SUPERFLUID TURBULENCE

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Acknowledgments:

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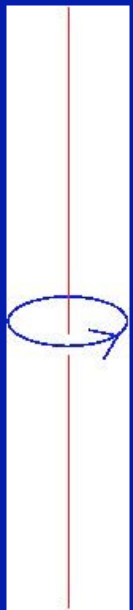
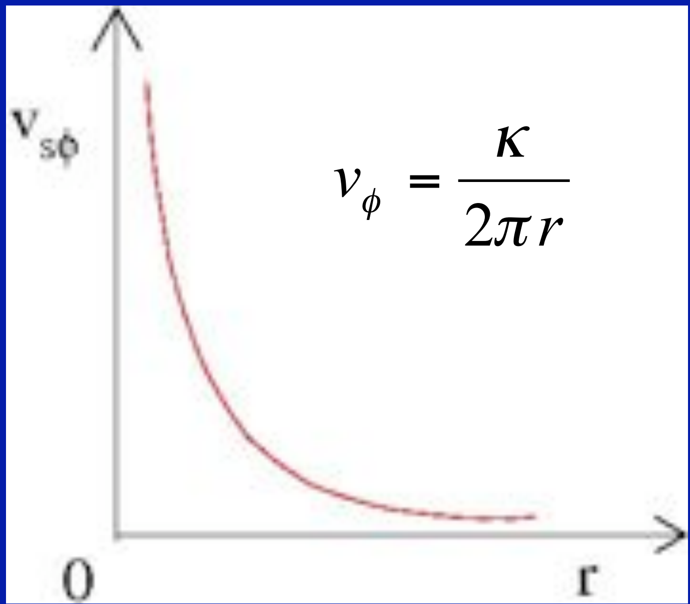
Joe Vinen (Birmingham),
Makoto Tsubota (Osaka)

Vortex line in an inviscid Euler fluid



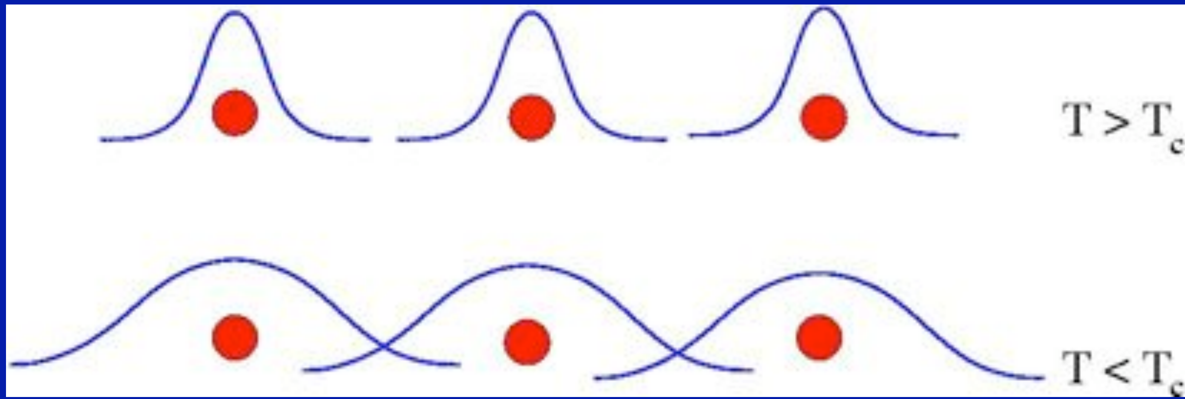


Too viscous
Core too thick

Bose-Einstein condensation

_<<d

_≈d



d = interatomic distance, _ =h/mv de Broglie wavelength

T=90 K   oxygen becomes liquid
T=77 K   nitrogen becomes liquid
T=20 K   hydrogen becomes liquid
T= 4 K   helium becomes liquid
T= 2.17 ^{4}He helium superfluid
T≈O(mK) ^{3}He helium superfluid
T≈O(_K)   atomic BEC

NLSE model: $$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V_0|\psi|^2\psi - E\psi$$

Let $\psi = Ae^{iS}$ and get:

1. continuity eq. $$\frac{\partial\rho_s}{\partial t} + \nabla\cdot(\rho_s\vec{v}_s) = 0$$

2. (quasi) Euler eq. $$\rho_s\left(\frac{\partial v_{sj}}{\partial t} + v_{sk}\frac{\partial v_{sj}}{\partial x_k}\right) = -\frac{\partial p}{\partial x_j} + \frac{\partial\Sigma_{jk}}{\partial x_k}$$

for density $\rho_s = mA^2$ and velocity $\vec{v}_s = \frac{\hbar}{m}\nabla S$

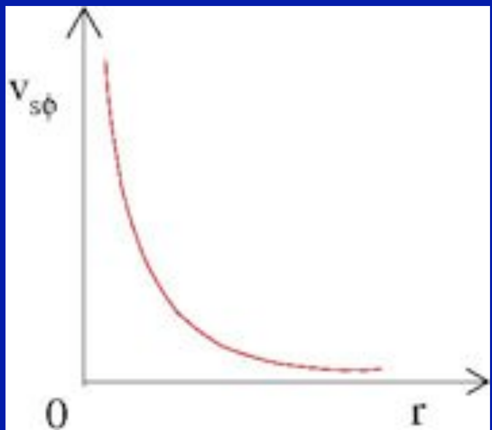
where $$p = \frac{V_0\rho_s^2}{2m^2}, \quad \Sigma_{jk} = \frac{\hbar^2}{4m^2}\rho_s\frac{\partial^2\ln\rho_s}{\partial x_j\partial x_k}$$

Vortex solution of the NLSE

Let S=_, then $\vec{v}_s = \frac{\hbar}{m}\nabla S = (0, \frac{\kappa}{2\pi r}, 0)$ and

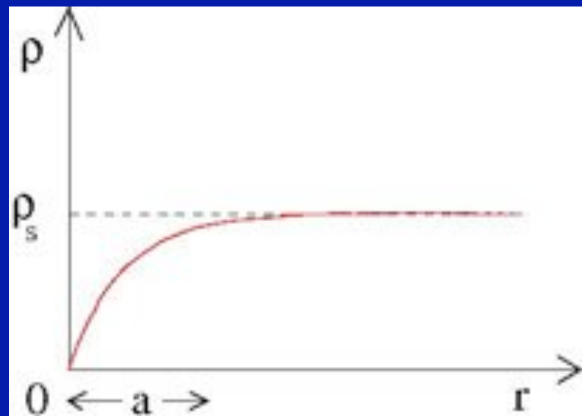
$$\oint_C \vec{v}_s \cdot d\vec{l} = \frac{h}{m} = \kappa = 9.97\times 10^{-4}\, cm^2 / s$$

quantisation of the circulation



velocity

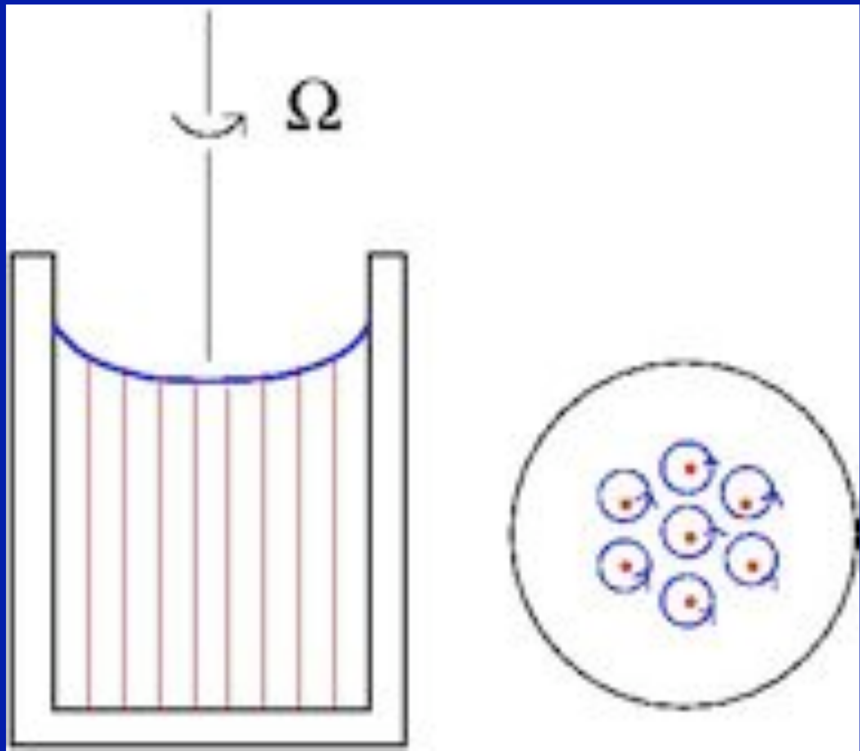
density



Hollow core of radius $a=10^{-8}$ cm << any other scale (eg typical vortex separation $\approx 10^{-4}$ or 10^{-3} cm).

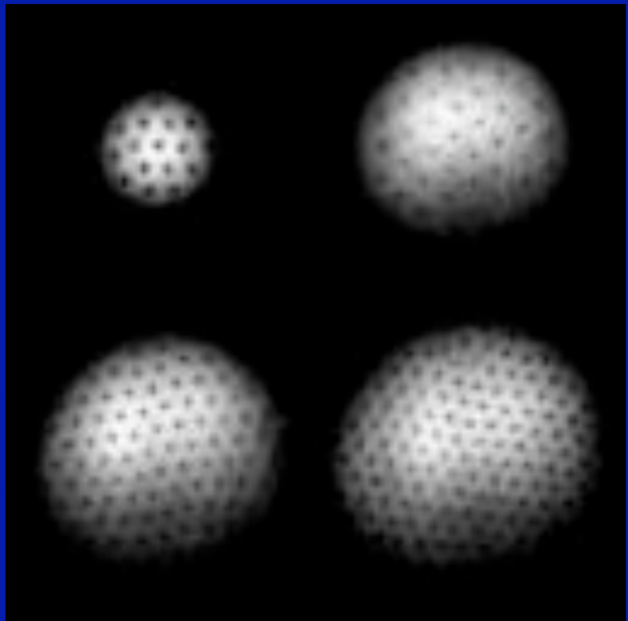
Length / core of vortex line as large is 10^9

Vortex lines in rotating helium

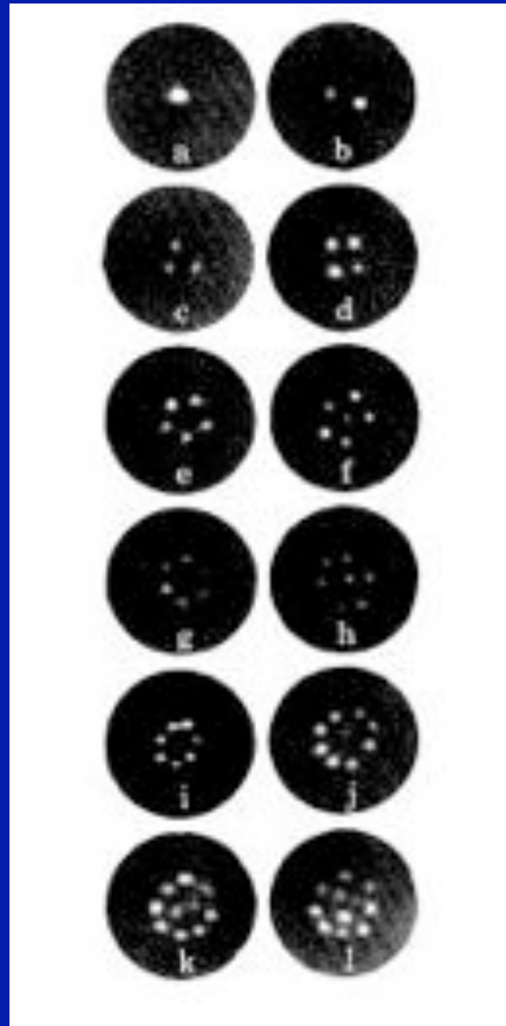


Number of vortex lines
per unit area n=2_/_

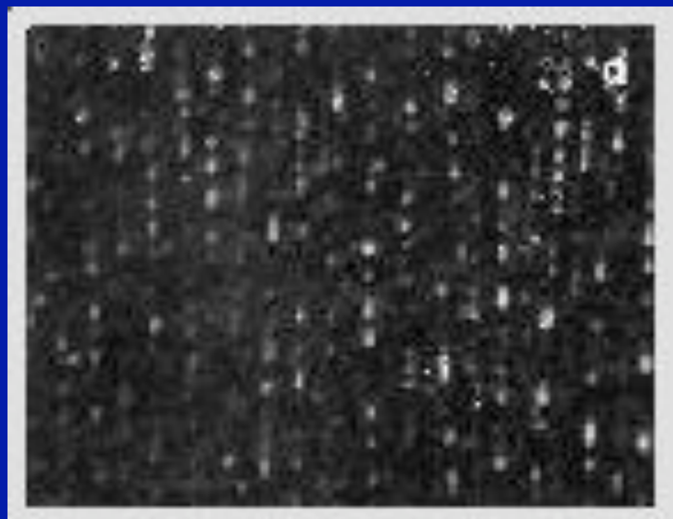
DIRECT VISUALIZATION OF VORTEX LINES



Atomic BEC: laser visualization
by Ketterle et al, MIT

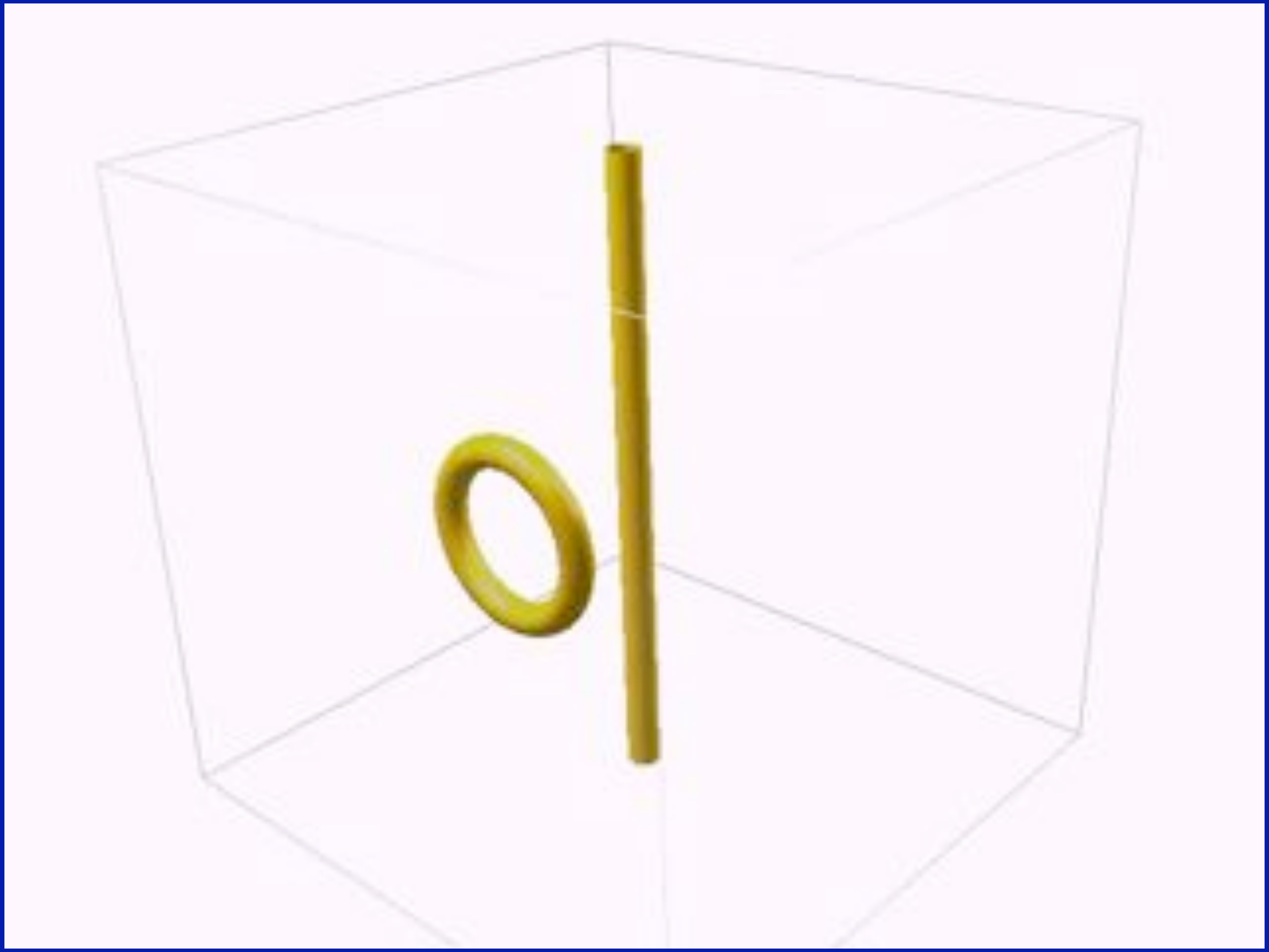


Helium: electron visualization by Packard et al, Berkeley



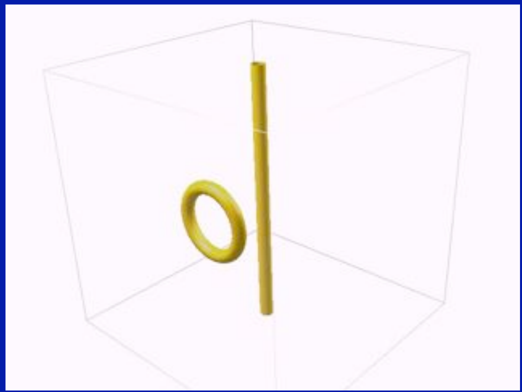
Helium: PIV by Sreenivasan, Lathrop et al, Maryland

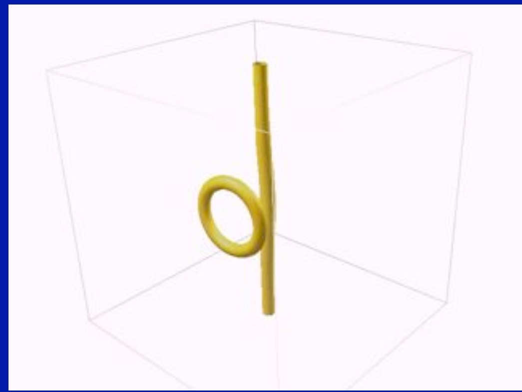
Vortex reconnection

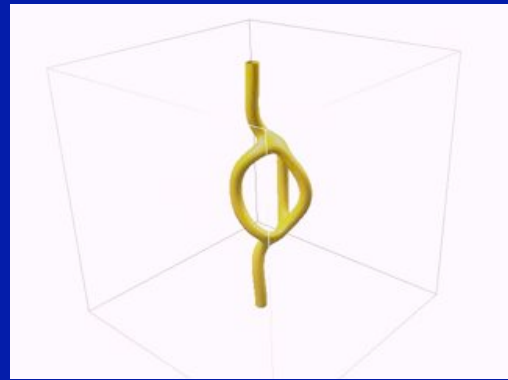


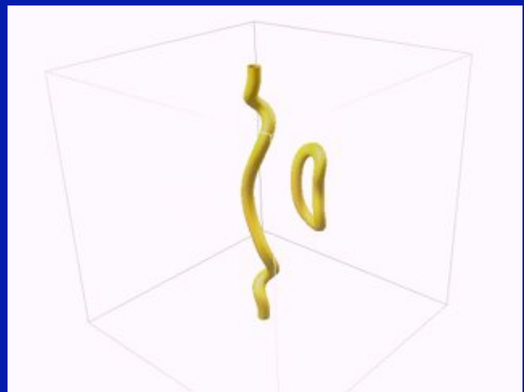
A.Youd

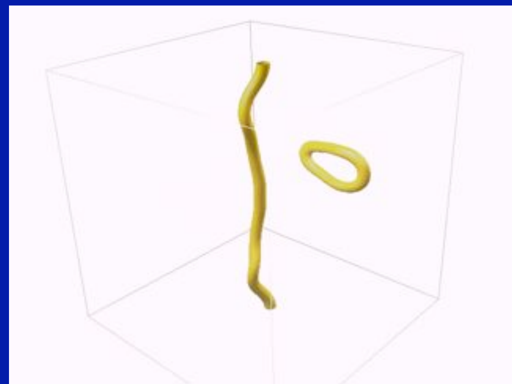
Vortex reconnection


(a)

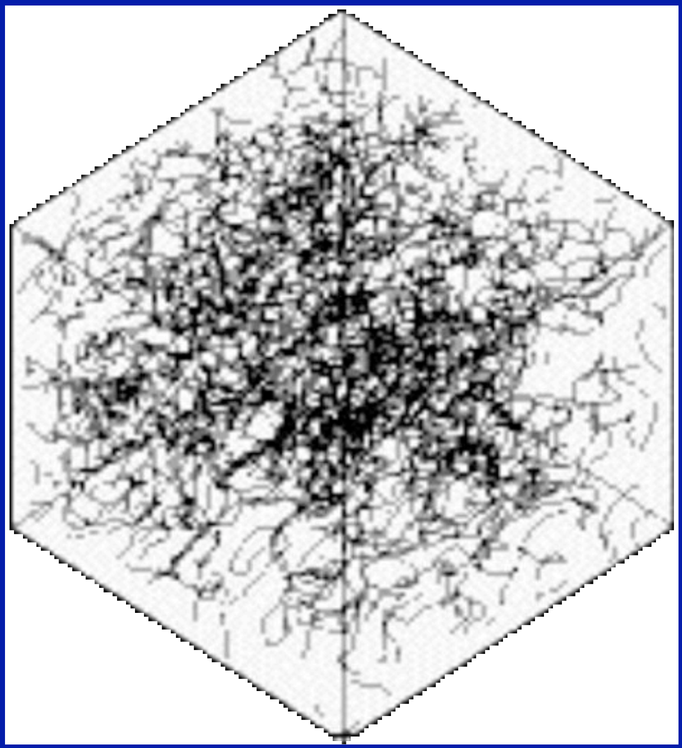

(b)


(c)


(d)

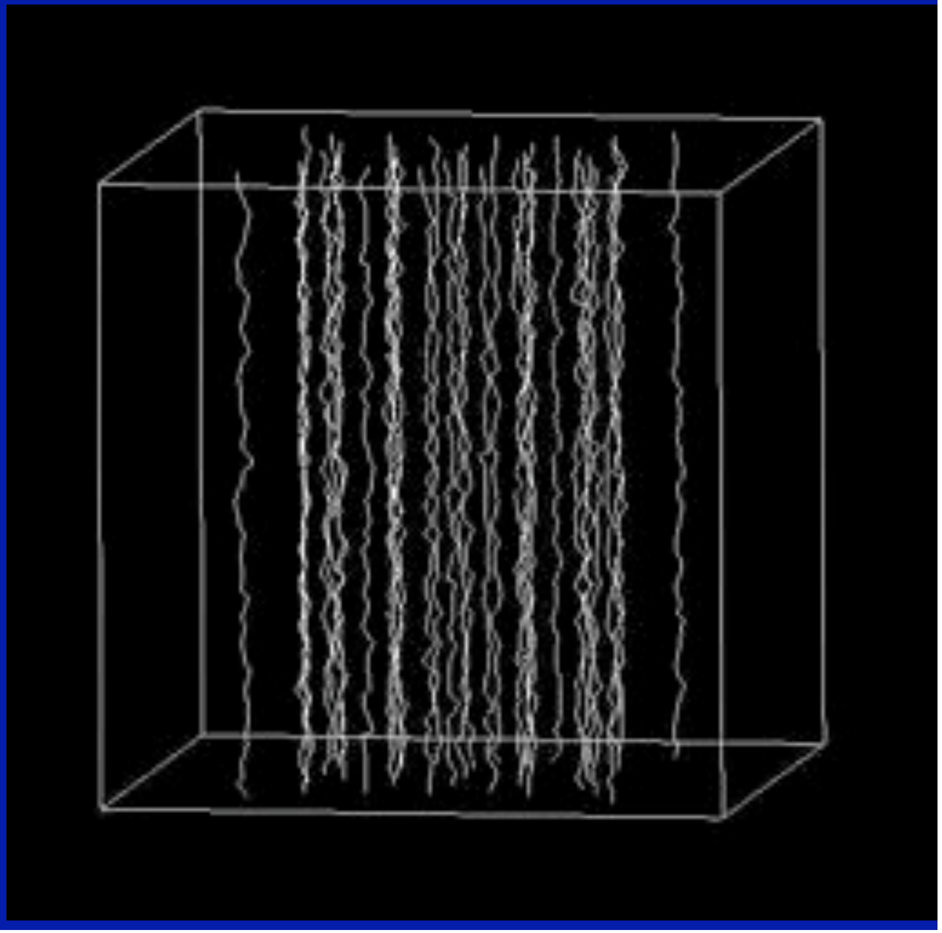

(e)

A.Youd



Superfluid turbulence

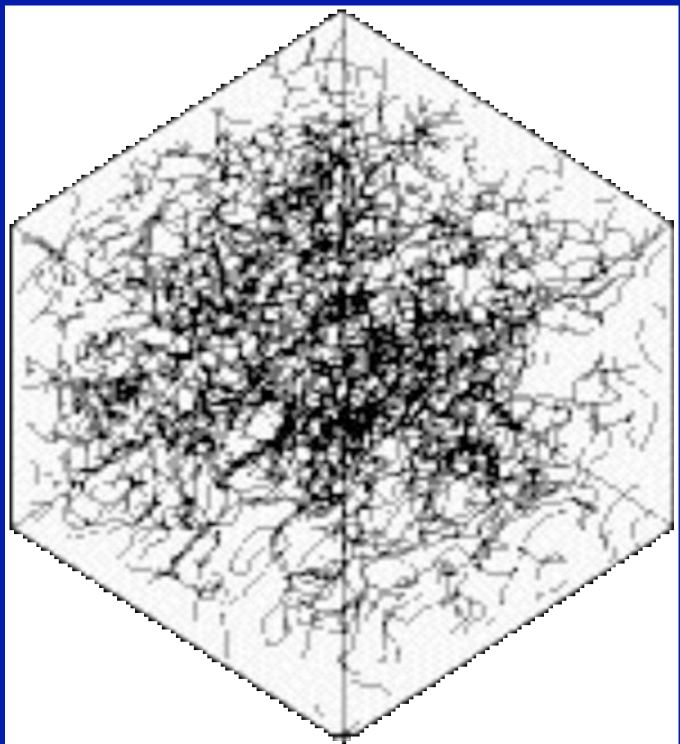
- No viscosity
- Vortex lines have fixed circulation and infinitesimal core radius



Rotation + axial flow = turbulence
Note the growth of Kelvin waves and vortex reconnections

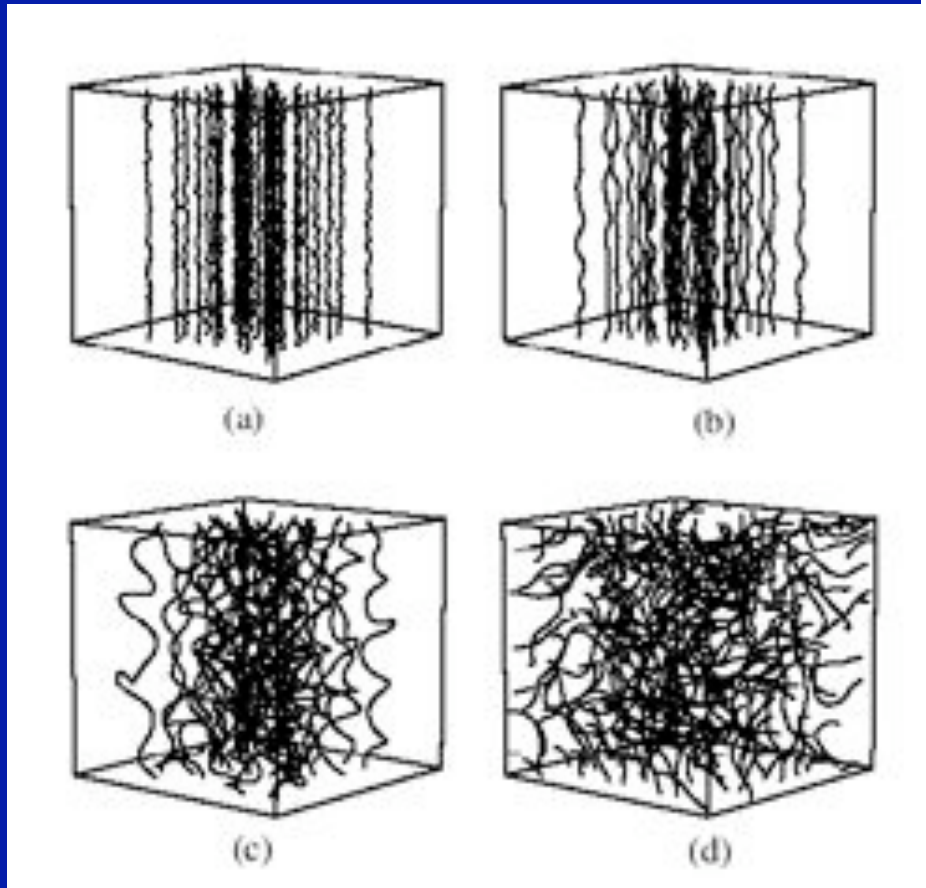
Tsubota, Araki & Barenghi,
Phys. Rev. Lett. 90, 20530 (2003)

Superfluid turbulence



- No viscosity
- Vortex lines have fixed circulation and infinitesimal core radius

Rotation + axial flow = turbulence
Note unstable Kelvin waves and vortex reconnections

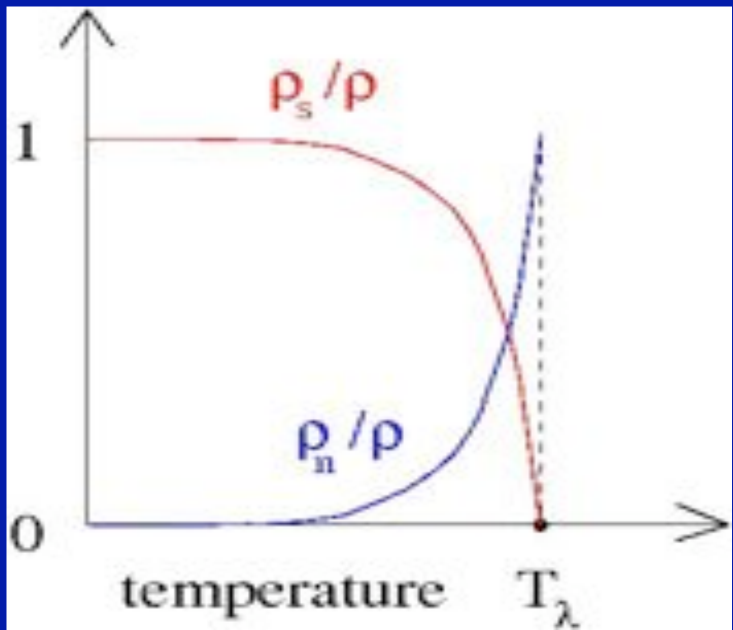


Tsubota, Araki & Barenghi,
Phys. Rev. Lett. 90, 20530 (2003)

Landau's two-fluid model

Superfluid: quantum ground state, density _$_s$, velocity v_s
no viscosity, no entropy,
almost like a classical inviscid Euler fluid

Normal fluid: thermal excitations, density _$_n$, velocity v_n
carries viscosity and entropy,
like a classical viscous Navier Stokes fluid



Total density _=_$_s$+_$_n$

Current issues (in ^{4}He):

High T:

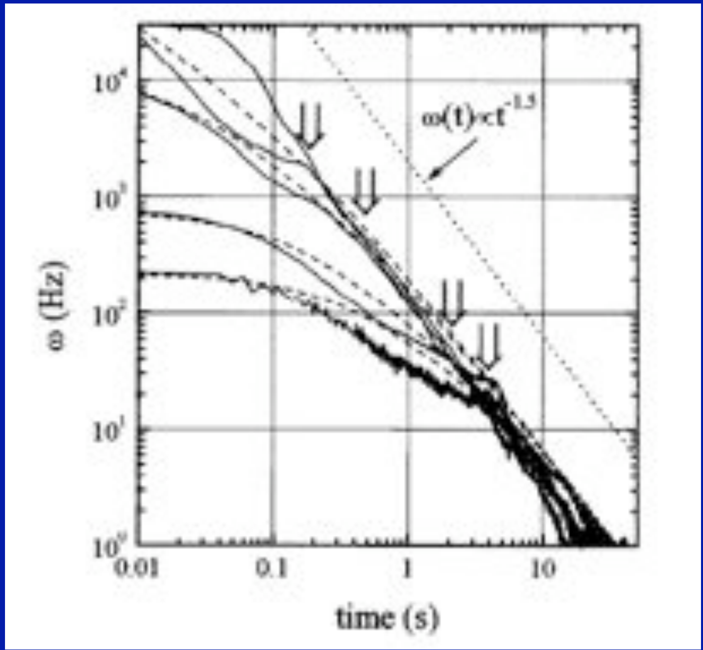
-similarities between quantum and classical turbulence:

- the same Kolmogorov $k^{-5/3}$ energy spectrum
- the same $t^{-3/2}$ decay rate
- the same pressure drops in channels
- the same drag crisis of a sphere

- double turbulence (both normal fluid and superfluid)

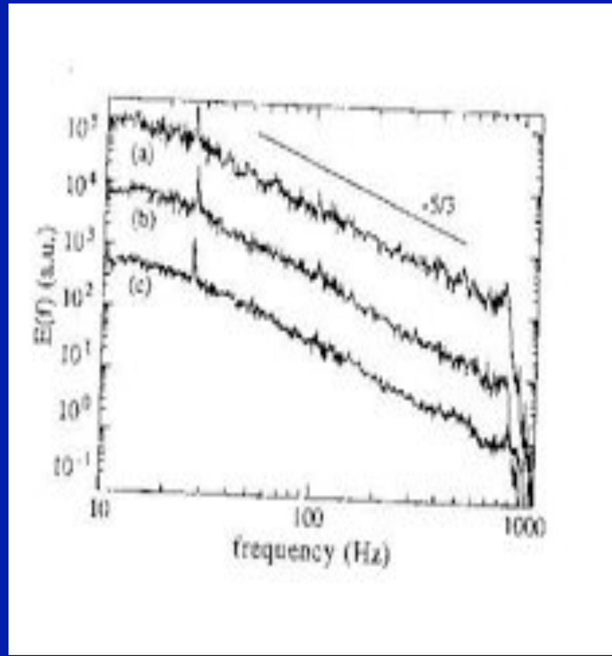
Low T:

- why does turbulence decay without viscosity ?
- nature of energy sink (acoustic rather than viscous)
- Kelvin waves turbulent cascade
- Kolmogorov $k^{-5/3}$ spectrum
- diffusion of vorticity without viscosity



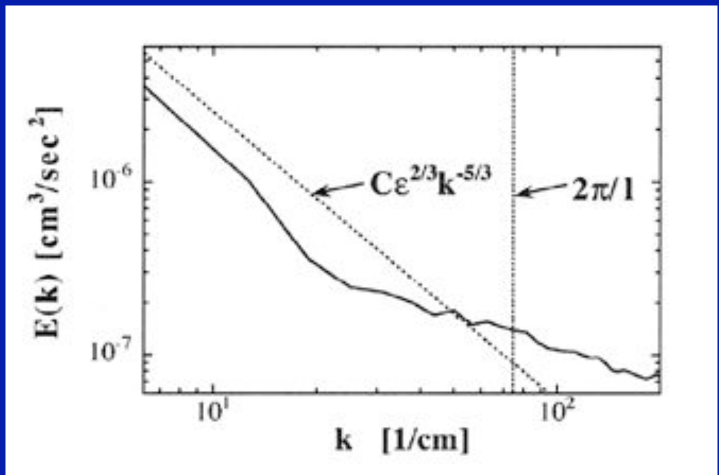
$t^{-3/2}$ decay

$k^{-5/3}$ spectrum
(a) T=2.18 K
(b) T=2.03 K
(c) T=1.4 K

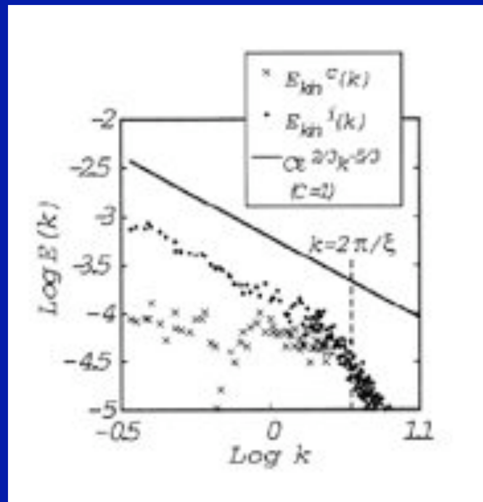


Stalp, Skrbek & Donnelly, PRL 82, 4831, 1999

Maurer & Tabeling, Europhys. Lett. 43, 29, 1998

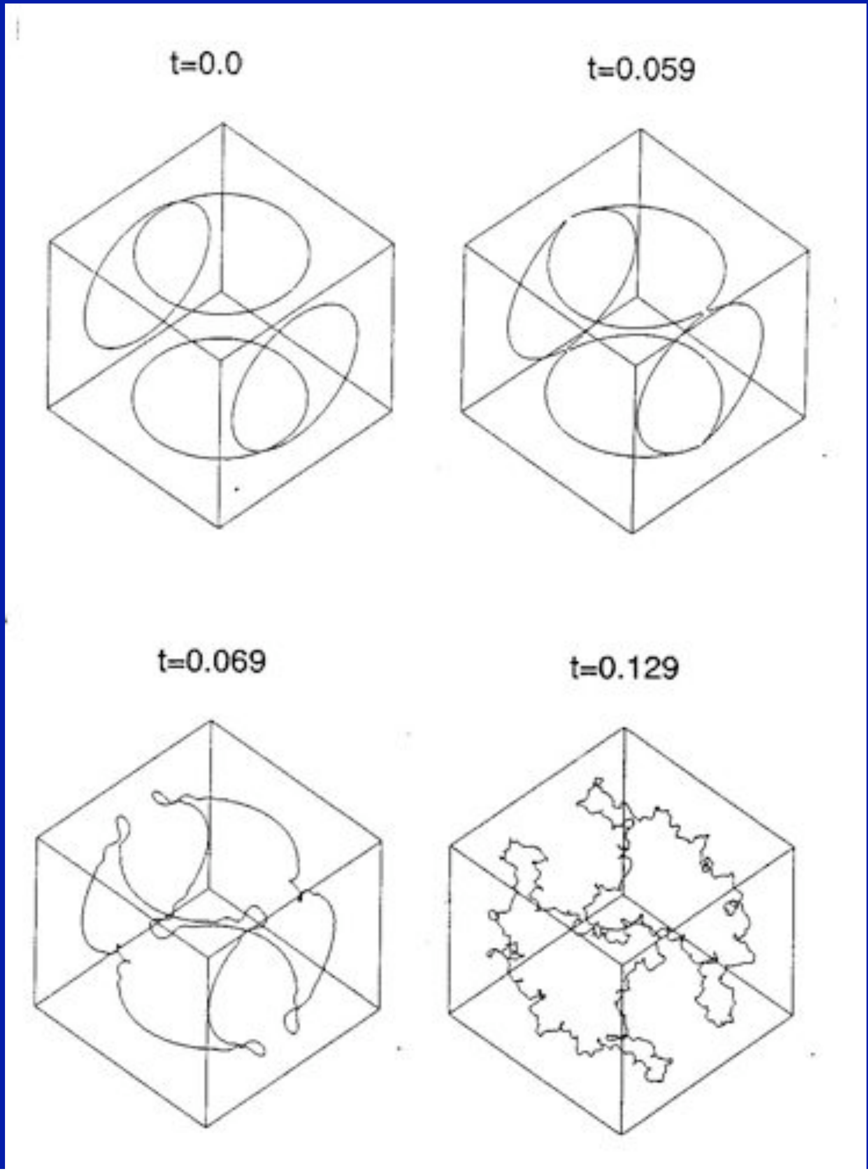


Araki et al, PRL 89, 145301, 2002



Kobayashi & Tsubota, PRL 94, 065302, 2005

Kelvin waves cascade



reconnections

↓

cusps

↓

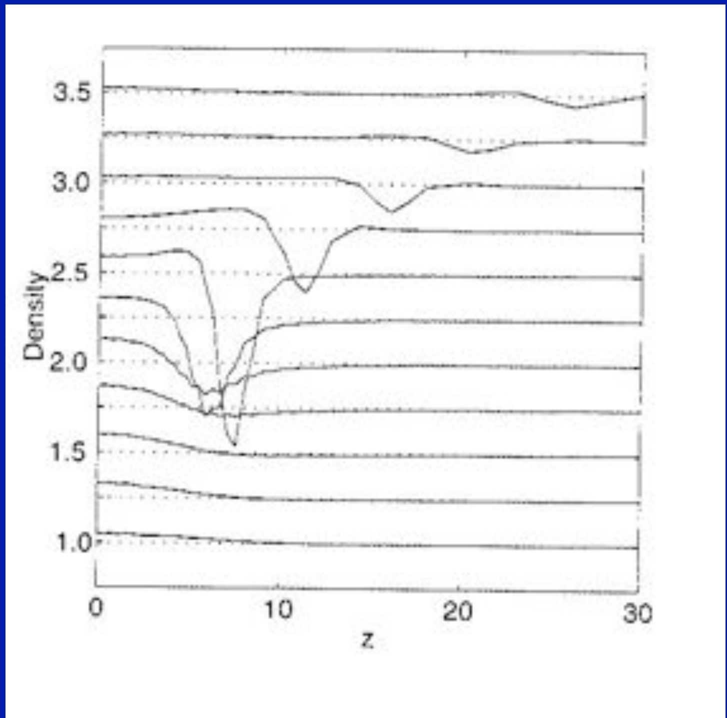
high k

↓

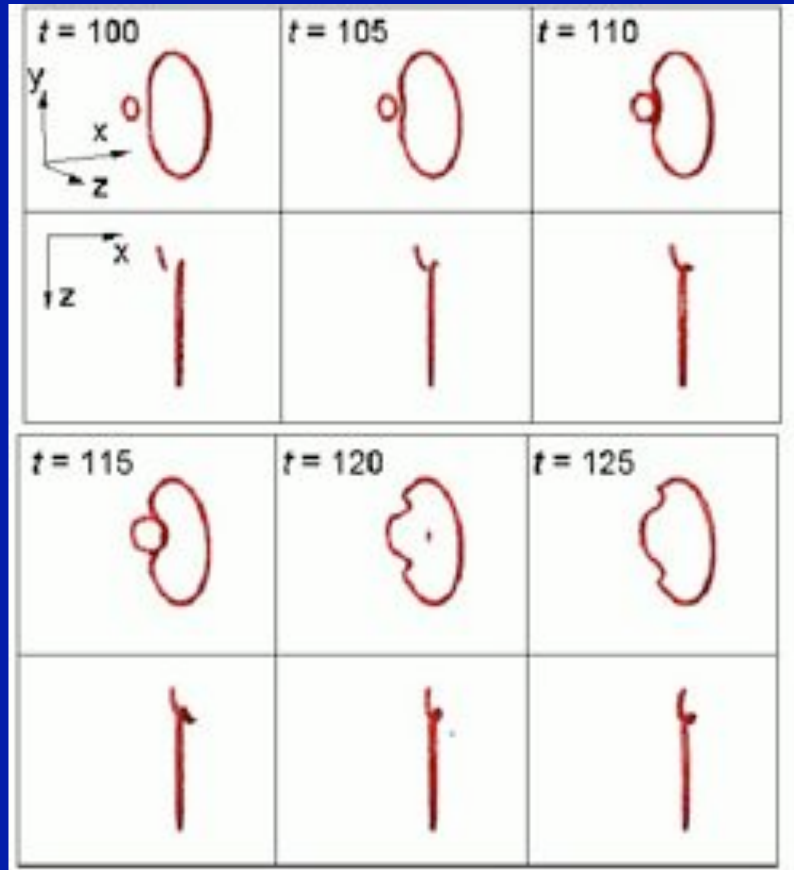
sound

Kivotides, Vassilicos, Samuels & Barenghi,
Phys. Rev. Lett. 86, 3080 (2001)
Vinen, Tsubota & Mitani,
Phys. Rev. Lett. 91, 135301 (2003)
Kozik & Svistunov,
Phys. Rev. Lett. 92, 035301 (2004)

Sound pulse at vortex reconnection



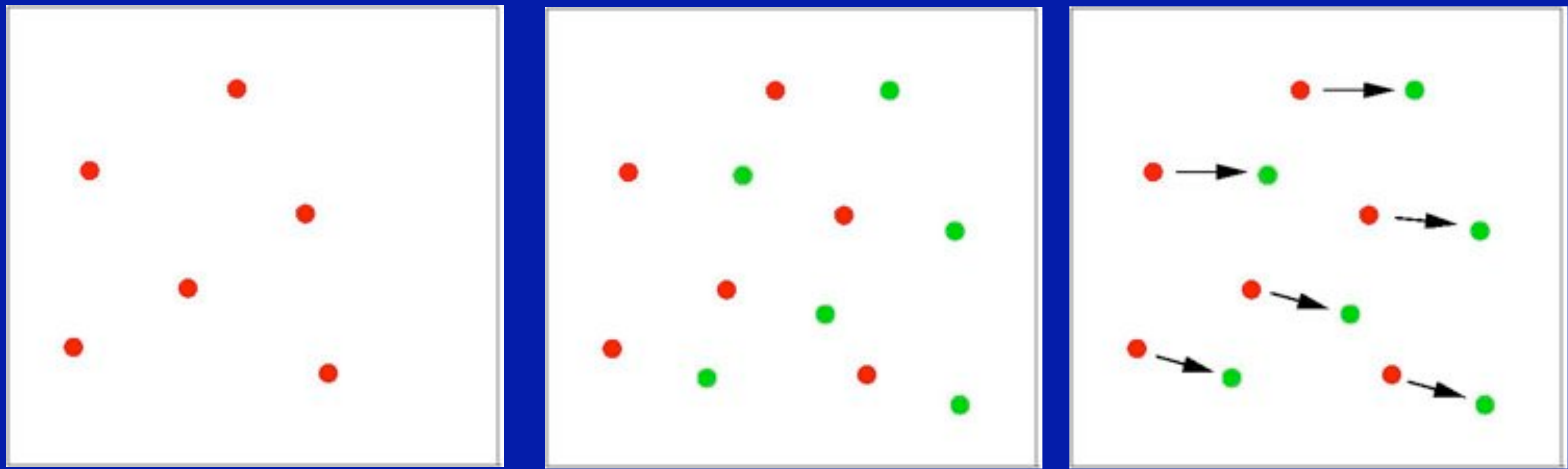
Leadbeater, Winiecki, Samuels,
Barenghi & Adams,
Phys. Rev. Lett. 86, 1410 (2001)



Leadbeater, Samuels, Barenghi &Adams,
Phys. Rev. A 67, 015601 (2003)

PIV (particle image velocimetry) in helium

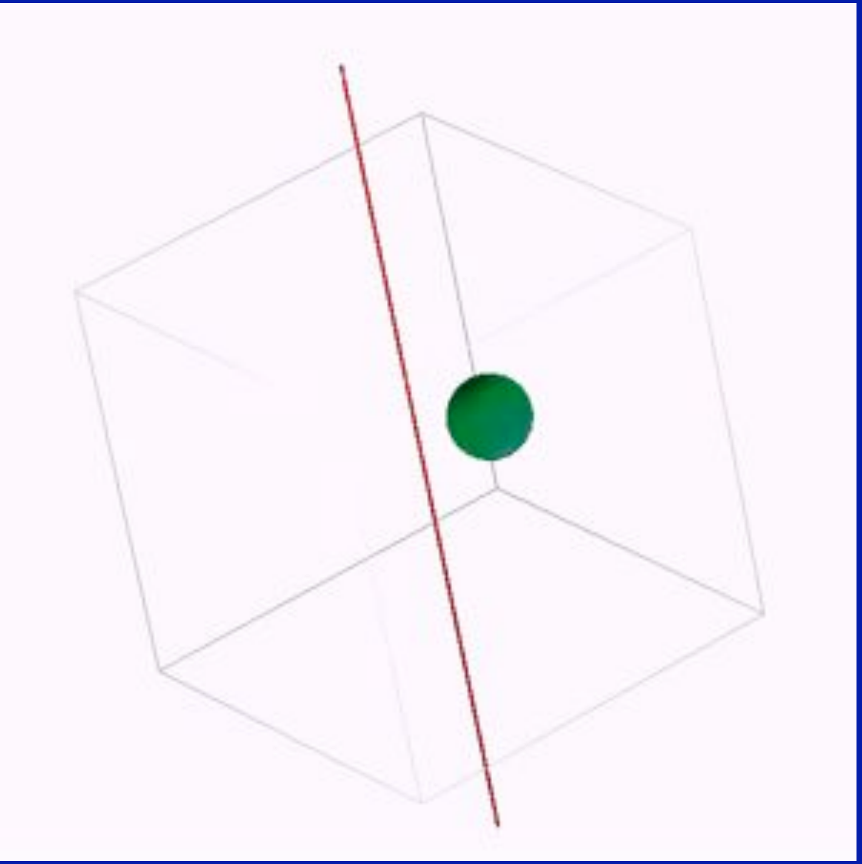
Donnelly et al, JLTP 126, 327, 2002
Zhang & Van Sciver, Nature Physics 1, 36, 2005
Bewley, Lathrop & Sreenivasan, Nature 44, 588, 2006



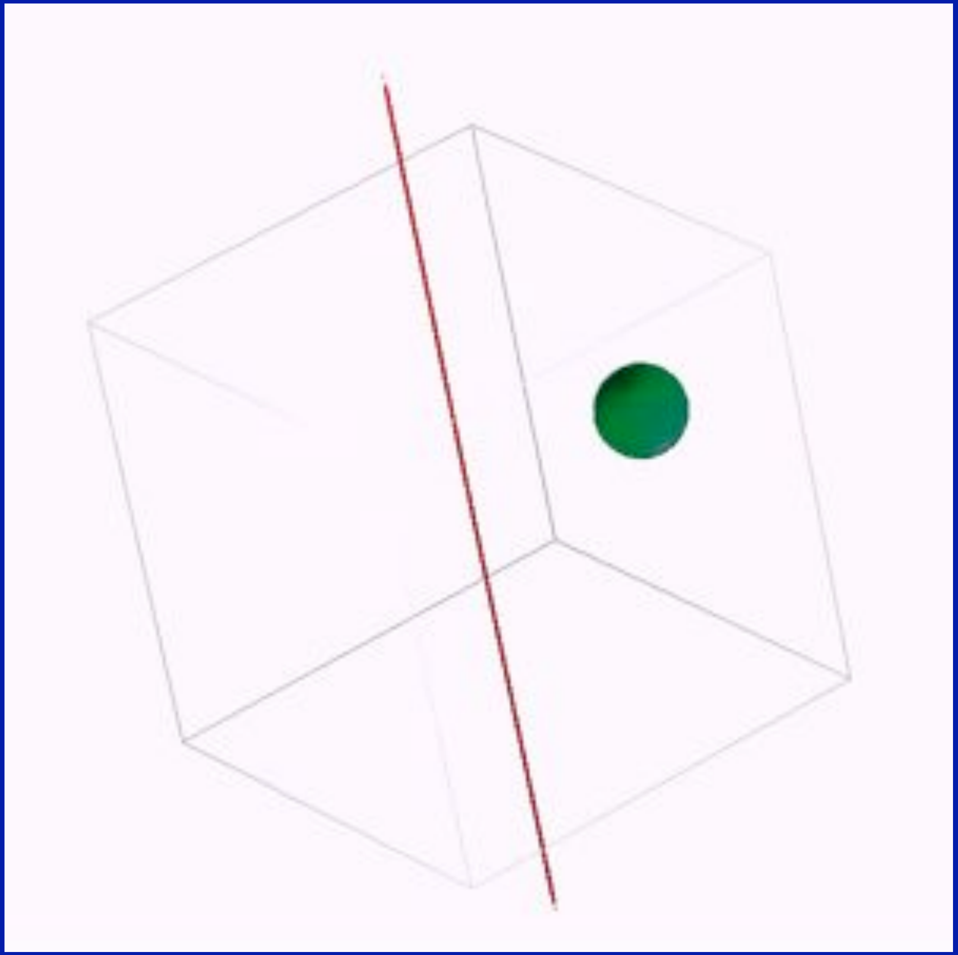
Interaction of vortex and tracer particle

Poole, Barenghi, Sergeev & Vinen, Phys Rev B 71, 0645141, 2005
Sergeev, Barenghi & Kivotides, Phys Rev B 74, 184506, 2006

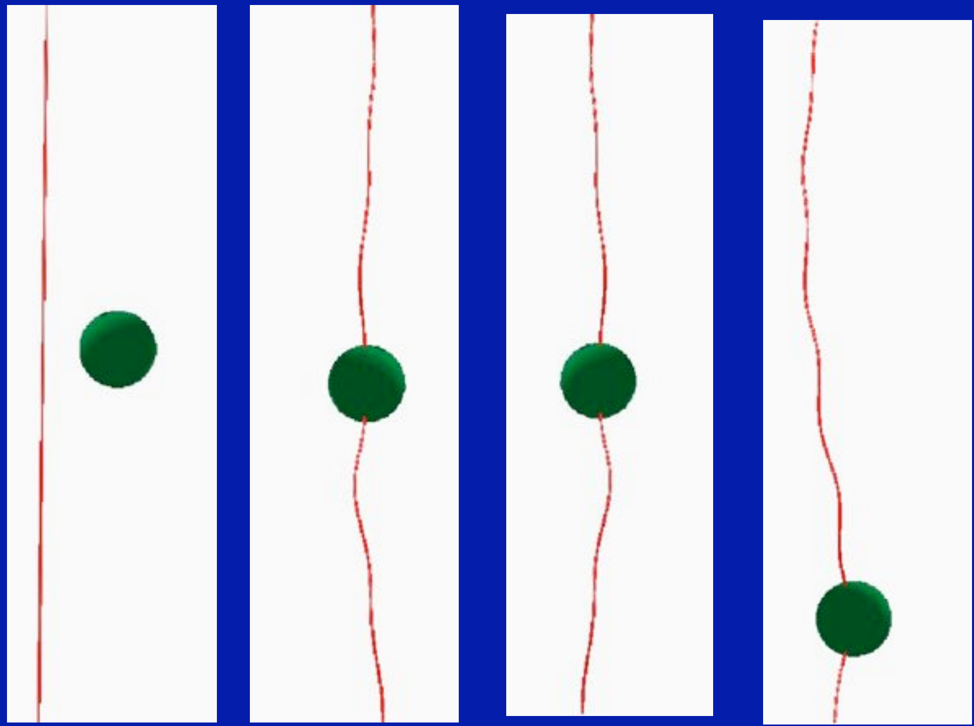
Sphere is trapped by vortex



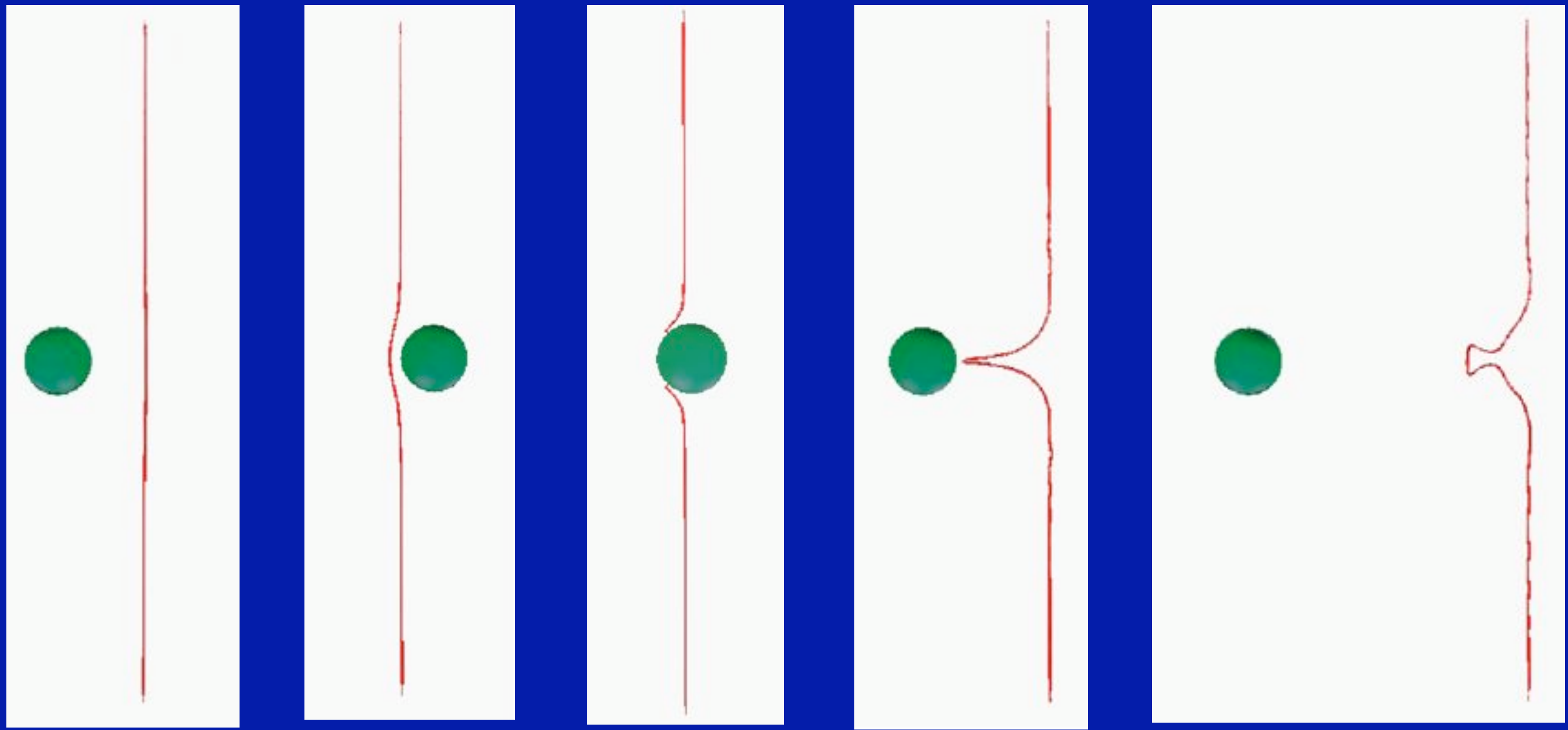
Sphere escapes vortex



Sphere is trapped by vortex



Sphere escapes vortex



CONCLUSIONS

- Superfluid vortices are Nature's best example of Euler's vortex lines

- Besides ^4He, there is current interest in superfluid vortex lines and turbulence in ^3He, atomic BEC and neutron stars

- Quantised vorticity is at the intersection of:
    - -low temperature condensed matter physics
    - -atomic and laser physics
    - -astrophysics
    - -fluid mechanics

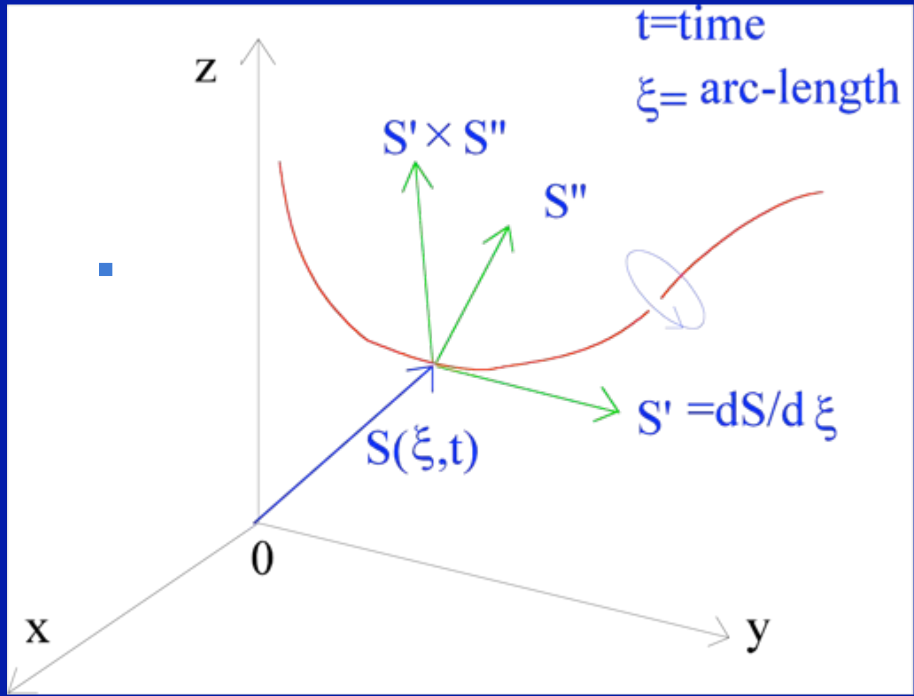
APPENDIX

Application of liquid helium as coolant

IRAS
720 litres
T=1.6 K

CERN: LHC
27 Km ring, T=1.8 K
700,000 litres

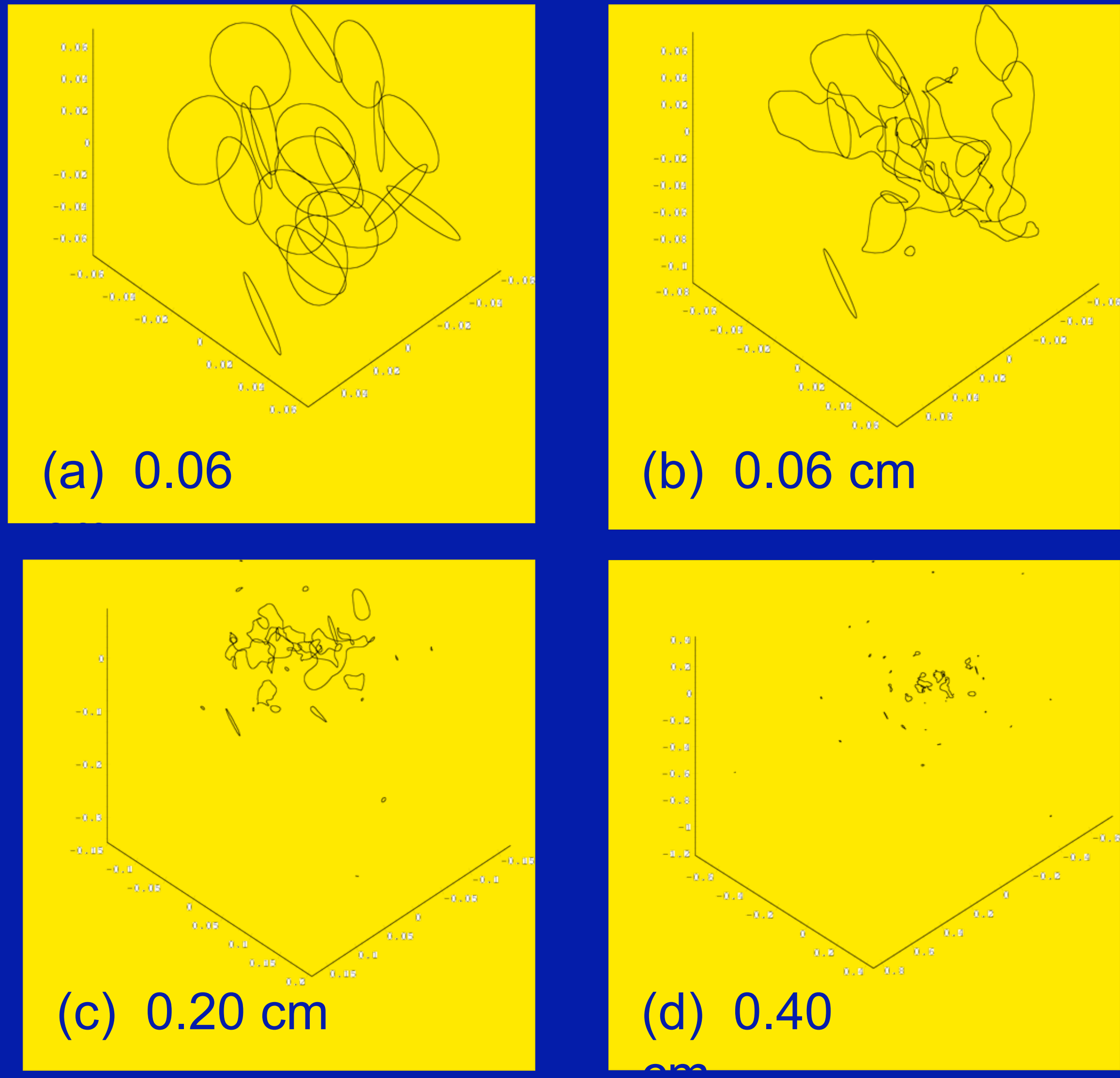
Small core _ vortex lines as space curves



$$\vec{V}_{self}(\vec{S}) = \frac{\kappa}{4\pi}\int d\xi \frac{\vec{S}'\times(\vec{S}-\vec{R}(\xi))}{|S-R(\xi)|^3} \approx \frac{\kappa}{4\pi}\ln(b/a)\,\vec{S}'\times\vec{S}''$$

“Evaporation” of a packet of vortex loops

The vortex cloud becomes larger with time (note the increasing box size) in agreement with experiments (Lancaster)



Barenghi & Samuels, Phys. Rev. Lett. 89, 155302 (2002)