Preferential concentration of inertial particles in turbulent flows

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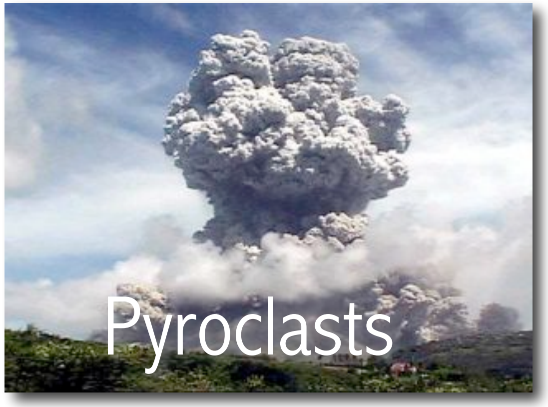
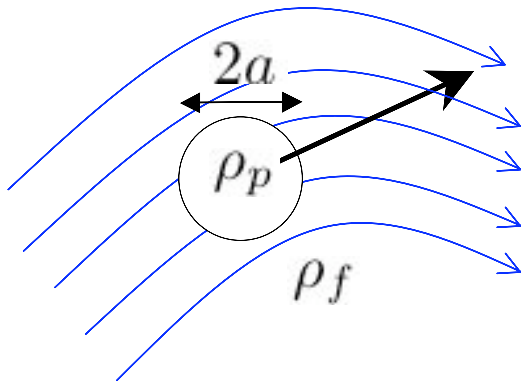
Particle laden flows



Finite-size and **mass** impurities advected by **turbulent** flow

Very heavy particles

- Spherical particles much smaller than the Kolmogorov scale η, much heavier than the fluid, feeling no gravity, evolving with moderate velocities: **one of the simplest model**



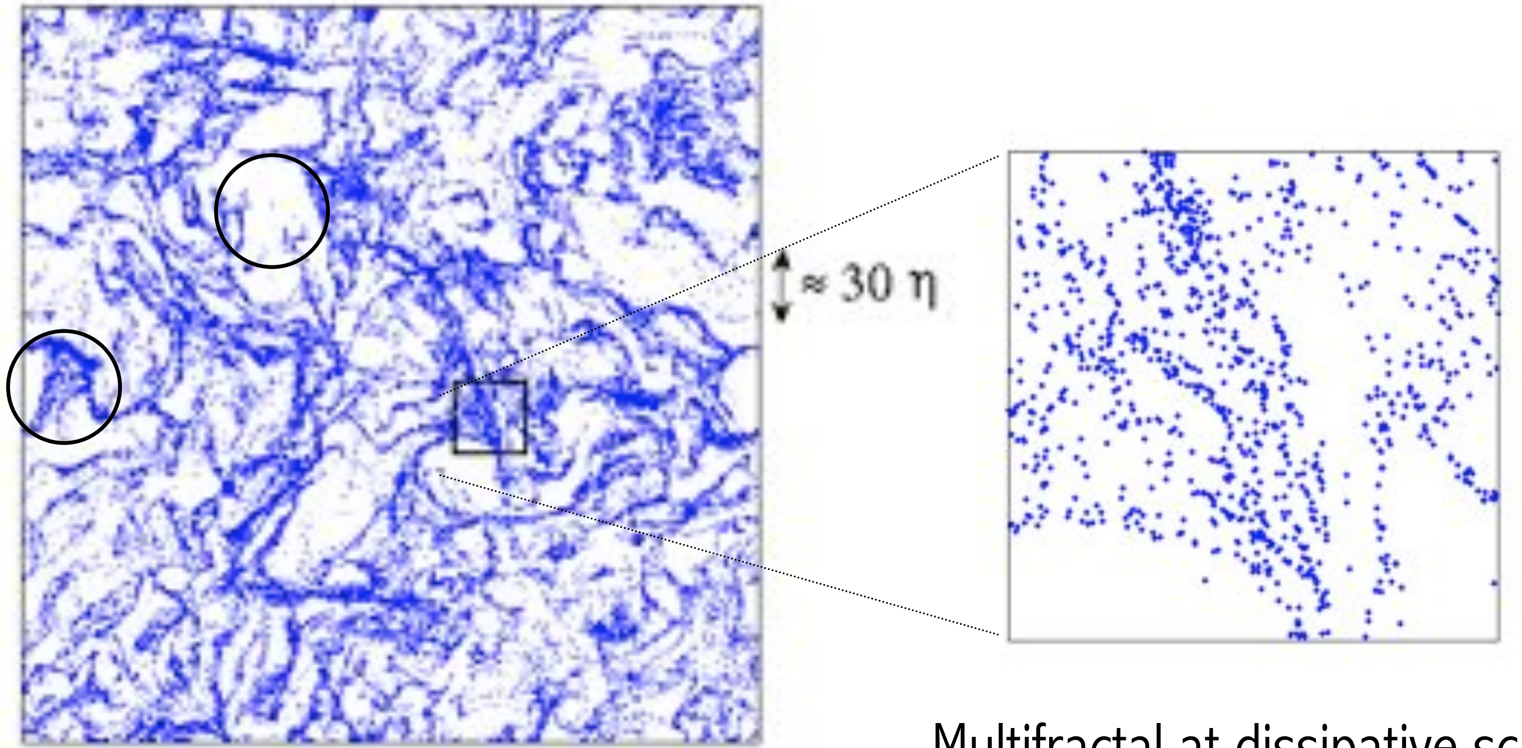
$$\frac{\mathrm{d}^2 \boldsymbol{X}}{\mathrm{d}t^2} = -\frac{1}{\tau_s}\left[\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} - \boldsymbol{u}(\boldsymbol{X},t)\right] \Rightarrow 2 \text{ parameters: } \begin{cases} \mathrm{St} = \tau_s/\tau_\eta \\ \mathrm{Re} = U\,L/\nu \end{cases}$$

Prescribed velocity field (random or solution to NS)

- **Dissipative dynamics** (even if $\boldsymbol{u}(\boldsymbol{x},t)$ is incompressible) Lagrangian averages correspond to an SRB measure that depends on the realization of the fluid velocity field.

Clustering of inertial particles

- Important for
  - the rates at which particles interact (collisions, chemical reactions, gravitation…)
  - the fluctuations in the concentration of a pollutant
  - the possible feedback of the particles on the fluid



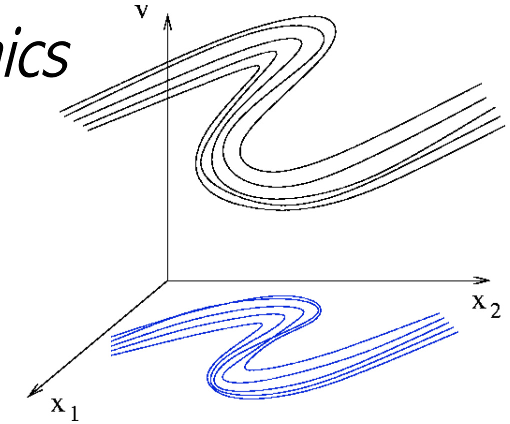
Inertial-range clusters and voids

Multifractal at dissipative scales

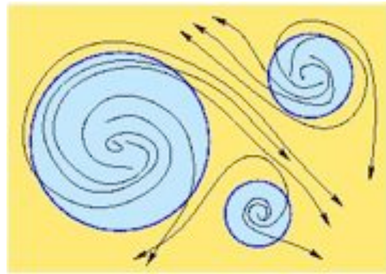
Phenomenology of clustering

- Different mechanisms:

Dissipative dynamics
⇒ ***attractor***



*Ejection from **eddies** by centrifugal forces*



- **Idea:** find models to disentangle these two effects

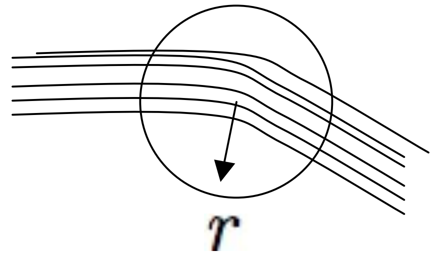
Random flows uncorrelated in time
(isolate effect of dissipative dynamics)
Lyapunov exponents
fractal dimensions
Wilkinson & Mehlig; Falkovich & Co.

Simple model for both the flow and the dynamics able to reproduce the typical shape of the **mass distribution**

Mass distribution

- Coarse-grained density

$$\bar{\rho}_r(\boldsymbol{X}, t) = \frac{1}{|\mathcal{B}_r|} \int_{\mathcal{B}_r(\boldsymbol{X}(t))} d\mu_t$$



- **Two asymptotics:**

$r \to 0$ (i.e. $r \ll \eta$): multifractal formalism $\quad p(\bar{\rho}) \propto r^{S(\ln \bar{\rho} / \ln r)}$

large deviations for the 'local dimension' $\ln \bar{\rho}_r / \ln r$

Question = dependence of the rate function S n the Stokes number St

tools = Lyapunov exponents and their large deviations

$r \to \infty$ (i.e. $r \gg \eta$): how is uniformity recovered at large scales?

use of the inertial-range properties of the flow

Problem = not scale invariant anymore

Question = how to account for a 'scale-dependent inertia'?

Small-scale clustering

- Linearized dynamics (tangent system)

$\boldsymbol{R}$ = infinitesimal separation between two trajectories

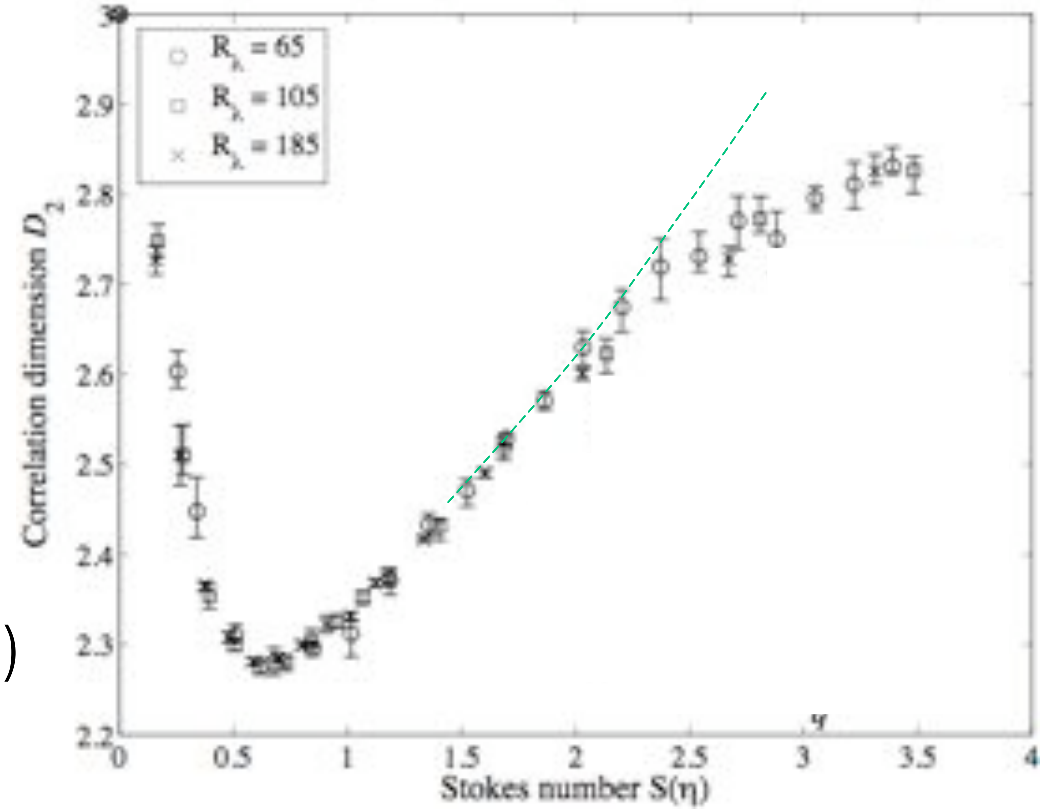
$$\frac{d^2\boldsymbol{R}}{dt^2} = -\frac{1}{\tau_s}\left(\frac{d\boldsymbol{R}}{dt} - \boldsymbol{\sigma}(t)\boldsymbol{R}\right) \qquad \sigma_{ij}(t) = \partial_j u_i(X(t),t)$$

$$\lim_{t\to\infty}\frac{1}{t}\ln|\boldsymbol{R}(t)| = \lambda$$

$$\mathrm{Prob}(|\boldsymbol{R}| < r) \propto r^{\mathcal{D}_2}$$

Correlation dimension

Related to the radial distribution function (see Sundaram & Collins 1997)



DNS (JB, Biferale, Cencini, Lanotte, Musacchio & Toschi, 2007)

Kraichnan flow

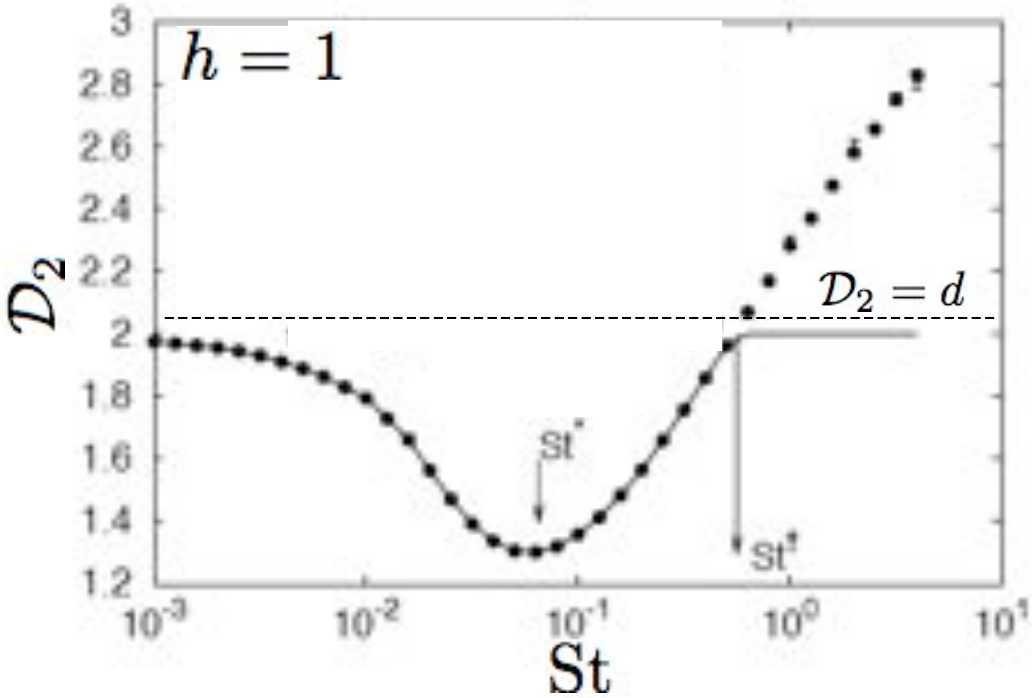
- Gaussian carrier flow with no time correlation

  Incompressible, homogeneous, isotropic

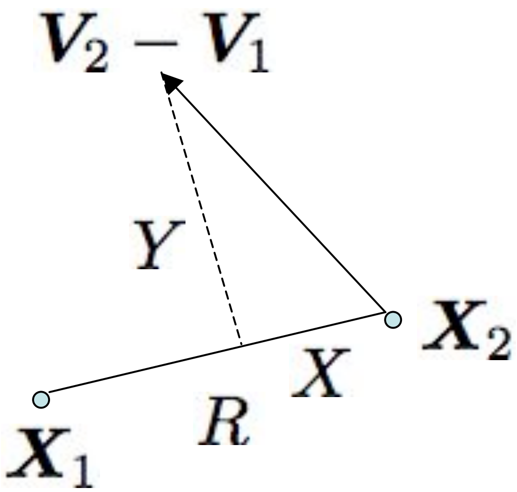
$$\langle u_i(\boldsymbol{x},t)\, u_j(\boldsymbol{x}',t')\rangle = [2D_0\delta_{ij} - B_{ij}(\boldsymbol{x}-\boldsymbol{x}')]\,\delta(t-t')$$

$$B_{ij}(\boldsymbol{r}) \simeq D_1\, r^{2h}\left[(d-1+2h)\,\delta_{ij} - 2h\, r_i r_j/r^2\right]$$

$0 \leq h \leq 1$ = Hölder exponent of the flow



Reduction of the dynamics



$$
\begin{aligned}
\mathrm{d}X &= -\left[X + X^2 - Y^2\right]\mathrm{d}s + \sqrt{2S}\,\mathrm{d}B_1 \\
\mathrm{d}Y &= -\left[Y + 2X\,Y\right]\mathrm{d}s + \sqrt{6S}\,\mathrm{d}B_2 \\
\mathrm{d}R &= X\,R\,\mathrm{d}s
\end{aligned}
$$

with $S = D_1 \tau_s$

- Maps to a problem of Anderson localization

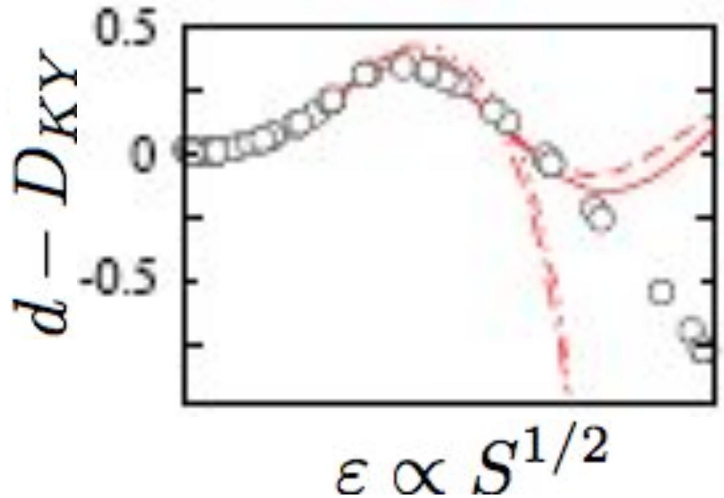
- Exponential separation

  Lyapunov exponent $\lambda = \langle X \rangle / \tau > 0$

Expansion in powers of the Stokes number

$=$ diverging series $\Rightarrow$ Borel resummation

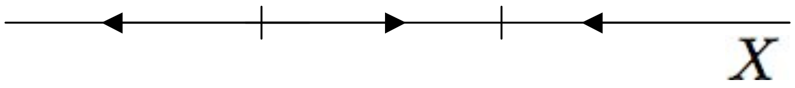
Duncan, Mehlig, Östlund & Wilkinson (2005)



"Solvable" cases

- **One dimension** (Derevyanko et al. 2006)

  Potential $U(X) = \dfrac{X^2}{2} + \dfrac{X^3}{3}$



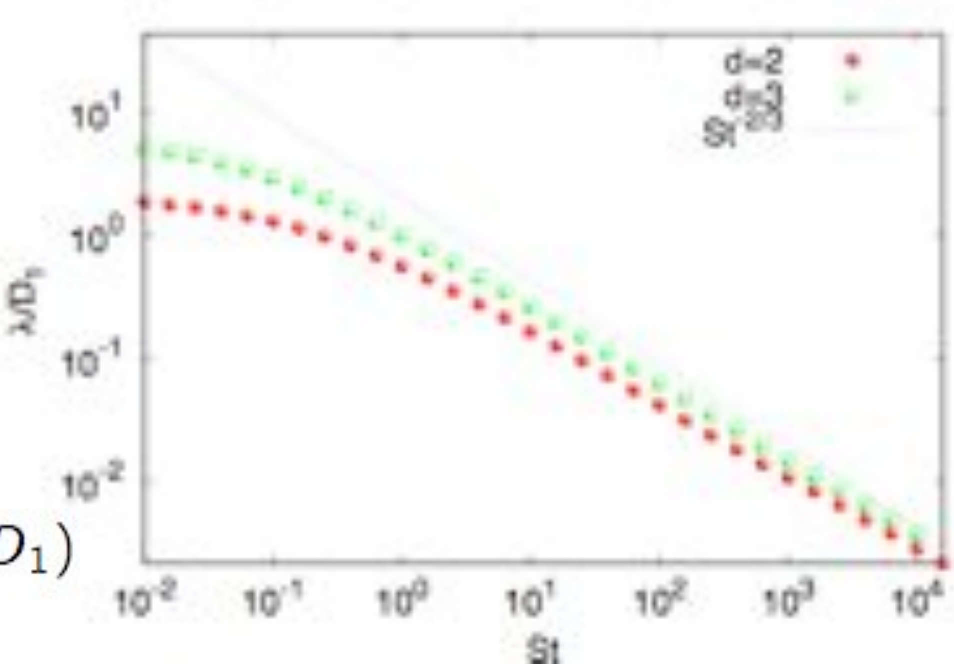
  Constant flux solution $p(X) \propto \mathrm{e}^{-U(X)/S} \int_{-\infty}^{X} dX' \, \mathrm{e}^{U(X')/S}$

  Lyapunov exponent: $\lambda = \dfrac{1}{2\tau} \left[-1 + c^{-1/2} \dfrac{\mathrm{Ai}'(c)}{\mathrm{Ai}(c)} \right] \qquad c \propto S^{-2}$

- **Large-Stokes asymptotics**

  (Horvai nlin.CD/0511023)

$$\lambda \propto D_1 S^{-2/3}$$



+ same scaling for FTLE

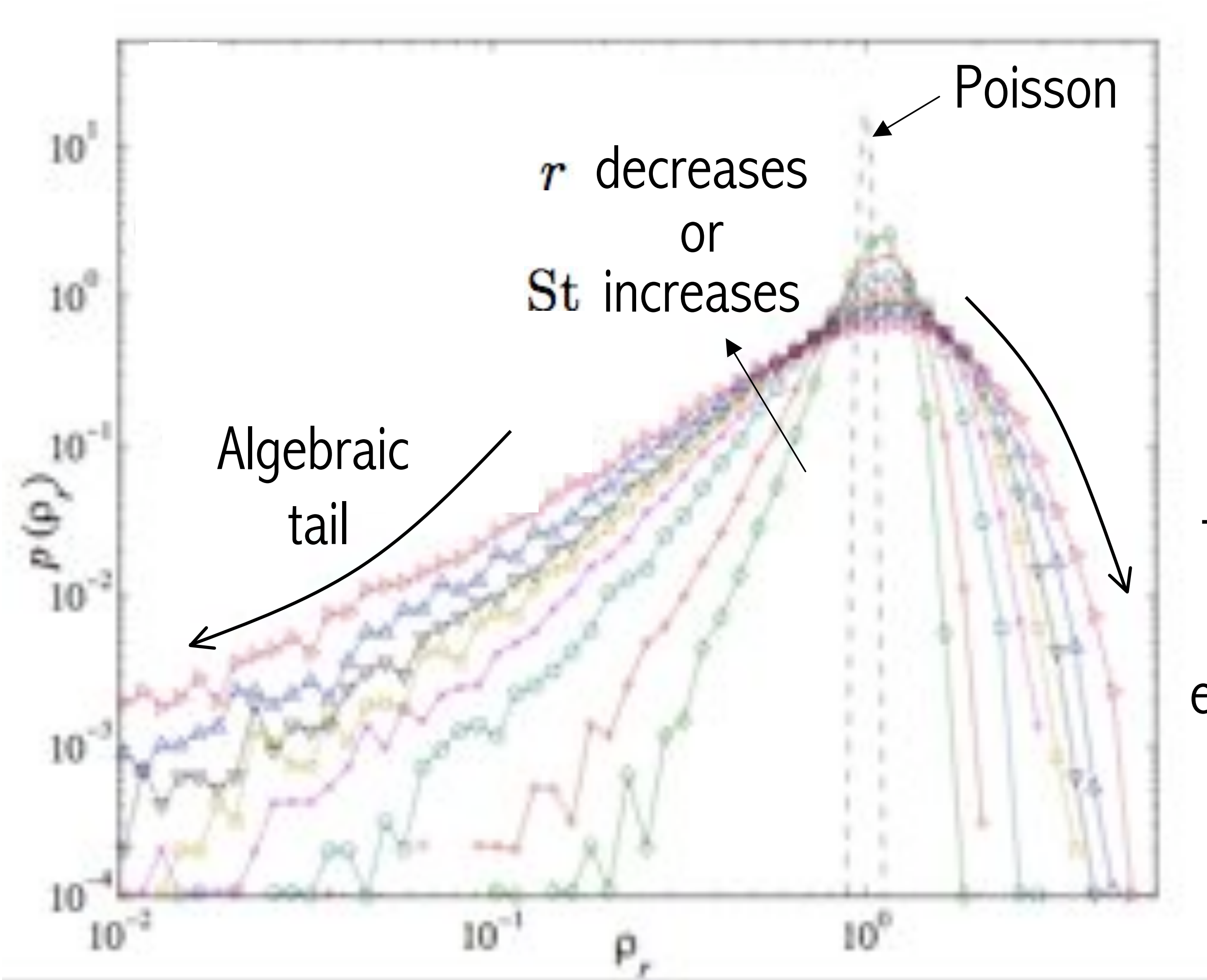
$$\mu(t) = \frac{1}{t} \ln[|\boldsymbol{R}(t)|/|\boldsymbol{R}(0)|]$$

$$p_t(\mu; S) \propto \mathrm{e}^{-tD_1 S^{-2/3} h(S^{2/3}\mu/D_1)}$$

(Bec, Cencini & Hillerbrand, 2007)

Inertial range distribution of mass

- Effective inertia decreases with r : scale invariance disappears



Small Stokes / Large box scaling

- The two limits $\tau_s \to 0$ and $r \to \infty$ are equivalent
- Naïve idea: **Local Stokes number** $St(r) = \dfrac{\tau_s}{\varepsilon^{-1/3} r^{2/3}}$
- Actually, scaling determined by the **increments of pressure**:
  Small inertia: Maxey's approximation $\dot{\boldsymbol{X}} \approx \boldsymbol{v}(\boldsymbol{X}, t)$
  synthetic compressible flow: $\boldsymbol{v} = \boldsymbol{u} - \tau_s(\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u})$
- Relevant time scale for the time evolution of a blob of particles

$$\Gamma = \frac{1}{r^3} \int_{B_r} \nabla \cdot \boldsymbol{v}\, d^3x \sim -\frac{\tau_s}{r} \Delta_r \nabla p$$

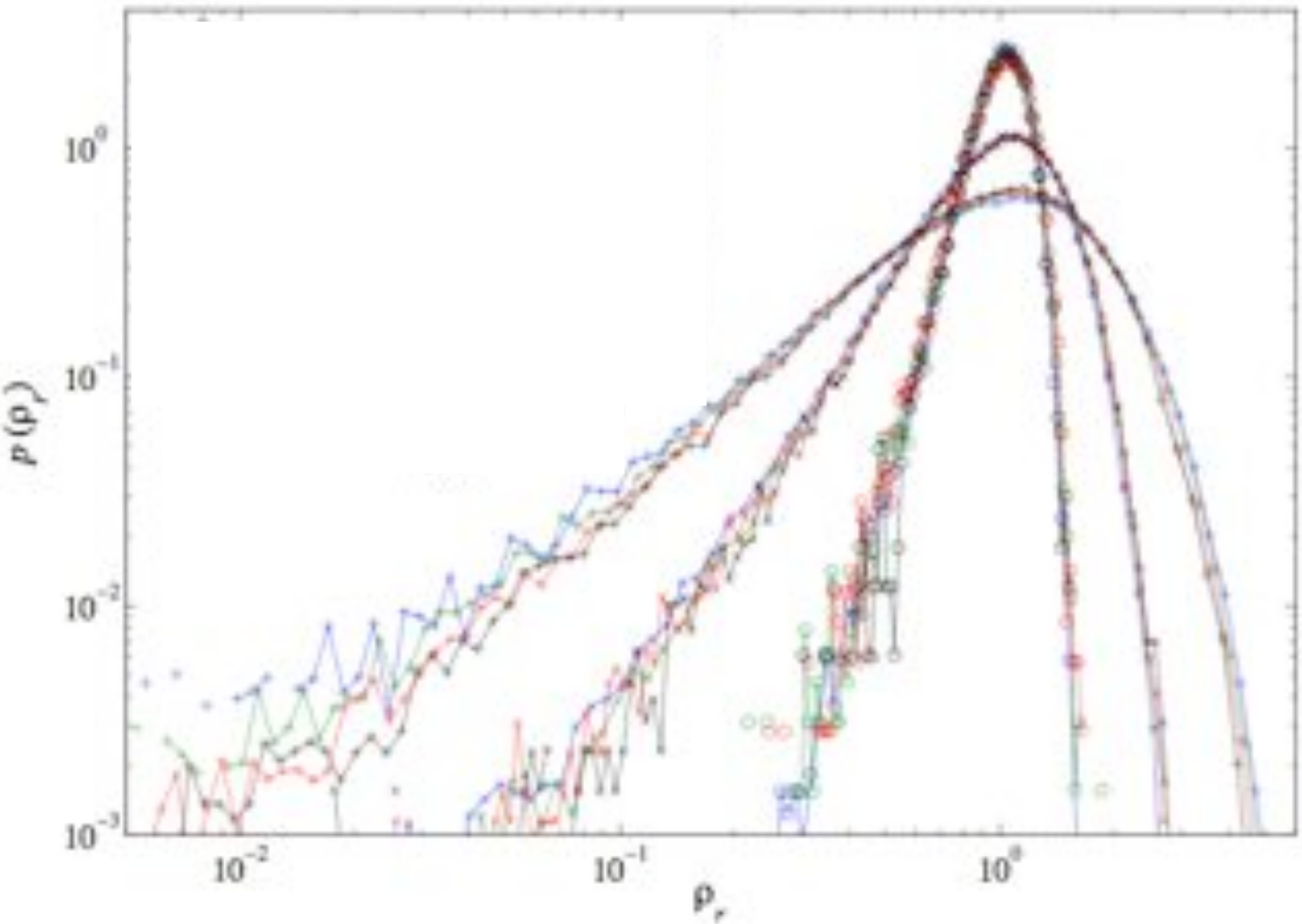
- Dimensional analysis: $\Delta_r \nabla p \sim \varepsilon^{2/3}\, r^{-1/3}$

  Observed: scaling dominated by sweeping

  $\Delta_r \nabla p \sim U \varepsilon^{1/3}\, r^{-2/3}$ so that $\boxed{\Gamma \propto \tau_s\, r^{-5/3}}$

Small Stokes / Large box scaling

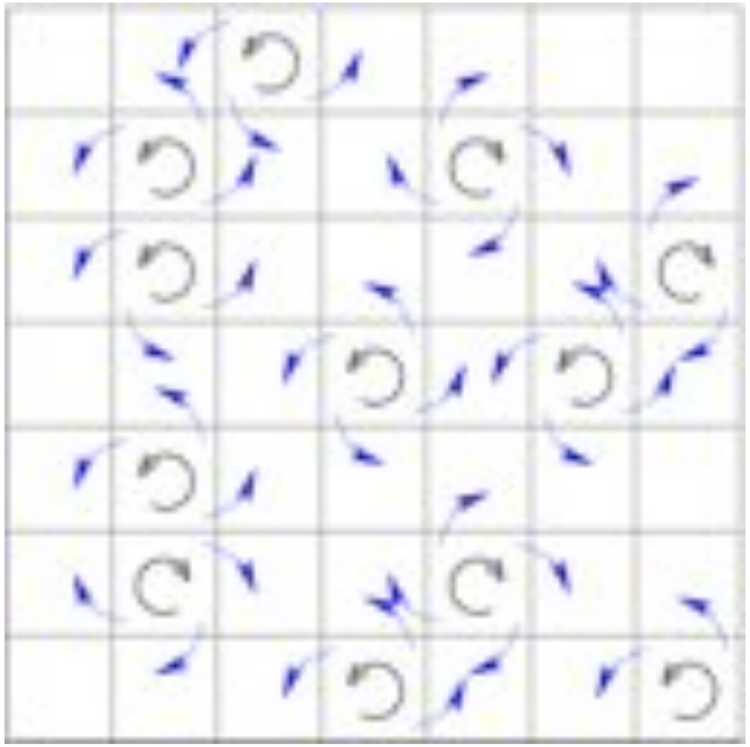
- The density distribution depends only on $\Gamma \propto \tau_s \, r^{-5/3}$



Model for vortex ejection

Between two time steps: t and $t+T$

- **Flow divided in cells.**
  With a probability p the cells are rotating and eject particles to their non-rotating neighbors



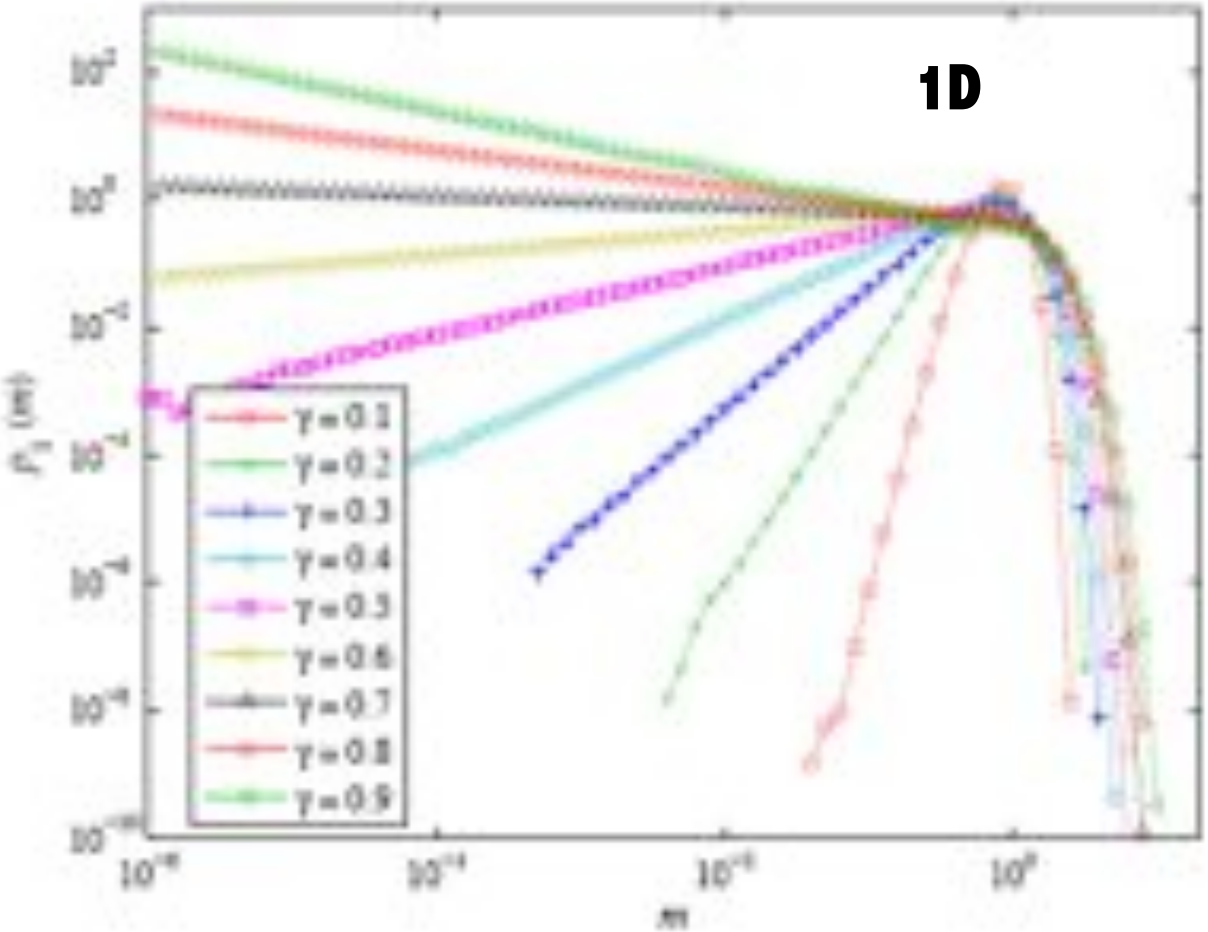
- Each cell contains a **continuous mass** m_j of particles.
  The mass ejected from the j th cell is at most γm_j .

(JB & Chétrite 2007)

One-cell mass distribution

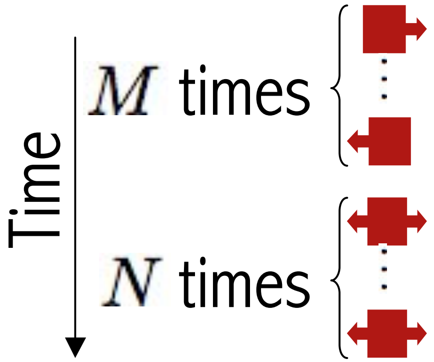
After sufficiently large time:
the system reaches a **non-equilibrium steady state**.

PDF of m_j very similar to that obtained in DNS (same tails)



Behavior of tails

- **Left tail:** algebraic $p(m) \propto m^{\alpha(\gamma)}$ when $m \ll 1$



Initial mass $m_j \approx 1$

Final mass $m_j \approx (1-\gamma)^N (1-\gamma/2)^M$

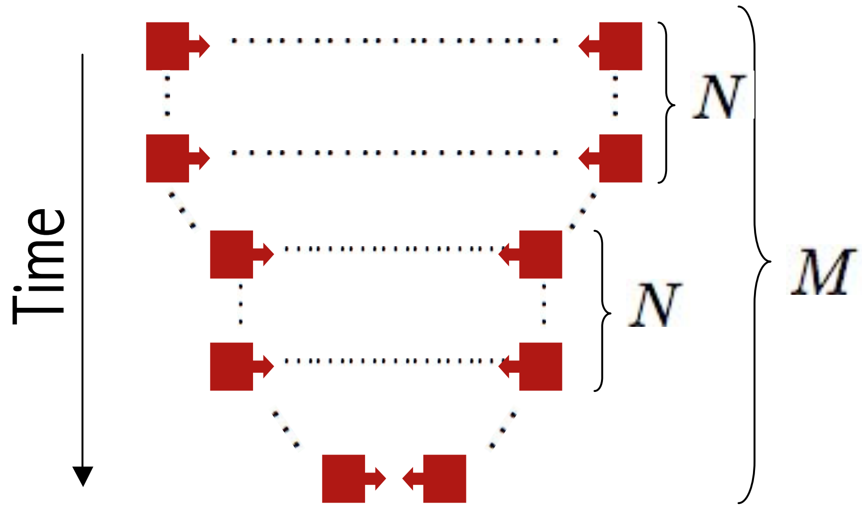
$$Prob = \left[p(1-p)^2\right]^N \left[p(1-p)\right]^M \quad \Rightarrow \quad \boxed{\alpha(\gamma, p)}$$

- **Right tail:** super exponential

$$m = \frac{1 - \left[1 - (1-\gamma/2)^N\right]^M}{(1-\gamma/2)^N}$$

$$Prob = \left[p^2(1-p)\right]^{N\,M}$$

$$\Rightarrow \boxed{p(m) \propto \exp(-C\, m \log m)}$$



Summary / Open questions

- Two kinds of clustering
  - **Dissipative scales:** scale invariant (multifractal)
    relevant time scale = Kolmogorov time τ_η
  - **Inertial range:** scale invariance broken
    relevant time scale = acceleration (pressure gradient) $\propto r^{5/3}$

- What can be **analytically quantified**?
  - **Fractal dimensions / Lyapunov exponents**:
    Short-correlated flows: Wilkinson & Mehlig
    1D telegraph: Falkovich and coll.
  - **Inertial-range distributions** (cell ejection models)

- Is the 5/3 scaling a **finite Reynolds number effect**?