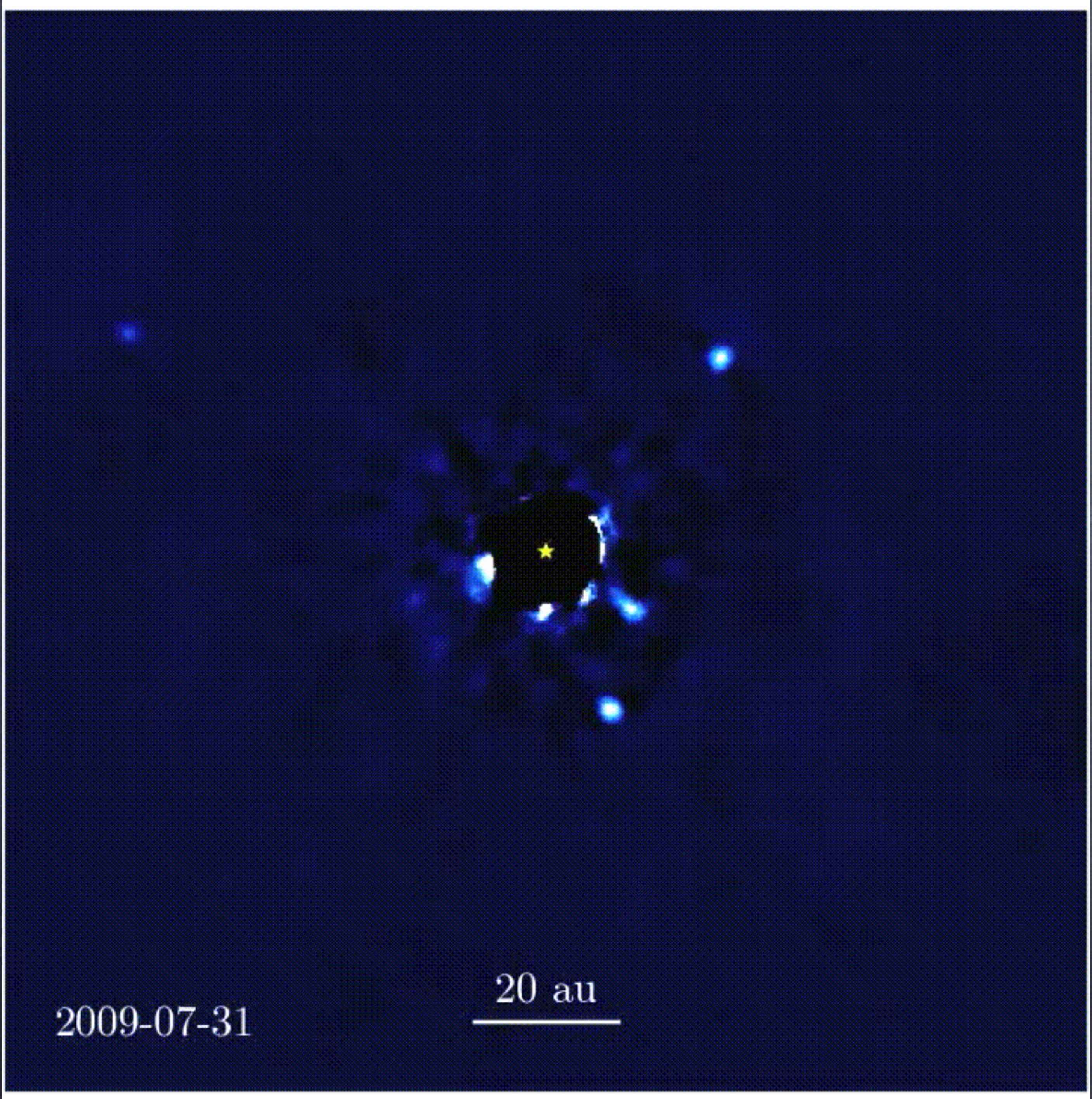
(HST vs. NACO/VLT)





(Neptune à 1.65 microns, Keck Observatory, mai et juin 1999)

Adaptive optics — 10



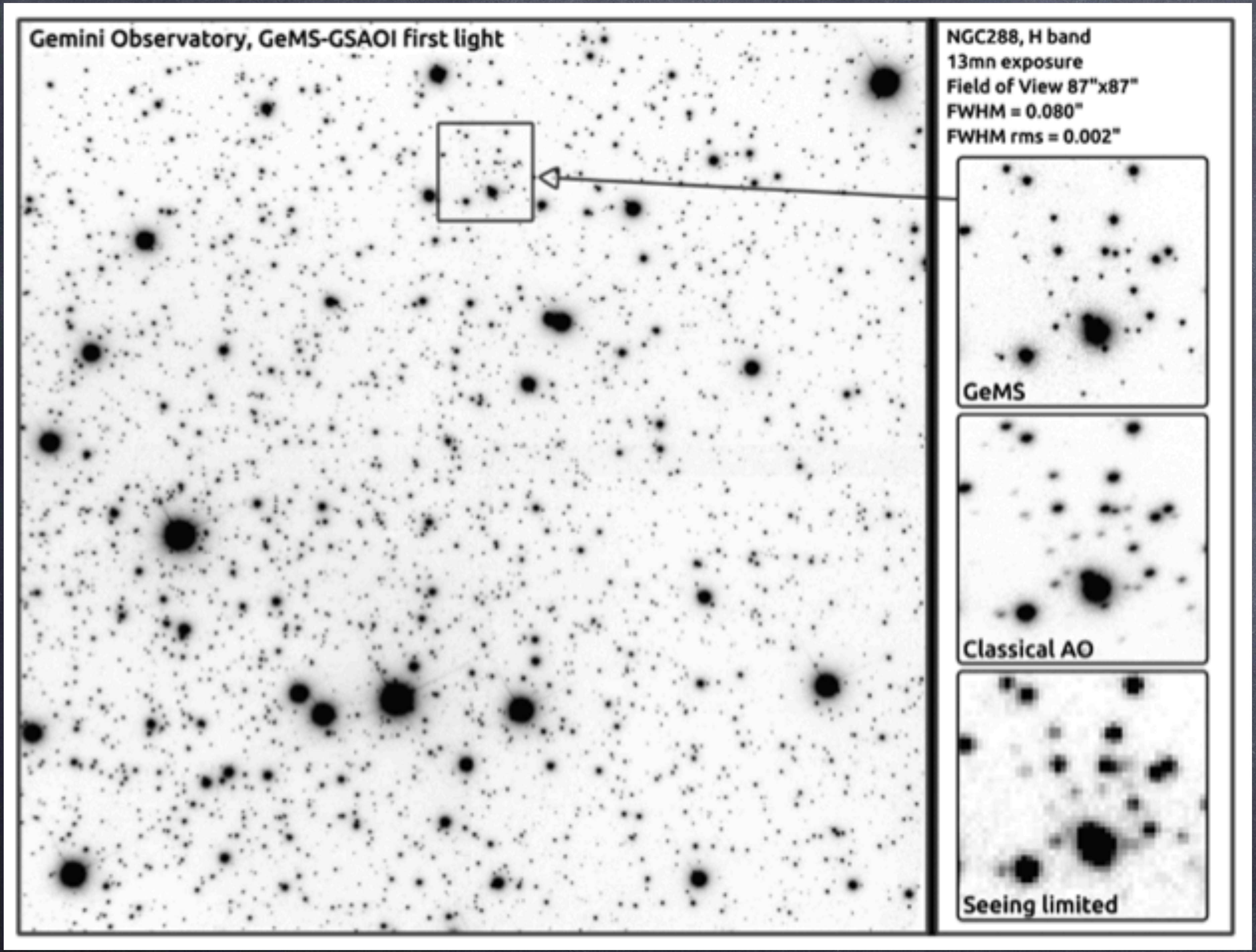
From Marois et al. 2010:
main sequence star HD8799, six exoplanets detected in 2013, from which 5 from (X)AO systems and 1 from HST.

Context: detection & characterisation of exoplanets

very high dynamic range
=> coronagraphy + extreme AO (XAO)

XAO usefull also for observing other types of faint objects (close to much brighter ones): circumstellar matter, (disks, jets), AGN, quasars, etc.

Adaptive optics — 11



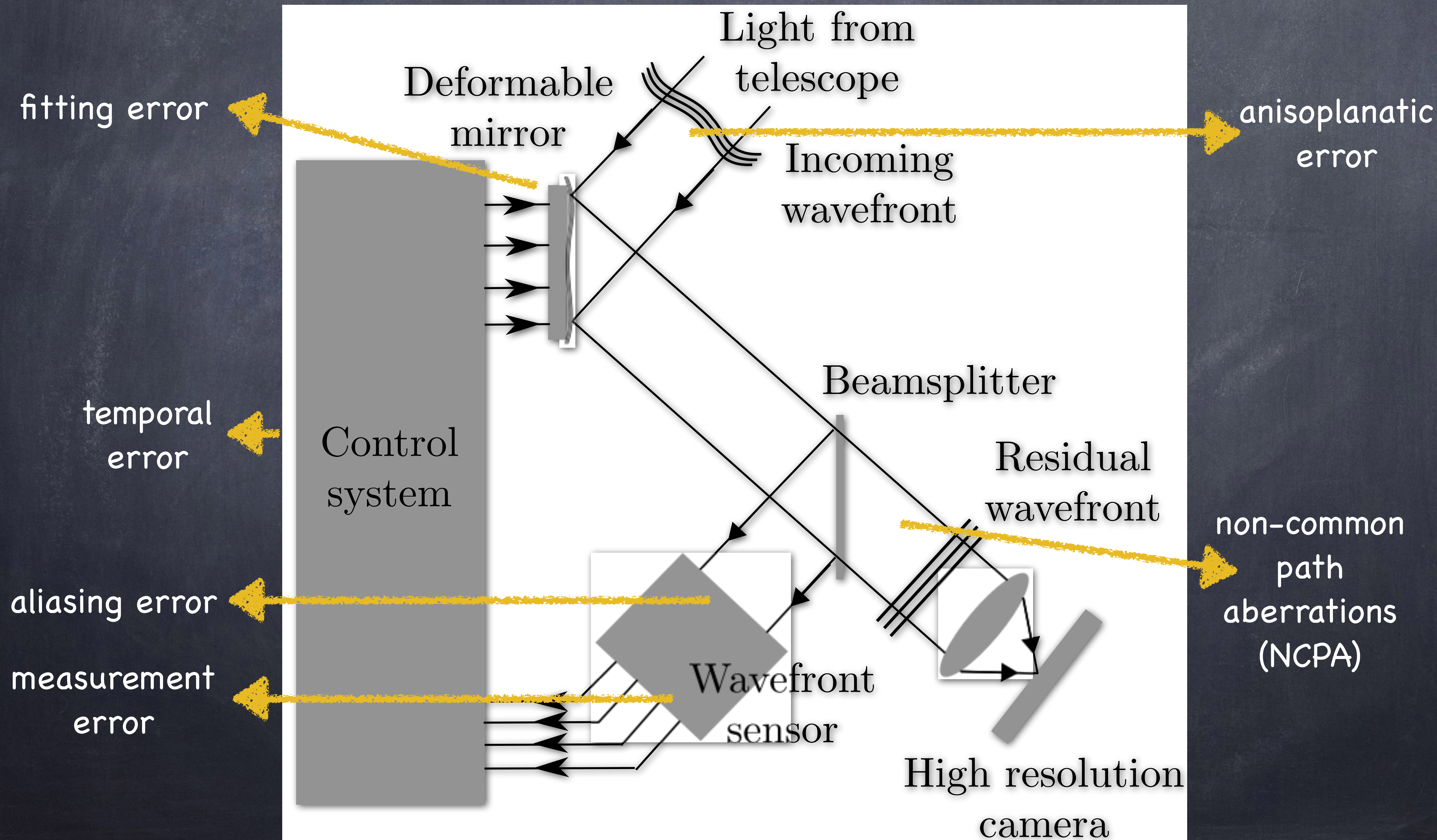
First-light image of GeMS, the MCAO system of Gemini
diffraction limit over a 2' square FoV - vs. a few arcsec !

Context: wide-field astronomical imaging

very wide fields
=> multi-reference
(& multi-conjugate)
AO systems...

-> Also read Rigaut's paper...

Post-AO error budget & PSF morphology — 1

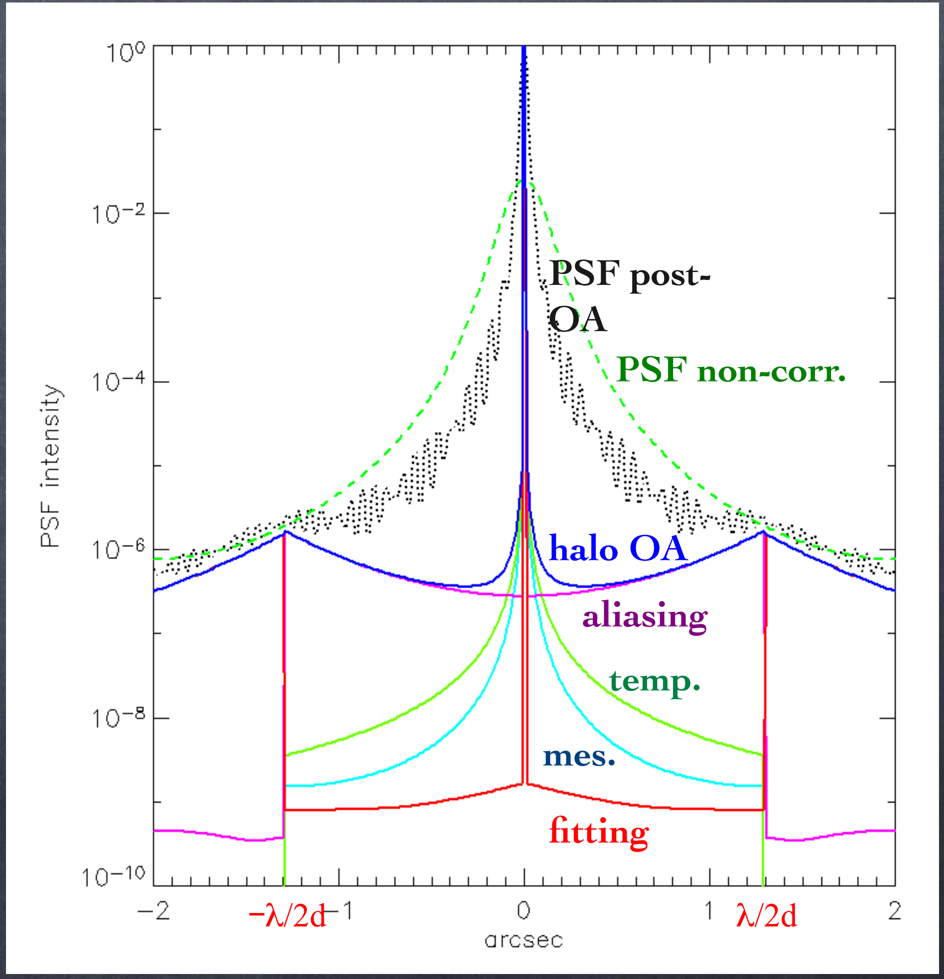


Post-AO error budget & PSF morphology – 2

$$\sigma^2_{\text{post-AO}} = \sigma^2_{\text{atm.}} + \sigma^2_{\text{AO syst.}} + \sigma^2_{\text{others}}$$

$$\sigma^2_{\text{atm.}} = \sigma^2_{\text{aniso.}} + \ldots$$

$$\sigma^2_{\text{others}} = \sigma^2_{\text{NCPA}} + \ldots$$



$$\sigma^2_{\text{AO syst.}} = \sigma^2_{\text{fitt.}} + \sigma^2_{\text{meas.}} + \sigma^2_{\text{alias.}} + \sigma^2_{\text{temp.}} + \ldots$$