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STRUCTURE OF THE SOLAR CORE: EFFECT OF ASYMMETRY OF PEAK PROFILES

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ABSTRACT

Recent studies have established that peaks in solar oscillation power spectra are not Lorentzian in shape but have a distinct asymmetry. Fitting a symmetric Lorentzian profile to the peaks, therefore, produces a shift in frequency of the modes. Accurate determination of low-frequency modes is essential to infer the structure of the solar core by inversion of the mode frequencies. In this paper we investigate how the changes in frequencies of low-degree modes obtained by fitting symmetric and asymmetric peak profiles change the inferred properties of the solar core. We use data obtained by the Global Oscillations at Low Frequencies (GOLF) project on board the *SOHO* spacecraft. Two different solar models and inversion procedures are used to invert the data in order to determine the sound speed in the solar core. We find that for a given set of modes no significant difference in the inferred sound speed results from taking asymmetry into account when fitting the low-degree modes.

Subject headings: Sun: interior — Sun: oscillations

1. INTRODUCTION

Accurate determination of the frequencies of low-degree oscillation of the Sun is essential in inferring the structure of the solar core, owing to the small effect of this part of the Sun on the oscillation frequencies (e.g., Turck-Chièze, Brun, & Garcia 1999). Thus, particular care is required in interpreting the data obtained. Important aspects, which have received considerable attention recently, are the effect on the frequencies of the variations of solar activity during the solar cycle (Dziembowski & Goode 1997; Dziembowski et al. 1997), as well as the different results obtained from intensity and velocity measurements (Toutain et al. 1997). These different observations have led to a better understanding of how the data must be analyzed, emphasizing, for example, the need to use the same period of time for low- and high-degree modes to avoid differences in the near-surface effects, and ideally considering data sets of one or a maximum of two years around the minimum of activity.

There are now a number of experiments designed specifically to investigate the properties of the solar core; one of these is the Global Oscillations at Low Frequencies (GOLF) instrument (e.g., Gabriel et al. 1997) on board the *Solar and Heliospheric Observatory* (*SOHO*) (e.g., Domingo, Fleck, & Poland 1995). A comparison of different data series still shows differences in the inferences in the solar core substantially exceeding the error bars (e.g., Fig. 6 of Turck-Chièze et al. 1997). In order to make progress on this point, we propose here a differential study, using GOLF data taken near the solar minimum, to show the influence of three ingredients of the investigation: (1) the asymmetry of the frequency peaks due to the localized nature of the source of the oscillation and its interaction with the noise; (2) the role of the methods used for inverting the data; and (3) the influence of the solar model used (with different prescriptions for the atmosphere) to perform the inversion. The purpose of the paper is to give quantitative estimates of these effects and a better estimate of the uncertainty they may introduce in our determination of the structure of the solar core.

It has been demonstrated that in general the peaks in solar oscillation power spectra are not symmetric (e.g., Duvall et al. 1993; Nigam & Kosovichev 1998; Toutain et al. 1998; Chaplin & Appourchaux 1999). This is believed to be a consequence of the localized nature of the source that drives the oscillations (e.g., Gabriel 1993; Abrams & Kumar 1996; Roxburgh & Vorontsov 1997; Nigam et al. 1998; Rosenthal 1998). Despite the evidence for asymmetry, most analyses of observed solar power spectra involve the fitting of symmetric Lorentzian profiles to the peaks in power. This leads to a systematic error in the inferred frequencies.

While there is no longer any doubt that the peaks are asymmetric and that there is a frequency shift, what is still not clear is whether the shift in the frequencies changes results obtained by inverting the frequencies. Tests by Christensen-Dalsgaard et al. (1998) and Rabello-Soares et al. (1999b) suggest that the frequency shift is a smooth function of frequency, which may possibly be removed while inverting the frequencies. Also, using frequencies determined from m-averaged spectra obtained by the Global Oscillation Network Group (GONG), Basu & Antia (2000) showed that frequencies obtained by using asymmetric fits to the peaks do not significantly change results obtained by inversion. However, Toutain et al. (1998) found that for low-degree data obtained by the Michelson Doppler Imager (MDI) on board *SOHO* for 679 days of observation, the shift in the frequencies does change the result substantially in the solar core.

In this paper, we use solar data obtained by GOLF to check whether there is a significant change in the inferred solar structure when one shifts from using frequencies from Lorentzian-fitted peaks to those from asymmetric profiles.

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The details of the fitting procedure and the results of the fits have been described by Thiery et al. (2000). We use only the $l = 0$, 1, and 2 modes from the GOLF data. Since it is essential to have intermediate- and high-degree modes to do a reliable inversion, we use the $l = 3$–250 data obtained from MDI (Schou et al. 1998). We note that only symmetrically fitted frequencies are available for these observations. This introduces an unfortunate inconsistency in our analysis, which, however, would most likely increase the error in the inferred sound speed (e.g., Rabello-Soares et al. 1999b).

The rest of the paper is organized as follows: we describe the inversion techniques and solar models used in this work in § 2, the results of the inversions are discussed in § 3, and our conclusions are stated in § 4.

2. THE INVERSION TECHNIQUE

Inversions to determine solar structure from solar oscillation frequencies proceed through the linearization of the equation for linear adiabatic oscillations around a known solar model. When the oscillation equation is linearized—under the assumption of hydrostatic equilibrium—the fractional change in the frequency can be related to the fractional changes in the squared sound speed (c^2) and density (ρ). Thus,

$$\frac{\delta\omega_i}{\omega_i} = \int K^i_{c^2,\,\rho}(r)\,\frac{\delta c^2}{c^2}\,(r)dr + \int K^i_{\rho,\,c^2}(r)\,\frac{\delta\rho}{\rho}\,(r)dr + \frac{F_{\mathrm{surf}}(\omega_i)}{E_i} \quad (1)$$

(cf. Dziembowski, Pamyatnykh, & Sienkiewicz 1990). Here $\delta\omega_i$ is the difference in the frequency ω_i of the ith mode between the solar data and a reference model. The kernels $K^i_{c^2,\rho}$ and K^i_{ρ,c^2} are known functions of the reference model that relate the changes in frequency to the changes in c^2 and ρ, respectively; and E_i is the inertia of the mode, normalized by the photospheric amplitude of the displacement. The term F_{surf} results from the near-surface differences between the Sun and models because of the difficulty in modeling the outer layers.

2.1. *The Inversion Methods*

We have used two different methods to invert the frequencies: the regularized least squares (RLS) and the subtractive optimally localized averages (SOLA) methods.

For RLS inversions, the sound speed and density differences between the Sun and the reference model are described by a set of basic functions in radius, r (in this case, splines). The surface term is described as a spline in frequency. The spline coefficients are found by minimizing the difference between the left-hand side of equation (1) and the right-hand side expanded in splines, subject to the condition that the resulting $\delta c^2/c^2$ is smooth.

The number of knots in radius r (a total of 120) and the number of knots in frequency to describe the surface term (25) are determined by the fact that for a proper inversion we need enough knots to ensure that the residuals of the fit are randomly distributed as a function of frequency and lower turning point of the modes for a proper inversion; on the other hand, the condition number of the system of equations (which increases with an increase in the numbers of knots) should be as small as possible to ensure that the system is sufficiently well conditioned to allow a stable numerical solution (see, e.g., Basu & Thompson 1996). The knots were distributed according to the density of turning points of the set of modes along the radius. The trade-off parameter, which controls the error in the solution and its smoothness, was determined by plotting the so-called L-curve, which gives the Tikhonov smoothing term (here the norm of the first derivative of the solution) as a function of χ^2 to find a compromise between a good fit of the data (small χ^2) and a rather smooth, physically acceptable solution (Gonczi et al. 1998).

The principle of the SOLA inversion technique (Pijpers & Thompson 1992) is to form linear combinations of equation (1) with weights $d_i(r_0)$ chosen so as to obtain an average of $\delta c^2/c^2$ localized near $r = r_0$ while suppressing the contributions from $\delta\rho/\rho$ and the near-surface errors when inverting for $\delta c^2/c^2$. In addition, the statistical errors in the combination must be constrained. The result of the inversion is then an average of $\delta c^2/c^2$, with a weight determined by the *averaging kernel*, defined by

$$\mathscr{K}(r_0, r) = \sum d_i(r_0) K^i_{c^2,\,\rho}(r) \quad (2)$$

and normalized so that $\int \mathscr{K}(r_0, r)dr = 1$. Details of the implementation were provided by Basu et al. (1996), and a procedure to find the parameters required in the inversion was discussed by Rabello-Soares, Basu, & Christensen-Dalsgaard (1999a).

2.2. *The Reference Solar Models*

We have used two reference models for this work. The first model (hereafter referred to as the "Saclay/Nice model") is an updated calculation of the standard model of Brun, Turck-Chièze, & Morel (1998) based on the CESAM code (Morel 1997). Nuclear reaction rates of Adelberger et al. (1998) were used with screening effects from Dzitko et al. (1995). The most recent OPAL opacity tables (Iglesias & Rogers 1996) and the OPAL equation of state (Rogers, Swenson, & Iglesias 1996) have been introduced in constructing the model. This model converged at the solar age with the observed abundances of 13 elements from Grevesse & Noels (1993), and microscopic diffusion of each of these elements was computed using diffusion coefficients from Michaud & Proffitt (1993). A reconstructed atmosphere was deduced from the ATLAS9 atmosphere code of Kurucz (1991). The computation included a pre–main-sequence evolution phase, and the model has an age of 4.6 Gyr, including this phase. A more detailed description of the model was given by Brun et al. (1998). Some results of comparisons of this model with others and preliminary comparisons with the Sun were made by Turck-Chièze et al. (1998).

The second model is model S of Christensen-Dalsgaard et al. (1996). This model is used because many helioseismological results in literature are based on this reference model. This is a standard solar model constructed with the Livermore (OPAL) equation of state (Rogers et al. 1996). For temperatures higher than 10^4 K, an early version of the OPAL opacities was used (Rogers & Iglesias 1992); at lower temperatures, opacities from the tables of Kurucz (1991) were taken. The model incorporates the diffusion of helium and heavy elements below the convection zone. The surface heavy element ratio is $Z/X = 0.0245$ (Grevesse & Noels

1993). The model has an age of 4.6 Gyr without pre–main sequence. The model was described in detail by Christensen-Dalsgaard et al. (1996).

3. RESULTS

The differences in frequencies obtained by fitting Lorentzian profiles and those obtained by fitting an asymmetric profile are shown in Figure 1. Note that the differences are systematic, not random, and hence one could expect that the result of inverting the two sets of frequencies will show systematic differences, too.

Figure 2 shows the sound speed difference between the Saclay/Nice model and the Sun, obtained using GOLF data combined with MDI data for $l \geq 3$. Both sets of GOLF frequencies, i.e., the one obtained by fitting a Lorentzian profile and the one obtained by fitting the asymmetric profile of Thiery et al. (2000), are shown.

The results of the SOLA and RLS inversions agree within errors in most of the Sun. The height of the bump at the base of the convection zone is higher in the case of RLS than SOLA and is a result of slightly different resolutions in the two inversions. For the same error magnification, RLS generally has a better resolution than SOLA, though with the drawback that the RLS averaging kernels have some structure far away from the target radius. There is some difference in the convection zone, which is most probably a reflection of differences in error correlation.

For a given inversion method, the results for the outer layers are almost identical regardless of the data set used. This is expected since the same set of high-degree modes is used in both cases. There are changes in the core when the symmetric data set is replaced by the asymmetric set; however, we can see that they are within the errors and

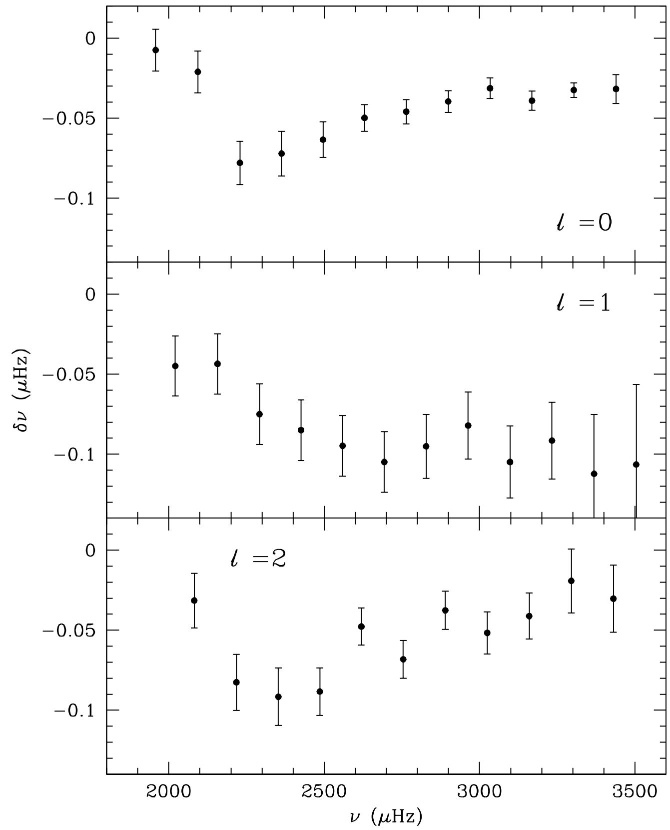


FIG. 1.—Difference between the frequencies obtained by fitting a Lorentzian profile to the peaks in the oscillation power spectrum and those obtained by fitting an asymmetric profile, in the sense of (symmetric fit) − (asymmetric fit). The differences are shown only for the frequency range used in the inversions.

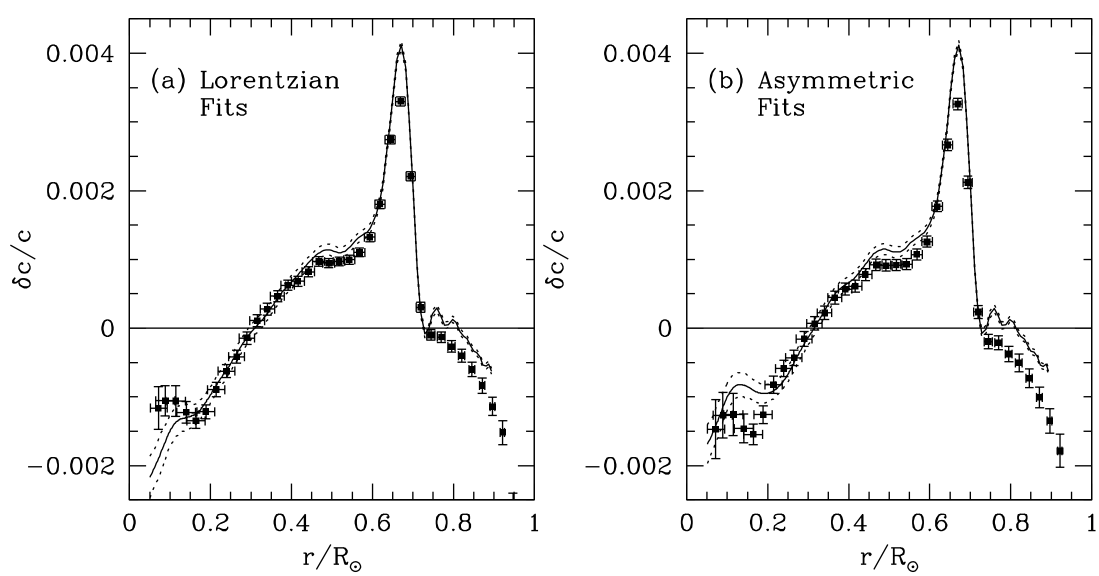


FIG. 2.—Relative sound speed differences, $\delta c/c$ between the Sun and the Saclay/Nice model. The differences are taken in the sense (Sun − model)/Sun. (*a*) Results of inverting frequencies obtained by fitting power spectrum peaks with a Lorentzian profile. (*b*) Frequencies obtained by fitting an asymmetric profile to the peaks. In both panels, data for models with $l = 0$, 1, and 2 were obtained from the GOLF instruments. Data for modes with higher degree were obtained by the MDI instrument. The points with error bars are the results of the SOLA inversions. The vertical error bars indicate the 1 σ error in the inversion due to data errors, while the horizontal error bars are a measure of the resolution. The solid line is the result of an RLS inversion, and the dotted lines show the 1 σ error limits on that inversion.

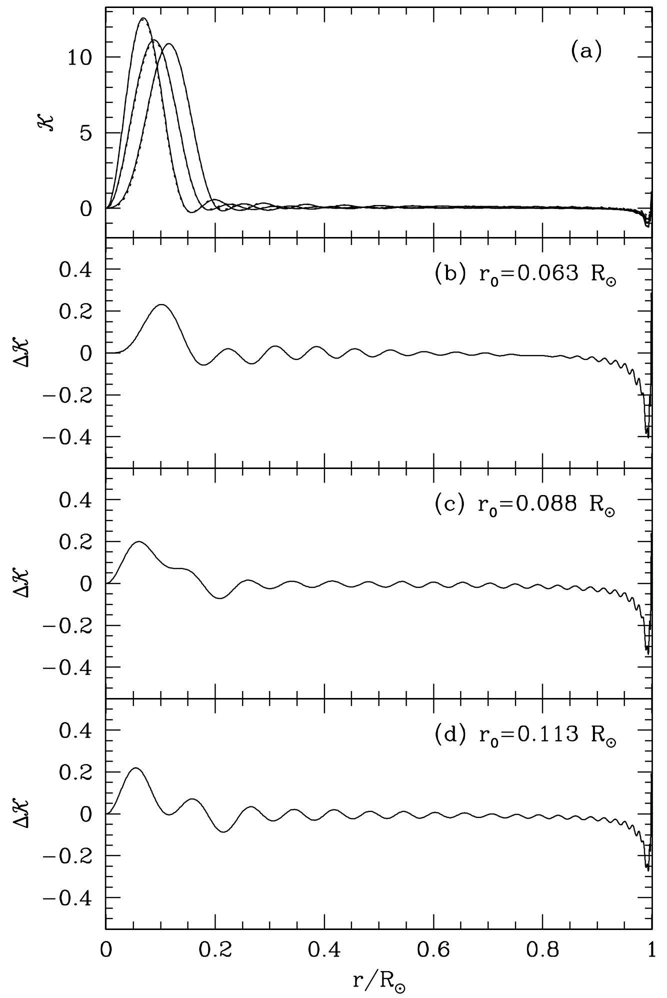


FIG. 3.—(*a*) Sample of SOLA averaging kernels. The target radii for the kernels are 0.063 $R_\odot$, 0.088 $R_\odot$, and 0.113 $R_\odot$. The continuous lines are for the mode set obtained by fitting a Lorentzian; the dotted lines—which can barely be distinguished from the continuous lines—are for the asymmetric set. Panels (*b*)–(*d*) show the differences, in the sense (Lorentzian − asymmetric), between the averaging kernels of the two sets. Note that the differences are much smaller than the peak height of the averaging kernels.

hence not significant. In Figure 3 we show some of the averaging kernels in the region of the core. The figure also shows the difference between the averaging kernels obtained for the inversion of the two data sets. We see that there is very little difference between them.

We get similar conclusions using model S (cf. Fig. 4): the introduction of the asymmetry does not significantly affect the inversion in the core. The results for the outer layers are identical, while the change in the core is very small when data sets are changed. The two models, however, have different sound speed profiles, which warrant some comments. The sound speed differences between the two models can be attributed completely to the physical inputs of the models. The differences are mainly due to the following three effects: (1) a reestimate of the solar age (about 4.55 Gyr without including the pre–main sequence) in the Saclay/Nice model, which lowers the relative sound speed difference in the very inner core and slightly increases the peak in the sound speed difference relative to the Sun below the base of the convection zone; (2) a reestimate of the nuclear reaction rates and the screening effect, which has similar consequences (see also the discussion of Turck-Chièze et al. 1998); and (3) the upgrade to the most recent OPAL opacity tables, which has little effect in the core but dominates the increase in the sound speed difference below the convection zone.

Our main result is that frequencies obtained using the asymmetric fits do not change inversion results. This is consistent with the estimates by Christensen-Dalsgaard et al. (1998) of the functional form of frequency shifts caused by asymmetry, based on artificial data, which indicated that such shifts would largely be eliminated together with the surface term in $F_{\rm surf}$. This conclusion was confirmed by the inverse analyses carried out by Rabello-Soares et al. (1999b) of such artificial data, which showed that asymmetric fits had a very modest effect on the results of structure inversion. However, the results are strikingly different from those obtained by Toutain et al. (1998) using only MDI data.

The question then arises as to why the MDI results of Toutain et al. are so discrepant. In Figure 5 we show the inversion of the symmetric MDI set (Schou et al. 1998), as well as the inversion of the low-degree MDI data obtained by fitting an asymmetric profile (Toutain et al. 1998) combined with the $l \geq 3$ data of the MDI data set; in both cases the reference model was model S. We see that while the symmetric MDI data give results quite similar to those based on GOLF, the asymmetric MDI data do indeed give quite different results.

The MDI asymmetric set has some modes with much lower errors than their adjacent modes. These modes thus get very large weights in the inversion process and, since there are very few low-degree modes anyway, they can indubitably influence the inversion result. This does indeed seem to be the case. Furthermore, the modes $l = 2$, $n = 6$ and $l = 2$, $n = 7$ are suspect because of the fact that they have extremely large residual ($>10\ \sigma$) in the RLS inversions. Removal of these modes reduces the difference in the results between the symmetric and asymmetric MDI sets (cf. Fig. 6). Therefore, we find no evidence even from the thus corrected MDI data that frequencies obtained with asymmetric profiles fitted to the peaks in the power spectrum cause significantly different results, compared with frequencies obtained with a Lorentzian fit.

It may be noted here that the $l = 1$ modes of the GOLF and MDI sets show a fairly large difference at high frequency. However, the errors on the modes are also very high, and the difference does not seem to cause substantial difference in the inversion results. There is, however, some remaining difference between the GOLF inversion results and the MDI inversion results. To check whether that is merely an artifact of having different numbers of modes in the different data sets, we have inverted a common set of modes from each data set. The set has $l = 0$ and $l = 1$ modes of $n = 13$ to $n = 25$, and $l = 2$ modes with $n = 13$ to $n = 23$. The higher degree modes are from the MDI set as before. The inferred sound speed differences in the inner parts of the Sun are shown in Figure 7. Note that the results are actually quite similar for both reference models.

4. CONCLUSIONS

We find very little evidence that the shift in frequencies between symmetric and asymmetric fits to solar oscillation power spectra changes our inferences concerning the sound speed in the solar core. The changes we see are well within

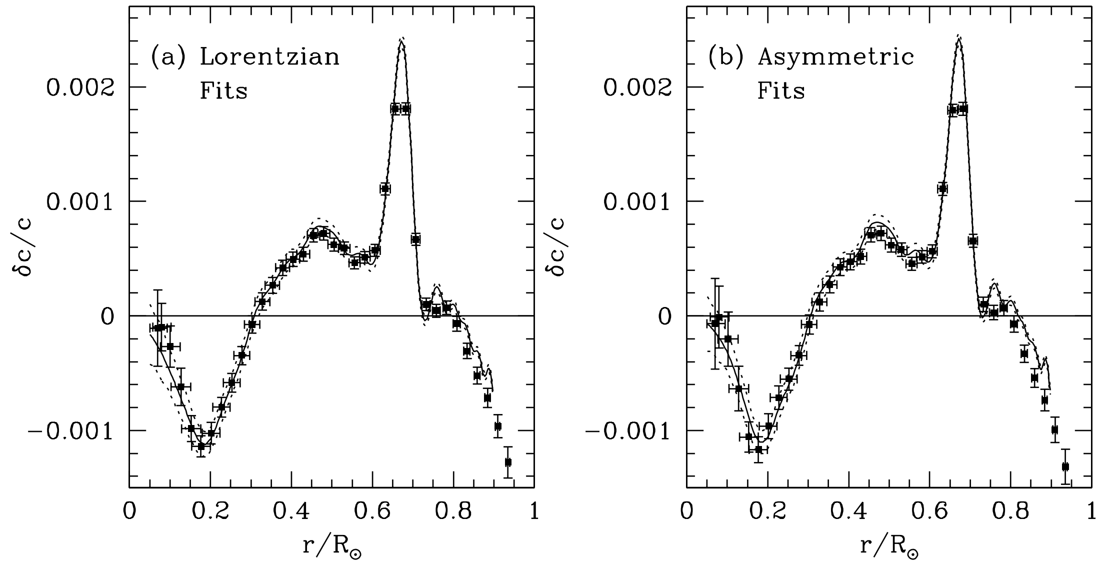


FIG. 4.—Relative sound speed differences, $\delta c/c$ between the Sun and model S using GOLF and MDI data. The differences are taken in the sense of (Sun − model)/Sun. (*a*) Results of inverting frequencies obtained by fitting power spectrum peaks with a Lorentzian profile. (*b*) Frequencies obtained by fitting an asymmetric profile to the peaks.

one standard deviation. Larger differences occur as a result of addition of modes to the set, or when using different inversion methods. It should be kept in mind that this conclusion is based on combining symmetrically or asymmetrically fitted low-degree data with higher degree frequencies obtained from symmetric fits. This introduces an inconsistency in the results based on the asymmetrical fits, which could be significant in view of the fact that the low-degree modes form a very small fraction of the total mode set. On the other hand, an inconsistency of this kind, by introducing a degree-dependent systematic error in the frequencies, would appear likely, if anything, to increase the effect on the inversions. We note also that Rabello-Soares et al. (1999b) found little effect of asymmetry in inversions of artificial data including asymmetry. Similarly, the analysis by Basu & Antia (2000), including both low- and intermediate-degree asymmetrically fitted modes, indicates that the results obtained in this work will not change; however, it should be kept in mind that Basu & Antia obtained the frequencies from *m*-averaged spectra, and errors in averaging may affect the results. Thus it is obvious that our study must be repeated when asymmetric fits to peaks of intermediate- and high-degree modes are also available. Of course, when this analysis is generalized, careful attention will need to be given to other sources of distortion, such as those resulting from the effect of the solar cycle on the outer layers, which will be considerable for data taken between 1998 and 2003.

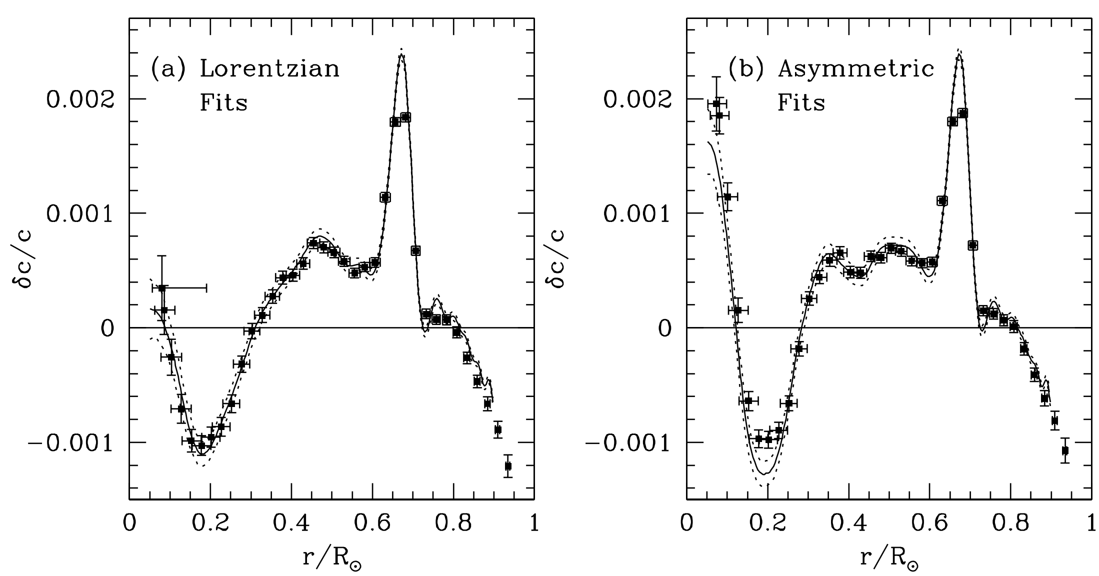
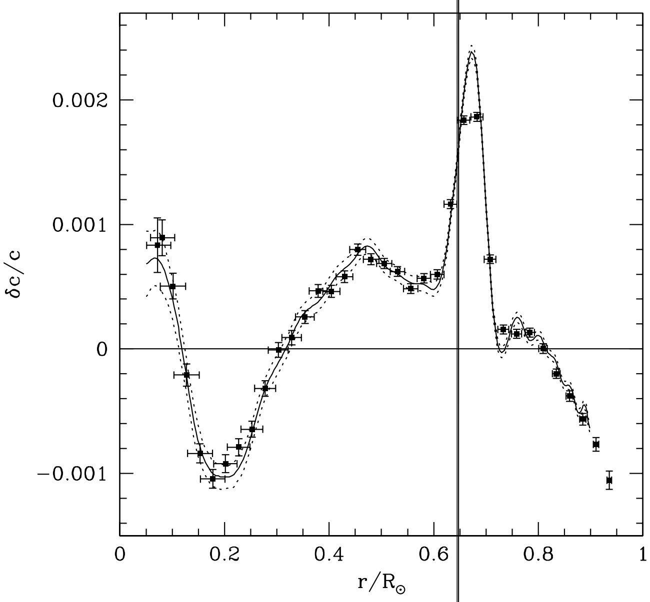
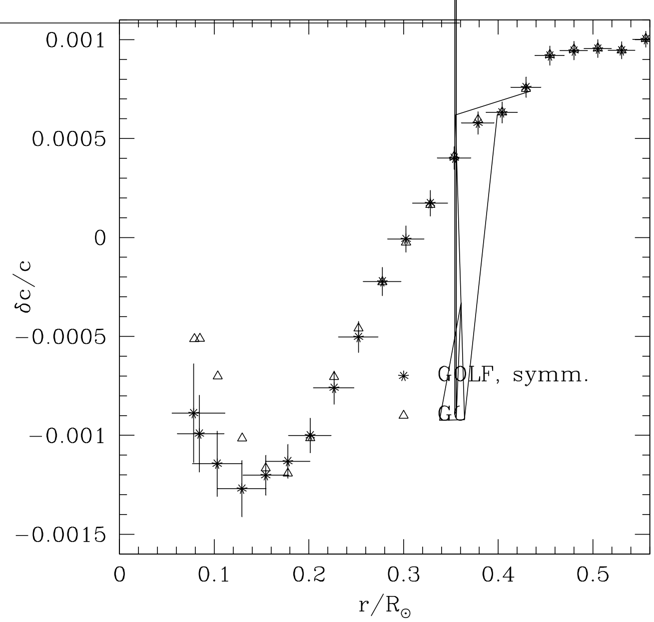


FIG. 5.—Relative sound speed differences, $\delta c/c$ between model S and the Sun, using MDI data only. (*a*) Result of using frequencies obtained by fitting symmetric profiles. (*b*) Results using the $l = 0$, 1, and 2 data from Toutain et al. (1998). Higher degree frequencies are from the MDI set.


0.002
0.001
0
−0.001
δc/c
0
0.2
0.4
0.6
0.8
1
r/R⊙


0.001
0.0005
0
−0.0005
−0.001
−0.0015
δc/c
0
0.1
0.2
0.3
0.4
0.5
r/R⊙
GOLF, symm.

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