

(Noll: residual error - 10)

Exercise 3: Find the (linear) number of sub-apertures (and actuators), considering a Fried configuration, corresponding to a fitting-error-only Strehl ratio in J of 30% [$D=8\text{m}$, $r_0@500\text{nm}=12\text{cm}$].

-> For next session: solve this exercise !!
(IDL batch, and also function)

(Noll: residual error - 11)

```
; call with: IDL> @Exo1      Exo_batch.pro
Diam  =8.0
r0    =0.12
Strehl=0.3
band  =1.25E-6

r0_band=r0*(band/500E-9)^(6./5)
J=(-.2944/alog(Strehl)*(Diam/r0_band)^(5./3))^2./sqrt(3))
N=(-3+sqrt(9+8*J))/2
if N ne floor(N) then N=floor(N)+1
; see result with: IDL> print, N
```

```
function Ncalc, Diam, r0, Strehl, band
; call with: IDL> N=Exo_function(Diam, r0, Strehl, band)

r0_band=r0*(band/500E-9)^(6./5)
J=(-.2944/alog(Strehl)*(Diam/r0_band)^(5./3))^2./sqrt(3))
N=(-3+sqrt(9+8*J))/2
if N ne floor(N) then N=floor(N)+1

return, N-1
end
```


Post-AO error budget - 9

Aliasing error

$$\sigma_{\text{AO syst.}}^2 = \sigma_{\text{fitt.}}^2 + \sigma_{\text{meas.}}^2 + \sigma_{\text{alias.}}^2 + \sigma_{\text{temp.}}^2 + \dots$$

$$\sigma_{\text{alias.}}^2 \propto \left(\frac{d}{r_0} \right)^{\frac{5}{3}}$$

$$\sigma_{\text{alias.}}^2 \approx 0.17 \left(\frac{d}{r_0} \right)^{\frac{5}{3}}$$

Reduce the aliasing error \Leftrightarrow increase the number of sensing elements (Shack-Hartmann sub-apertures, pixels of the pyramid) within the WFS

Post-AO error budget - 10

Temporal error

$$\sigma_{\text{AO syst.}}^2 = \sigma_{\text{fitt.}}^2 + \sigma_{\text{meas.}}^2 + \sigma_{\text{alias.}}^2 + \sigma_{\text{temp.}}^2 + \dots$$

$$\sigma_{\text{temp.}}^2 \propto \left(\frac{\Delta t_{\text{AO}}}{\tau_0} \right)^{\frac{5}{3}}$$

$$\sigma_{\text{temp.}}^2 \approx \left(\frac{\Delta t_{\text{AO}}}{\tau_0} \right)^{\frac{5}{3}}$$

Reduce the temporal error \Leftrightarrow make a faster system
(exposure time of the WFS, computing time for the
wavefront reconstruction, actuating time for the DM)

Post-AO error budget - 11

Measurement error

$$\sigma_{\text{AO syst.}}^2 = \sigma_{\text{fitt.}}^2 + \sigma_{\text{meas.}}^2 + \sigma_{\text{alias.}}^2 + \sigma_{\text{temp.}}^2 + \dots$$

$$\sigma_{\text{mes.}}^2 = \sigma_{\text{phot.}}^2 + \sigma_{\text{RON}}^2 + \dots$$

The measurement error has many origins:

- photon noise
- read-out noise (RON)
- dark-current noise
- sky background and possibly instrumental background
- in case of EMCCD: (almost) no RON but additional noises (exotic dark, « excess noise factor » => Gamma-distributed noise)

Post-AO error budget - 12

Photon noise error term

$$\sigma_{\text{AO syst.}}^2 = \sigma_{\text{fitt.}}^2 + \sigma_{\text{meas.}}^2 + \sigma_{\text{alias.}}^2 + \sigma_{\text{temp.}}^2 + \dots$$

$$\sigma_{\text{mes.}}^2 = \sigma_{\text{phot.}}^2 + \sigma_{\text{RON}}^2 + \dots$$

$$\sigma_{\text{phot.}}^2 \propto \frac{1}{N_{\text{phot.}}}$$

$$\sigma_{\text{phot.}}^2 \propto \frac{1}{N_{\text{phot.}} r_0^2 \tau_0}, \quad \tau_0 \propto r_0$$

(with N_{phot} =number of photons/exposure time/subaperture)

Reduce the photon noise error term \Leftrightarrow

- 1- reduce the number of WFS elements \Rightarrow increase the aliasing error !!
- 2- increase the exposure time \Rightarrow increase the temporal error !!

Post-AO error budget - 13

Read-out noise error term

$$\sigma_{\text{AO syst.}}^2 = \sigma_{\text{fitt.}}^2 + \sigma_{\text{meas.}}^2 + \sigma_{\text{alias.}}^2 + \sigma_{\text{temp.}}^2 + \dots$$

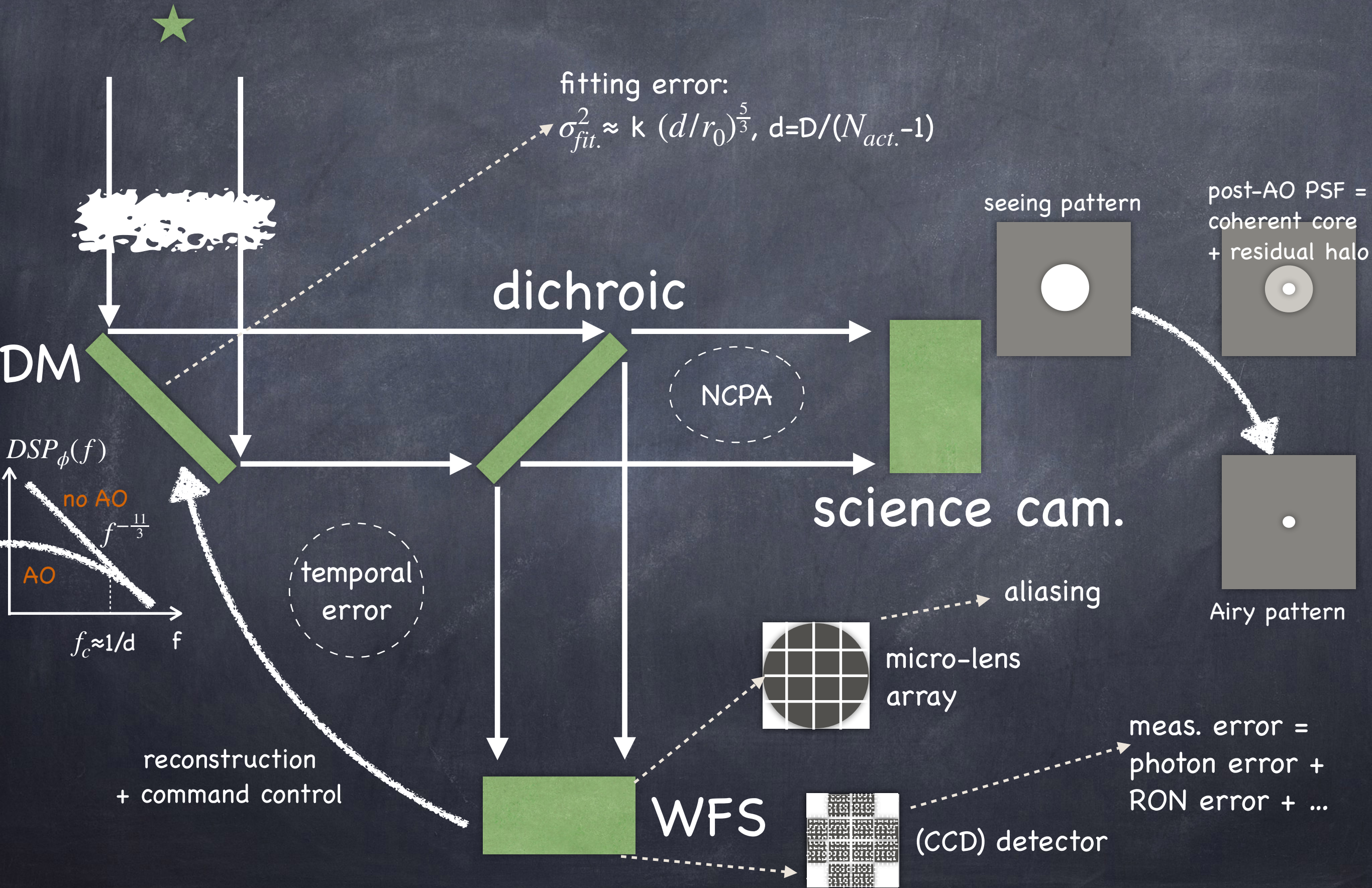
$$\sigma_{\text{mes.}}^2 = \sigma_{\text{phot.}}^2 + \sigma_{\text{RON}}^2 + \dots$$

$$\sigma_{\text{lect.}}^2 \propto \frac{\sigma_e}{N_{\text{phot.}}^2}$$

Reduce the RON error term \Leftrightarrow

- 1- reduce the number of WFS elements \Rightarrow increase the aliasing error !!
- 2- increase the exposure time \Rightarrow increase the temporal error !!
- 3- but also: reduce the impact of RON \Rightarrow use of EMCCDs...

Post-AO error budget - 14



Post-AO error budget - 15

Generic case: observe as much sources as possible

Problem: most sources are (obviously!) too faint

1- find and use brighter NGS nearby...

=> anisoplanatic error !

=> use more than one brighter NGS nearby...

=> multi-reference AO system (GLAO, MCAO, MOAO)

=> yes, but: specific errors !

=> limited quality of correction

2- create a brighter source (LGS)

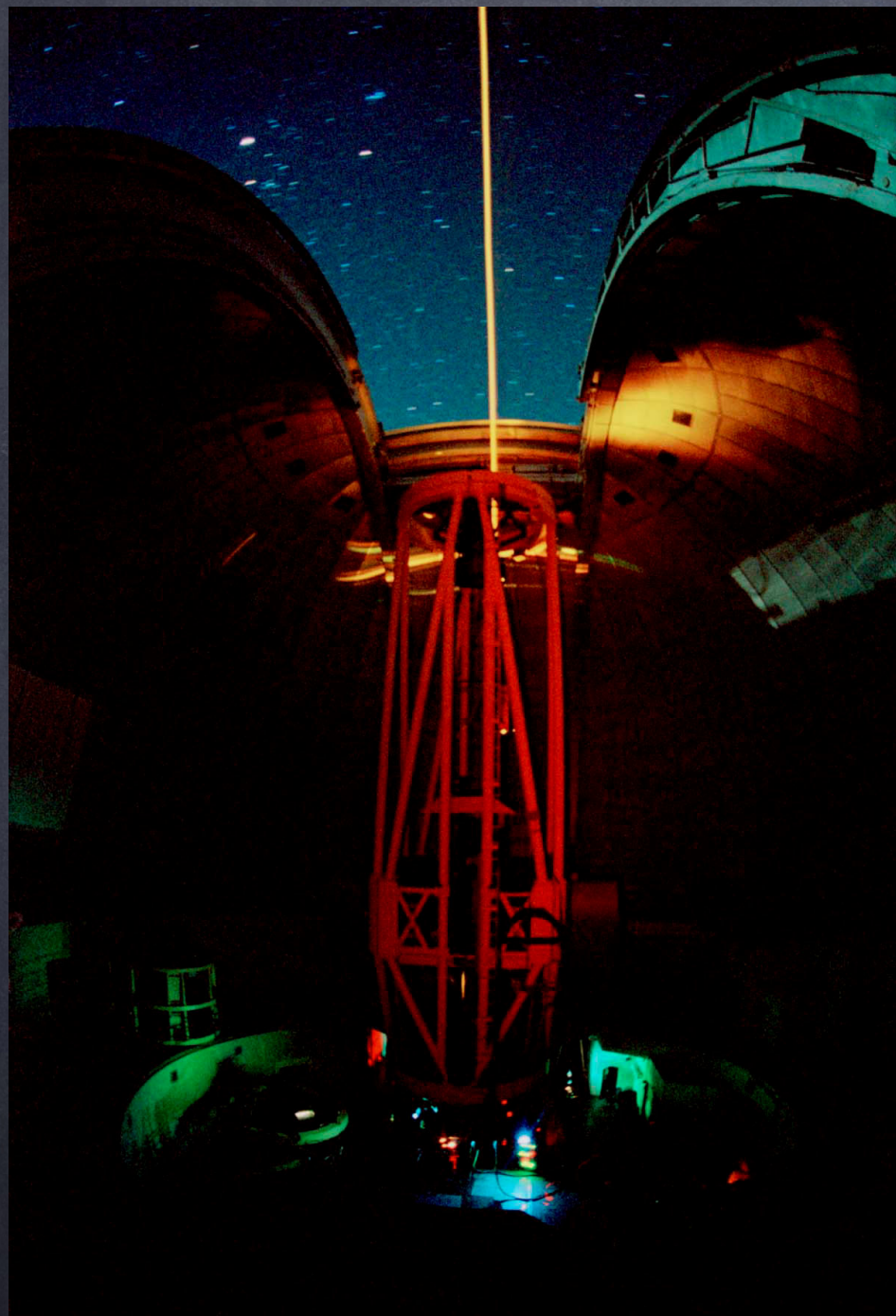
=> 100% sky coverage

=> yes, but here again: specific errors !

=> limited quality of correction

Laser Guide Star AO - 1

[Happer 1982, Foy & Labeyrie 1985]



(Keck Observatory)

+ https://www.youtube.com/watch?v=3BpT_tXYy_I

Laser Guide Star AO - 2

Specific LGS errors:

- 1- Cone effect (focus aniso.)
- 2- Tip-tilt indetermination
- 3- Perspective elongation

In addition (Na LGS case):

- Rayleigh backscattering until $\sim 35\text{km}$,
- variation of the Na layer column density (seasonally: a factor $\sim 5!$ but also short time) and of the centroid height.

Science w/LGS: solar system bodies, YSO (circumstellar material), brown dwarves, Galactic novae, starburst galaxies, AGN, radio galaxies, gravitational lenses, clusters of galaxies, etc.

