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Pupil plane wavefront sensing with an oscillating prism

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Abstract. A compact pupil plane wavefront sensor is described, which is able to image on a single detector four images of the pupil, containing information on the gradient of the incoming wavefront. The wavefront sensor consists of a lens relay and an oscillating pyramidal-shaped prism. The gain of the device is driven by the amplitude of the oscillations, while the sampling is determined by the focal length of the lens relay. This wavefront sensor can be conveniently used for astronomical adaptive optics purposes because of its flexibility to match the brightness of the reference source used (varying the sampling) and the seeing conditions (varying the gain).

1. Introduction

Wavefront sensors used in adaptive optics for astronomy belong, essentially, to the Shack–Hartmann, curvature and shearing interferometers [1]. Of these devices in a particular implementation of the curvature sensor [2] it is possible to change the gain easily and in a continuous way, i.e. the response versus the measured wavefront deformation. The sampling is usually taken as a fixed parameter, especially for Shack–Hartmann wavefront sensors where the spots produced by a lenslet array are usually matched with the intersection of four adjacent pixels in the detector. In this way it is hard to change the lenslet array for sampling modification, because of alignment problems.

This paper describes a concept for a wavefront sensor where both gain and sampling can be easily changed in a simple and continuous manner.

2. Description of the wavefront sensor

The wavefront sensor layout is sketched in figure 1 while a cross section of a *true* implementation adopting a polychromatic source and an achromat doublet as lens relay is shown in figure 2.

The nominal focal plane of the telescope illuminating the wavefront sensor lies approximately on the vertex of the pyramidal prism. The four faces of this prism deflect the light in slightly different directions and, as seen from the lens relay, the exit pupil position of the telescope will appear slightly shifted in four different directions. Provided there are negligible aberrations over a wide enough field of view, the lens relay is able to conjugate the four apparent exit pupils onto four pupil images on the detector surface. The vertex angle of the pyramid is to be kept very low. It must be as slightly less than 180° to give the four pupils enough relative displacement so that they do not overlap (as in the figures). This allows for only very tiny displacements of the pupils when the pyramid is moved slightly

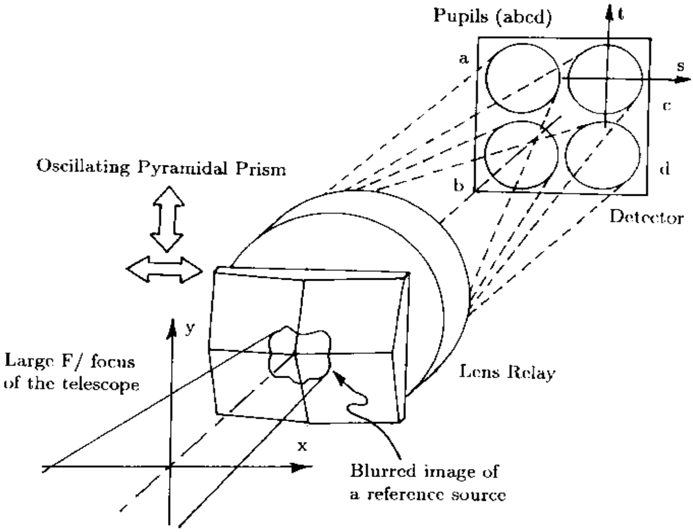

Figure 1. The overall layout of the wavefront sensor concept described in the text.

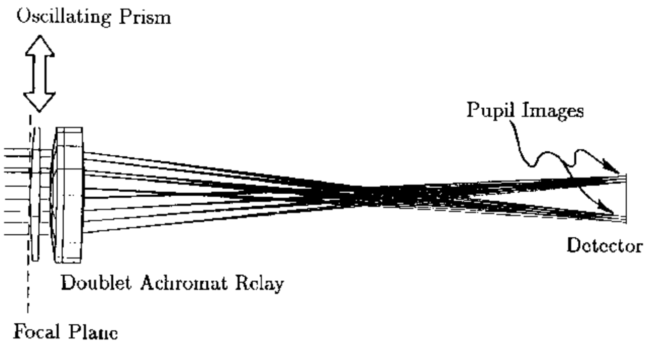

Figure 2. A *true* optical ray-tracing section of the proposed wavefront sensor. For clarity the vertex angle of the pyramid and the distance between the two pupils has been greatly exaggerated.

in the direction of the arrows due to a much larger distance of the exit pupil with respect to the focal length of the lens relay.

Let us consider a section of the wavefront sensor (see also figure 2). The amount of light that is collected by one pupil is given by the amount of light that hits the prism on the related face. Being the prism on the focal plane and the position depending upon the derivative of the wavefront W, one can easily see that each pupil is illuminated by rays with a given range ∇W. Establishing a normalized coordinate system (s, t) on the pupil plane ($s^2 + t^2 = 1$ on the edges of the pupil, both in the real and in the reimaged ones), and a coordinate system aligned with (s, t), on the image plane at the pyramid vertex (x, y), one can easily verify the relationships

$$x = \frac{\partial W}{\partial s} F ; \quad y = \frac{\partial W}{\partial t} F , \qquad (1)$$

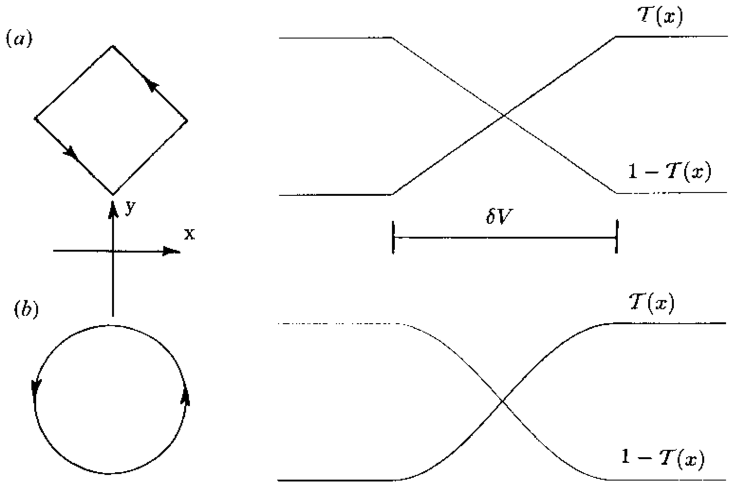

Figure 3. Two ways to vibrate the pyramidal prisms. See text for details.

where F is the focal ratio of the telescope at the wavefront sensor foci. Moreover, adding the images (a) and (b) together one exactly obtains the image collected via a knife-edge test (or Foucault test). Adding images (c) and (d) one obtains the complementary knife-edge test, while adding (a) and (c) images or (b) and (d) images the same test is obtained along the orthogonal direction.

If the pyramidal prism is vibrated (with a frequency equal to, an integer multiple of, or simply much higher than $1/\tau$, where τ is the integration time adopted for the detector) one can obtain the relative (in one vibration cycle) *transparency* of each of the four faces.

In figure 3(*a, b*) one can see two different cases, where the amplitude of vibration is noted as δV and the corresponding relative transparencies $T(x)$ or $T(y)$ can be derived.

The mode shown in figure 3(*a*) will provide a linear T, but at the turning edges will produce very high accelerations on the system (this can be realized with piezo-drivers for small δV amplitudes). Adopting the mode shown in figure 3(*b*) one can more easily realize it with some eccentric spinning mechanism, but the resulting T is nearly linear only in the central portion of the curve.

Provided the max blur is smaller than δV, the amount of light collected in the pupils (*a*) and (*b*) co-added together after the proper integration time τ, will be given by:

$$I_{\mathrm{ab}}(s,\ t) = I_{\mathrm{o}}(s,\ t)\, T\left(\frac{\partial W\ (s,\ t)}{\partial s}\, F\right), \tag{2}$$

while I_{cd} can be obtained by substituting $1 - T$ for T, and I_{ac} and I_{bd} can be evaluated along the orthogonal t direction.

The scintillation effect can be ruled out by using as an estimator of the wavefront derivative the normalized differences between opposite pairs of pupils. Finally, if the transparency function T is (or can be assumed) linear over the whole interval

δV, the following relationships (where $x = I_x(s, t)$) will hold:

$$\frac{\partial W(s,t)}{\partial s} = \frac{(a+b)-(c+d)}{a+b+c+d} \times \frac{F}{\delta V}; \qquad \frac{\partial W(s,t)}{\partial s} = \frac{(a+c)-(b+d)}{a+b+c+d} \times \frac{F}{\delta V}, \qquad (3)$$

where it can easily be seen that the gain is driven by the vibration amplitude δV. The sampling, i.e. the size of the pupil on the detector surface, is driven by the focal length of the relay lens and this can be easily changed in a continuous manner with a zoom optical relay. The required field of view is very small and the working F ratio of this optical relay is so large that such an option can probably be obtained with a few optical elements, while not affecting the throughput performances. As a final remark, changing position of the focus of this last element (but still keeping the foci of the telescope on the vertex of the prism) one could also image onto the detector some turbulent layer located kilometres away from the telescope.

3. Comparison of performances with Shack-Hartmann sensor

The performance of this detector, i.e. photon efficiency, accuracy and sensitivity [3], are the same as a Shack–Hartmann wavefront sensor, provided the following are true:

(1) the motion of each subaperture image on the Shack–Hartmann sensor is measured with a 4 CCD pixel quadrant [4];
(2) the sampling of the pupil from the CCD pixel size in the wavefront sensor here proposed and the sampling of a single lens of the lenslet array for the Shack–Hartmann case are the same on the input pupil;
(3) the integration time is the same for both sensors;
(4) the equivalent angular size, as projected in the sky, of the spot collected on the Shack–Hartmann sensor's detector is the same as the equivalent angular size, as projected in the sky, of δV.

In the case of a small departure of the wavefront from a plane one, the statement obviously derives from the fact that the photons collected for each subaperture and for the same integration time are equal and split into a 4 CCD pixel for both sensors, because, with the given assumptions, and the derivative of the wavefront as in equation (3), the effects of readout noise and photon noise are the same. Moreover, the saturation effect experienced by this sensor when, occasionally, some portion of the input pupil spreads over the δV range is equivalent to the Shack–Hartmann case when the spot is shifted in such a way that the intersection of the four-quadrant pixel is no longer inside the spot.

The required number of pixels, i.e. the size of the CCD, can be the same if care is taken such that the four pupils are as close as possible without overlapping (this is done by the proper choice of the vertex angle of the pyramid).

4. Conclusions

The wavefront sensor here described closely resembles the one illustrated by Horwitz [5] and the concepts illustrated by Sprague and Thompson [6] and Horwitz [7]. It is curious to point out that a similar concept is the basis of the modulation contrast microscope [8]. In some cases in microscopy, in fact, the object to be observed has a very low contrast and can only be imaged through the deformation of the wavefront of the light used for its illumination [9].

In these devices, however, the transparency T is to be generated with a gradient beam-splitter, in order to recover all the light collected and to avoid scintillation bias. Such filters are very difficult to realize, having δV of the order of $D \times F \times \lambda / r_0$. The proposed layouts, moreover, require additional optics and four different detectors, leaving little space for flexibility. In contrast, the key parameters of the wavefront sensor here proposed can easily be changed and the compactness of the design allows for a simpler and more cost-effective realization. It has also been shown that its performance is the same as a Shack–Hartmann wavefront sensor used with a four pixels per spot centroiding mode.

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