(Noll: residual error - 4)

Exercice 1: Compute the Noll error and then the corresponding maximum Strehl ratio in J (1.25 μm) for a 10x10 AO system [D = 1m, r0@500nm = 10cm]

(Noll: residual error - 5)

$$\boxed{J = (N+1)\,(N+2)\,/\,2}$$

Here: N=10 => J = 11x12/2= 66 Zernike modes (piston included)

$$\boxed{\Delta_J \simeq 0.2944\ J^{-\sqrt{3}/2}\left(\frac{D}{r_0}\right)^{5/3},\ J \geq 20}$$

Here: $\Delta_J = 0.2944\ J^{-\sqrt{3}/2}\ (1/0.3)^{5/3} \approx 0.0589\ \mathrm{rad}^2$ (for λ=1.25μm)

$$\boxed{S = \exp(-\Delta_J)}$$

Here: S ≈ exp(-0.0589) ≈ 0.94, i.e. ≈94% Strehl ratio (at λ=1.25μm).

(Noll: residual error - 6)

```
; call with: IDL> @Exo2
Diam  =1.0
r0    =0.3
N     = 10

J = (N+1)*(N+2)/2-1
Noll = .2944*J^(-sqrt(3)/2)*(Diam/r0)^(5./3)
S = exp(-Noll)
; see result with: IDL> print, S
```

batch: all the variables defined are accessible

```
; call with: IDL> .rn Exo2_main
Diam  =1.0
r0    =0.3
N     = 10

J = (N+1)*(N+2)/2-1
Noll = .2944*J^(-sqrt(3)/2)*(Diam/r0)^(5./3)
S = exp(-Noll)

end
; see result with: IDL> print, S
```

main: idem (« .rn » : run new)

```
; call with: IDL> .rn Exo2_proc
;            IDL> Exo2_proc, Diam, r0, N, S
; with, e.g: Diam=1.0, r0=0.3, N=10, S undefined
pro Exo2_proc, Diam, r0, N, S

J = (N+1)*(N+2)/2-1
Noll = .2944*J^(-sqrt(3)/2)*(Diam/r0)^(5./3)
S = exp(-Noll)

end
; see result with: IDL> print, S
```

procedure: the input/output parameters are accessible, not the variables defined inside the procedure

```
; call with: IDL> .rn Exo2_func
;            IDL> print, Exo2_func(Diam, r0, N)
; with, e.g: Diam=1.0, r0=0.3, N=10
function Exo2_func, Diam, r0, N

J = (N+1)*(N+2)/2-1
Noll = .2944*J^(-sqrt(3)/2)*(Diam/r0)^(5./3)
S = exp(-Noll)

return, S
end
```

function: no output parameters, variables defined not accessible, results of the function returned.

(Noll: residual error - 7)

Exercice 2: Which mirror configuration for a (minimum, other errors excluded) goal Strehl ratio of 30% in band J (1.25um) ?

[knowing that: r0@500nm=10cm, D=8m]

[and: nb of Z modes=(n+1)(n+2)/2, n=radial order]

(Noll: residual error - 8)

- Fried parameter in band J:

$$r0[J] = 0.1\ (1.25/0.5)^{6/5} \approx 0.3$$

- What we want is hence:

$$0.3 = \exp\{-0.2944\ J^{-\mathrm{sqrt}(3)/2}\ (D/r0)^{5/3}\}$$

(Thanks to Maréchal and Noll...)
Then: $J \approx 109$ (minimum)

- But: $J = (N+1)(N+2)/2-1 \Rightarrow 13<N<14$

Hence: N=14 (which corresponds to J=119) in order to have the minimum required…

(Noll: residual error - 9)

Remark:

One will often prefer to start from the evaluation of the *rms* on the residual wavefront (σ[m])…

$$\boxed{\sigma[\mathrm{m}] = \lambda/2\pi \sqrt{\Delta_J}}$$

Here: σ[m] = *rms* of the residual wf

$= 1.25\ 10^{6} / 2\pi \sqrt{\Delta_J} \approx 4.83\ 10^{8}$ m $\approx$ 48.3 nm

And then:

$$\boxed{S_\lambda = \exp\{-(2\pi/\lambda\ \sigma[\mathrm{m}])^2\}}$$

Here: $S_{1.25\mu m} = \exp(-(2\pi/1.25\mu m\ \sigma[\mathrm{m}])^2) \approx 0.94$.

(Noll: residual error - 10)

Exercice 3: Find the (linear) number of sub-apertures (and actuators), considering a Fried configuration, corresponding to a fitting-error-only Strehl ratio in J of 30% [D=8m, r0@500nm=12cm].

-> For next session: solve this exercice !!
(IDL batch, and also function)