Coriolis effects on the elliptical instability in cylindrical and spherical rotating containers

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The elliptical instability corresponds to the 3D destabilisation of 2D rotating flows with elliptical streamlines. It takes place in a large range of industrial and natural systems, where the ellipticity is generated either by vortices interactions or by tidal effects. It is for instance expected in the wake vortices behind aircrafts, in the intense vortical structures of the atmosphere and the ocean, in planetary liquid cores, in binary stars and accretion disks, and more generally in any turbulent flow exhibiting some coherent structures with elliptical motion. Since its discovery in the mid-1970s, the elliptical instability has thus received considerable attention, theoretically, experimentally and numerically (see for instance the review by Kerswell 2002).

Nevertheless, previous studies have mostly focused on the purely hydrodynamical aspects of the elliptical instability: their results must thus be used with cautions when applied to natural situations, where additional effects such as thermal (Le Bars & Le Dizès 2006) or magnetic fields (Lacaze et al. 2006) significantly perturb the idealised system. In the present work, we study analytically and experimentally the effects of a global rotation of the strain field responsible for the elliptical pattern, as induced for instance by the Earth's rotation on atmospheric vortices or by the relative motions of moon-planet systems and binary stars. Both cylindrical and spherical geometries are studied.

Theoretical approaches

We use two different but complementary analytical approaches to tackle this problem. In the global approach, the elliptical instability is associated with the triadic resonance between the elliptical deformation and two normal modes of the undistorted inviscid circular flow (Waleffe 1990). This method then permits to calculate explicitly the conditions of resonance for a given geometry and provides information on the structure of the eigenmodes. Taking into account the Coriolis force, it demonstrates that (i) the elliptical instability cannot develop when Ω^G (i.e. the ratio between the global and the flow rotation rates) ranges between -3/2 and -1/2, and (ii) that resonances with various structures can be excited in a given container by changing Ω^G only. In addition to these conclusions, the local approach allows the analytical determination of the growth rate of the instability. It is based on the inviscid short-wavelength Lagrangian theory developed by Bayly (1986) and Craik & Criminale (1986). There, perturbations are assumed to be sufficiently localised in order to be advected along flow trajectories and are searched as local plane waves. Resolution of the Euler equations at first order in the elliptical deformation then gives the exponential growth rate of the instability. In both methods, volumic and surfacic viscous effects can be added afterwards as first order corrections to the Euler solutions.

Experimental study

In our experimental set-up, a deformable and transparent container - either a cylinder of radius R=2.75cm and H=21.4cm or a hollow sphere of radius R=2.175cm - is set in rotation versus its axis (Oz) with an angular velocity Ω^{F*} up to 300rpm and is simultaneously deformed elliptically by two fixed rollers parallel to (Oz). The container is filled with water seeded by anisotropic particles (Kalliroscope) and a light sheet is formed in a plane containing the rotation axis: the elongated and flat shape of these reflective flakes forces them to align in the strain field and allows to visualise the velocity field. In particular, the rotation axis and its undulations are clearly visible, and wavelengths and frequencies of excited modes can be measured. The whole set-up (with also the camera and light projector) is placed on a 0.5m-diameter rotating table, which allows rotation with angular velocity Ω^{G*} up to +/-60rpm.

Series of experiments were first performed in the cylinder and in the sphere to observe the various possible resonances by changing Ω^{F*} and Ω^{G*}. Good agreement is found with the linear inviscid global approach: stationary mode with a sinusoidal rotation axis and various wavelengths as well as other more exotic modes recognised by their complex radial structure and/or by their periodic behaviour can be selected by changing the dimensionless ratio Ω^G only, providing the Reynolds number is large enough (see figure 1a).

The stationary mode is especially interesting since its growth rate can be determined experimentally by measuring the maximum amplitude of the sinusoidally deformed rotation axis. The variations of the growth rate with respect to Ω^G are presented in figure 1b, together with the theoretical estimate. A good agreement is found: when decreasing progressively the global rotation rate, we observe that various bands of resonance coexist for $\Omega^G>\Omega^G_c\sim-1/2$, first separated by large regions of stability (especially for cyclones), then progressively overlapping (especially for anticyclones). Simultaneously, the instability wavenumber and its growth rate significantly increase and reach a maximum just before Ω^G_c, where all resonances sharply disappear. Two limitations are to be noted when comparing experiments and theory. First, one can see in figure 1b that analytical values always overestimate experimental values. This is mainly due tot the fact that non-linear effects were not taken into account in the theory, but are expected to be stabilising (Eloy et al. 2003). Besides, it is also worth recalling that the theoretical estimate is based on a short-wavelength asymptotic analysis: the discrepancy could therefore be associated with finite size effects. Secondly, because of the limited rotation rate of the rotating table (up to 1Hz only), it was not possible to explore experimentally the range below Ω^G=-1: we can only expect in the light of the good agreement between theoretical predictions and experiments for Ω^G>-1 that this will also be the case for Ω^G<-1, and in particular, that instabilities will reappear for Ω^G<-3/2.

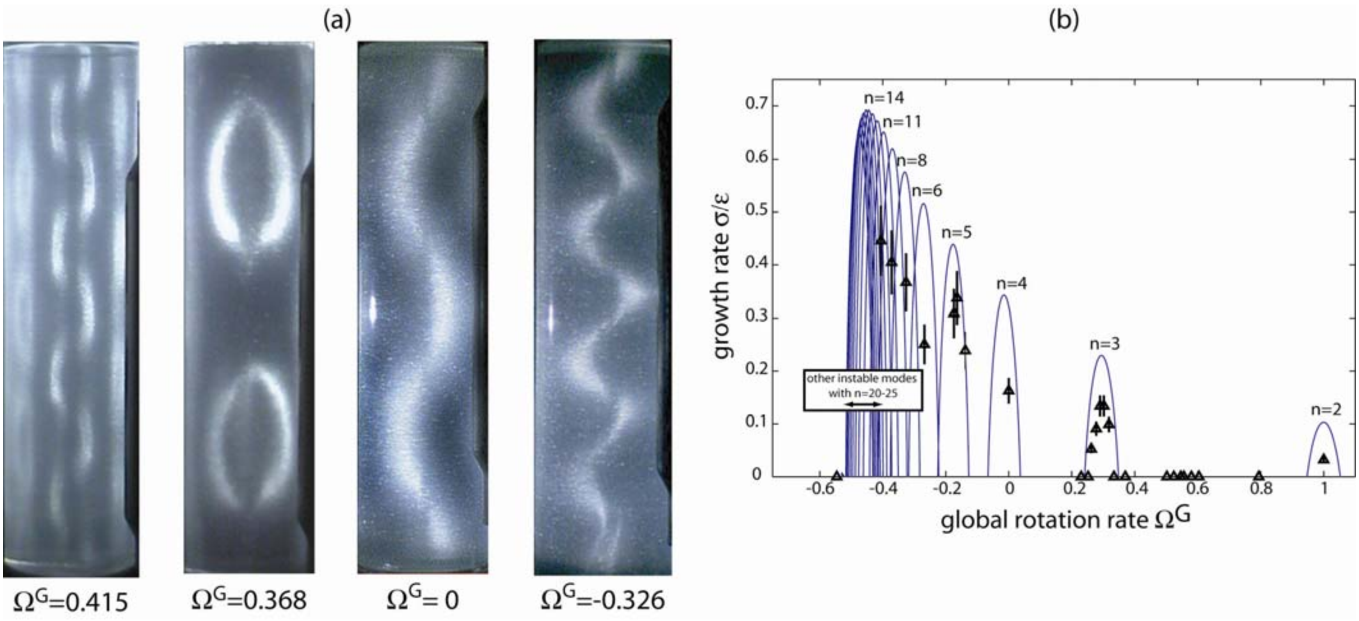

Figure 1- (a) Pictures of some of the observed modes of elliptical instability in the cylinder depending on the global rotation Ω^G*. (b) Growth rate of the stationary mode of the elliptical instability in the cylinder determined by the local analysis as a function of the global rotation rate* Ω^G*: triangles stand for experimental measurements and solid lines for theoretical predictions. The predicted number n of axial half-wavelengths increases by 1 from the right to the left on each resonant band, starting from n=2; measured values are indicated above each experimental point. Note that additional resonances were observed for* Ω^G *in the range [-0.507;-0.403]; nevertheless, because of their small wavelength and their rapid growth rate, quantitative measurements were not accurate.*

Conclusion

Our systematic conclusions agree qualitatively with the general trend observed by Afanasyev (2002) in vortex pairs on a rotating table and by Stegner et al. (2005) in Karman vortex streets on a rotating table, even if our experimental set-up is totally different (i.e. our vortices are confined axially and radially, and our set-up permits to analyse the growth and the saturation of the elliptical instability, without being perturbed by competing centrifugal instabilities). Indeed, all studies report the systematic destruction of elliptical anticyclones by a sinusoidal mode with a decreasing wavelength when Ω^G decreases up to a certain critical value. We thus argue that this behaviour is universal, i.e. independent of the considered vortical structure. We also argue that conclusions in the spherical geometry are especially interesting in the geophysical and astrophysical contexts. For instance, complex motions can be expected in the Earth's core in addition to the simple spin-over excited by both precession and elliptical instability. More generally, one can imagine that binary stars and moon-planet systems where the elliptical instability is expected to take place, encounter various bands of resonance during their evolution: depending on the relative changes in their rotation and revolution rates, different and complex histories regarding energy dissipation and flow motions can thus be expected.

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