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Fluid Mechanics and Climate Dynamics: *Observations, Simulations and (Maybe) Predictions*

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Please see these sites for collaborators and further details:

<http://www.environnement.ens.fr/>

<http://e2c2.ipsl.jussieu.fr/>

<http://www.atmos.ucla.edu/tcd/>

Special thanks to Mickaël Chekroun and Eric Simonnet!

Motivation

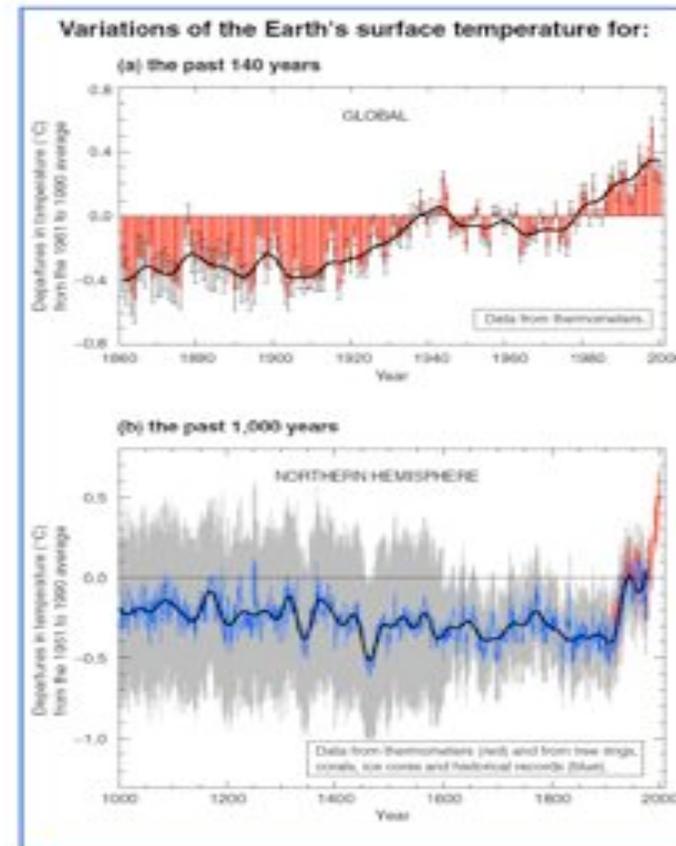
- The *climate system* is highly *nonlinear and* quite *complex*.
- Its *major components* — the atmosphere, oceans, ice sheets — *flow* on many time and space scales.
- Its *predictive understanding* has to rely on the system's physical, chemical and biological modeling, but also on the mathematical analysis of the models thus obtained.
- The *hierarchical modeling* approach allows one to give proper weight to the understanding provided by the models vs. their realism, respectively.
- This approach facilitates the evaluation of *forecasts (pognostications?)* based on these models.
- Back-and-forth between *“toy”* (conceptual) and *detailed* (“realistic”) *models*, and between *models* and *data*.

Global warming and its socio-economic impacts

Temperatures rise:

- What about impacts?
- How to adapt?

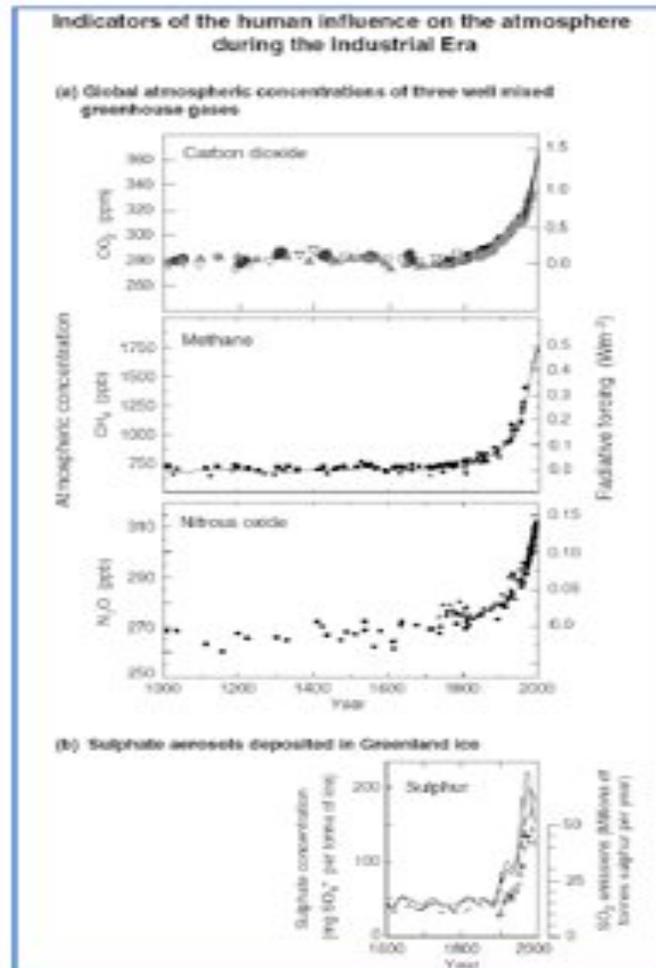
*Source : IPCC (2001),
TAR, WGI, SPM*



GHGs rise

It's gotta do with us, at least a bit, ain't it?

IPCC (2001)



But things aren't that easy!

What to do?

- Natural variability introduces additional complexity into the anthropogenic climate change problem.
- The most common interpretation of observations and GCM simulations of climate change is still in terms of a scalar, linear ordinary differential equation (ODE):

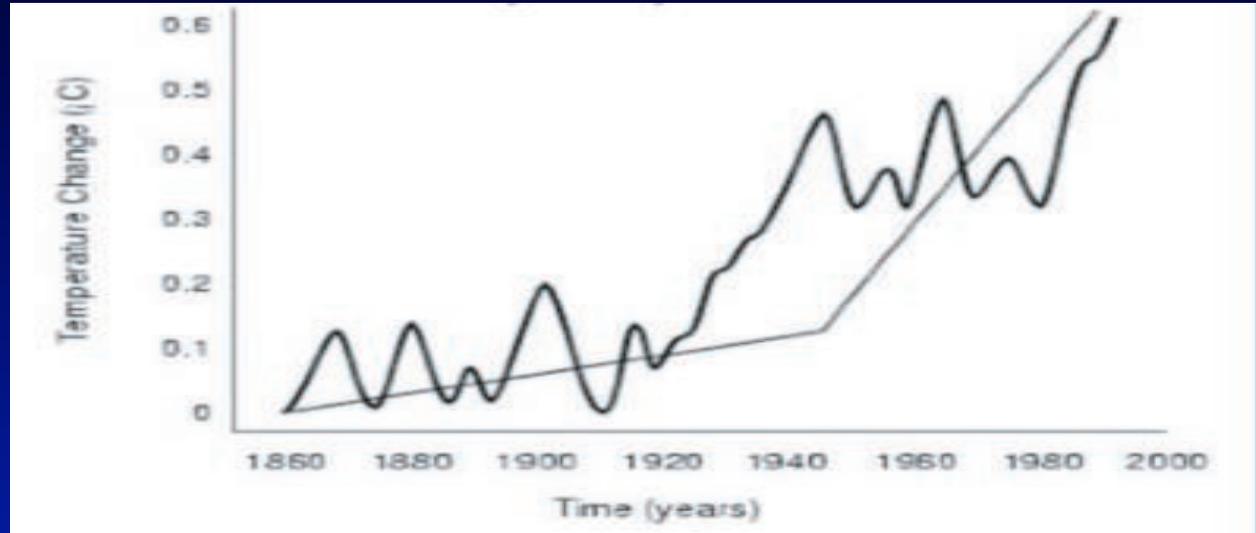
$$c \frac{dT}{dt} = -kT + Q,$$

where

$$k = \sum k_i - \text{feedbacks (+ve and -ve);}$$

$$Q = \sum Q_j - \text{sources \& sinks, } Q_j = Q_j(t)$$

Linear response vs. observations



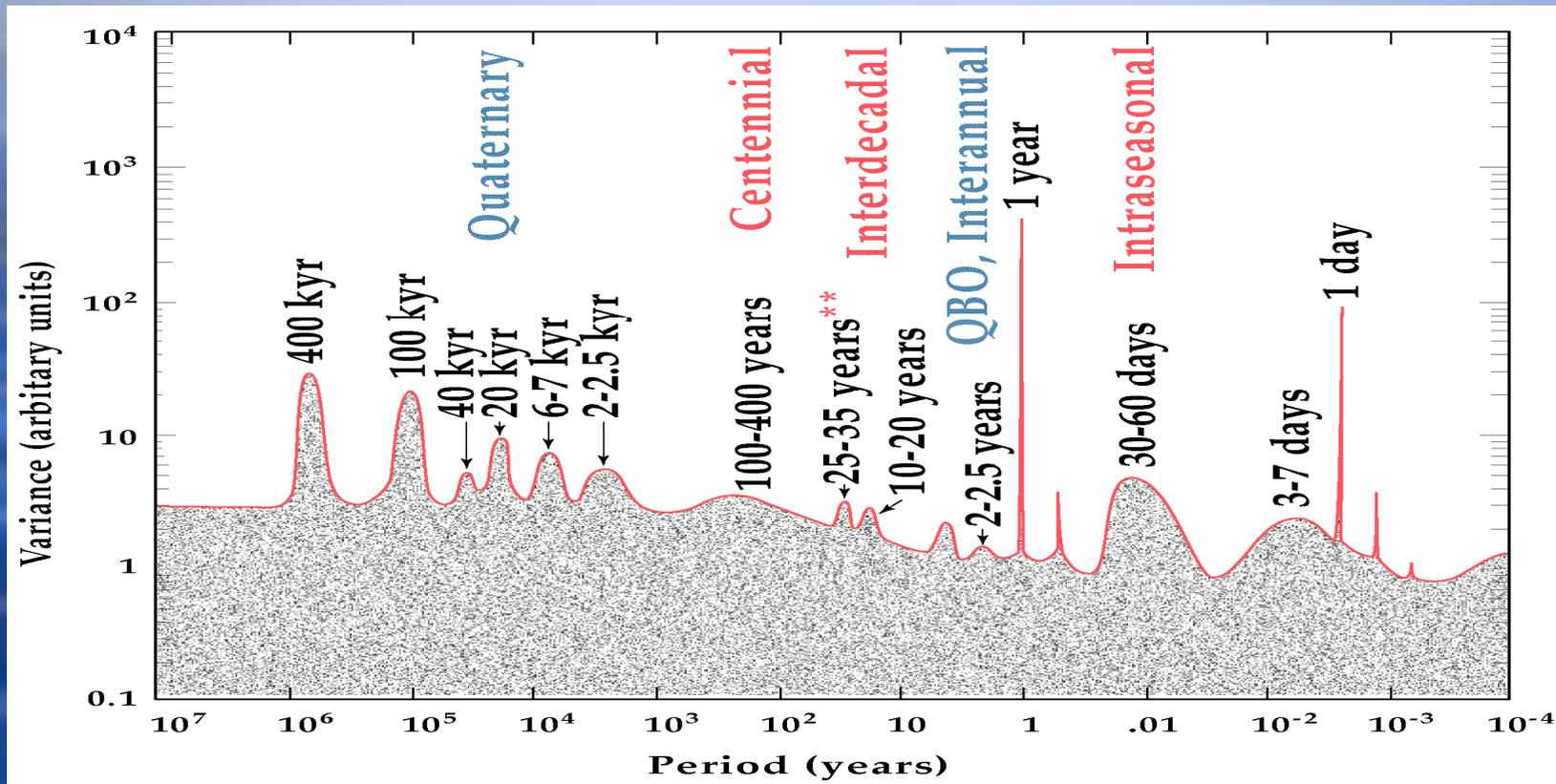
- Linear response to change in atmospheric CO₂ concentration vs. observed change in global temperature T .
- Hence we need to consider instead a system of nonlinear partial differential equations (PDEs), with parameters and multiplicative, as well as additive forcing (deterministic + stochastic)

$$\frac{dX}{dt} = N(X, t; \mu, \beta)$$

Composite spectrum of climate variability

Standard treatment of frequency bands:

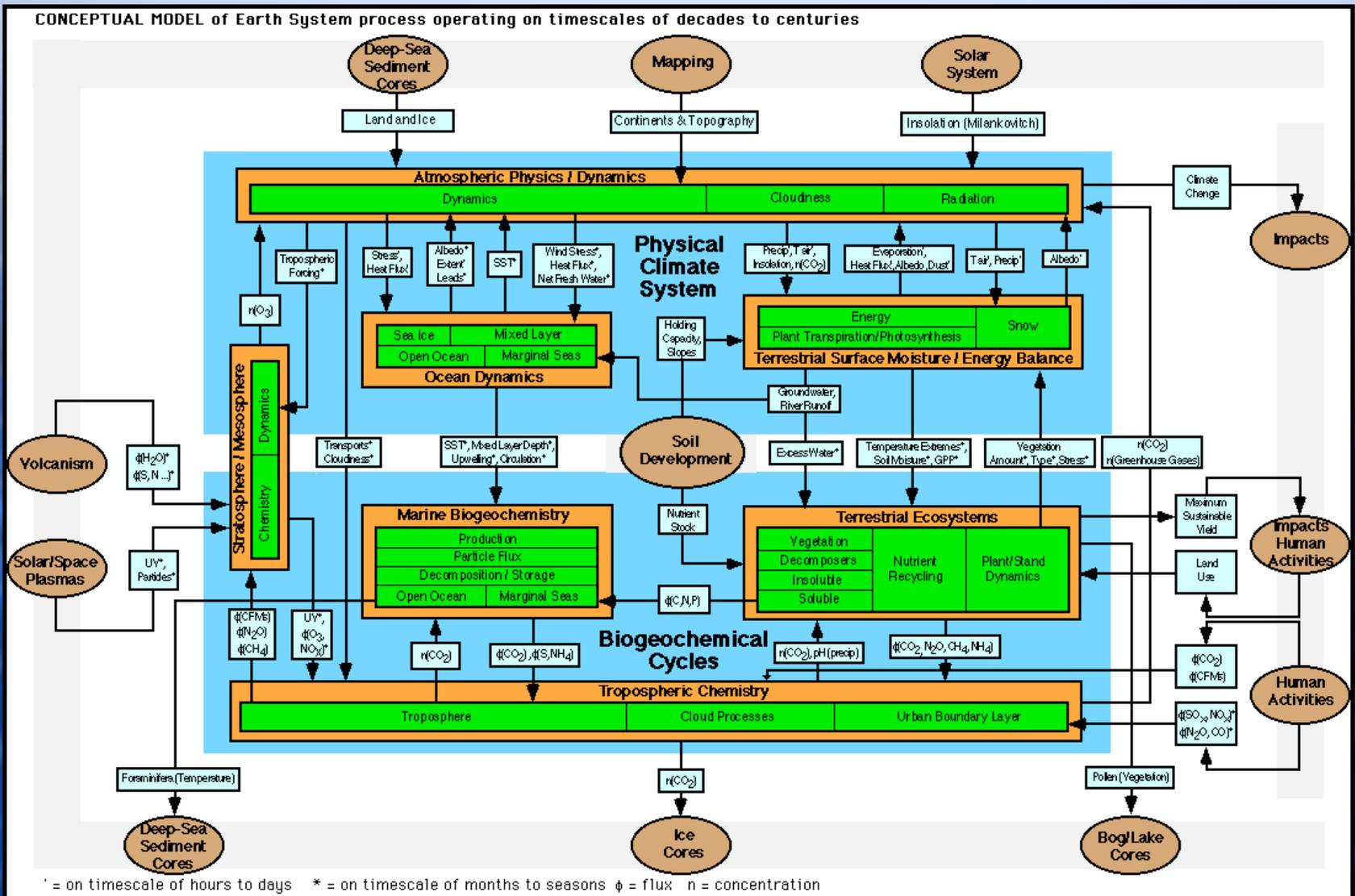
1. High frequencies – white (or “colored”) noise
2. Low frequencies – slow (“adiabatic”) evolution of parameters



From Ghil (2001, EGEN), after Mitchell* (1976)

* “No known source of deterministic internal variability”

F. Bretherton's "horrendogram" of Earth System Science



Climate models (atmospheric & coupled) : A classification

• *Temporal*

- stationary, (quasi-)equilibrium
- transient, climate variability

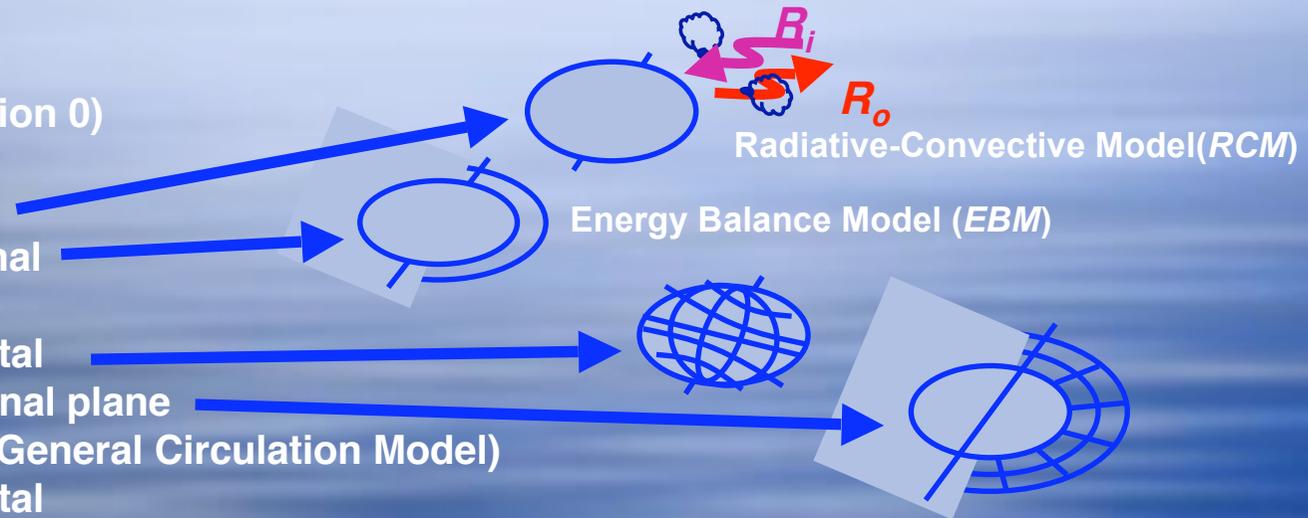
• *Space*

- 0-D (dimension 0)
- 1-D
 - vertical
 - latitudinal
- 2-D
 - horizontal
 - meridional plane
- 3-D, GCMs (General Circulation Model)
 - horizontal
 - meridional plane
- Simple and intermediate 2-D & 3-D models

• *Coupling*

- Partial
 - unidirectional
 - asynchronous, hybrid
- Full

Hierarchy: from the simplest to the most elaborate,
iterative comparison with the observational data



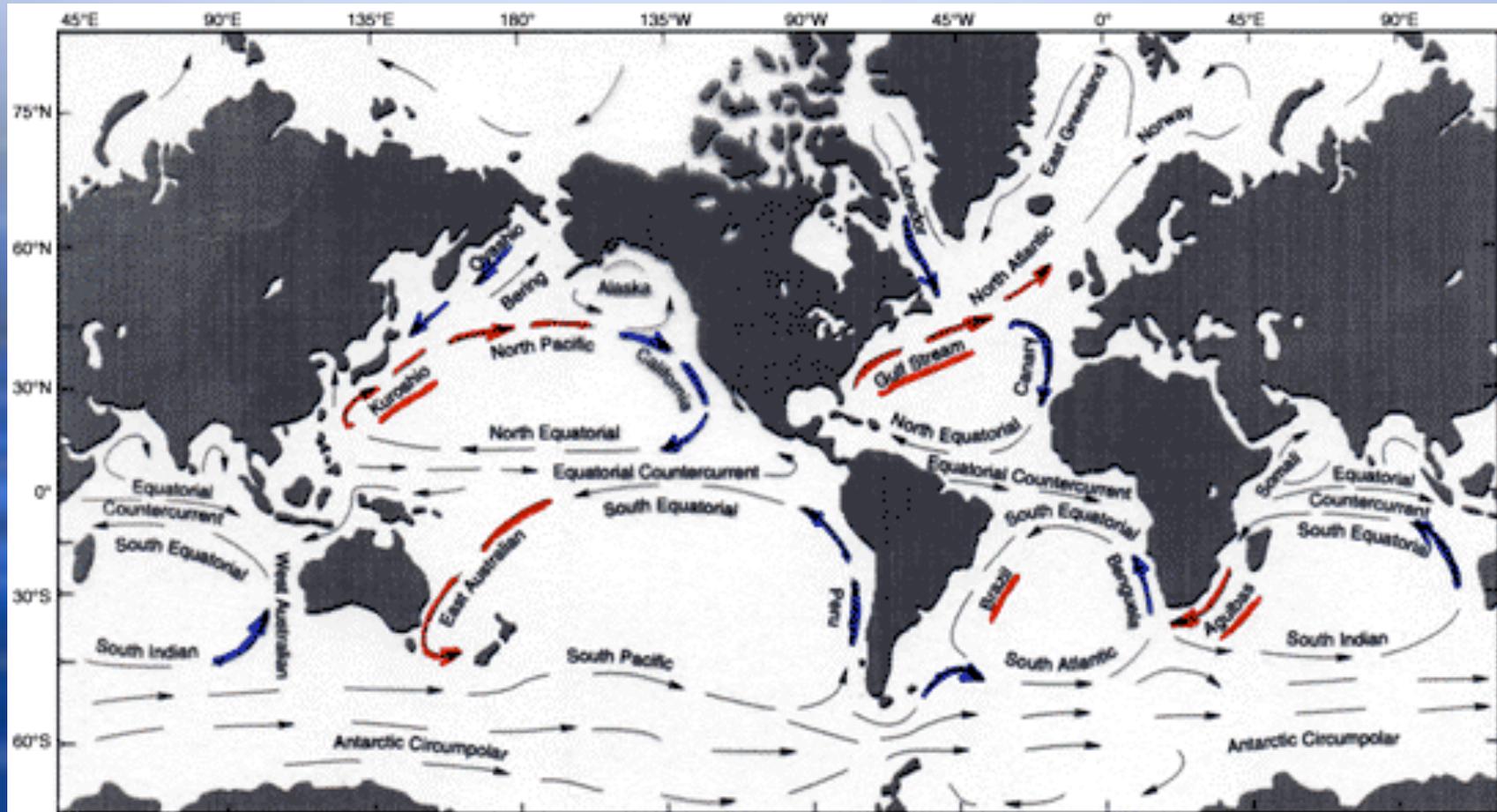
Climate and Fluids

The coupled climate system is dominated by its fluid components: the atmosphere and hydrosphere (oceans, rivers, lakes)

L. Euler's portrait courtesy of Georgi S. Golitsyn (IFARAN, Moscow); formerly in the collection of the Imperial Academy of Sciences, St. Petersburg (till 1918)



An example of bifurcations and hierarchical modeling: The oceans' wind-driven circulation



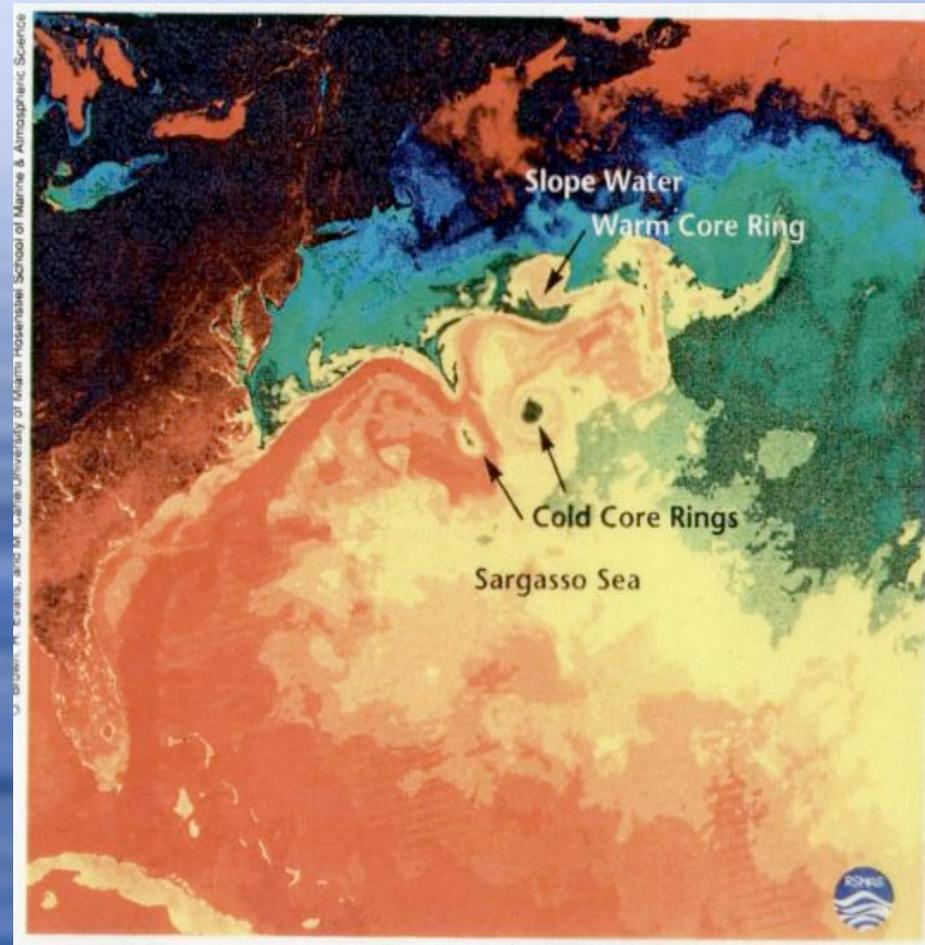
J. Apel (1987), Principles of Ocean Physics

The mean surface currents are (largely) wind-driven

The gyres and the eddies

Many scales of motion, dominated in the mid-latitudes by
(i) *the double-gyre circulation*;
and (ii) *the rings and eddies*.

Much of the focus of physical oceanography over the '70s to '90s has been with the "*meso-scale*": the meanders, rings & eddies, and the associated two-dimensional and quasi-geostrophic *turbulence*.



Based on SSTs, from satellite IR data

The double-gyre circulation and its low-frequency variability

Shallow-water model: An "intermediate" model of the mid-latitude, wind-driven ocean circulation, with 20-km resolution \Rightarrow about 15 000 variables.

$$\begin{cases} U_t + \nabla \cdot (\mathbf{u}U) = -g'h h_x + fV + \alpha_A A \nabla^2 U - RU - \alpha_\tau \frac{\tau^x}{\rho} \\ V_t + \nabla \cdot (\mathbf{u}V) = -g'h h_y - fU + \alpha_A A \nabla^2 V - RV \\ h_t = -(U_x + V_y) \end{cases}$$

where $U\hat{e}_x + V\hat{e}_y = h\mathbf{u} = h(u\hat{e}_x + v\hat{e}_y)$,

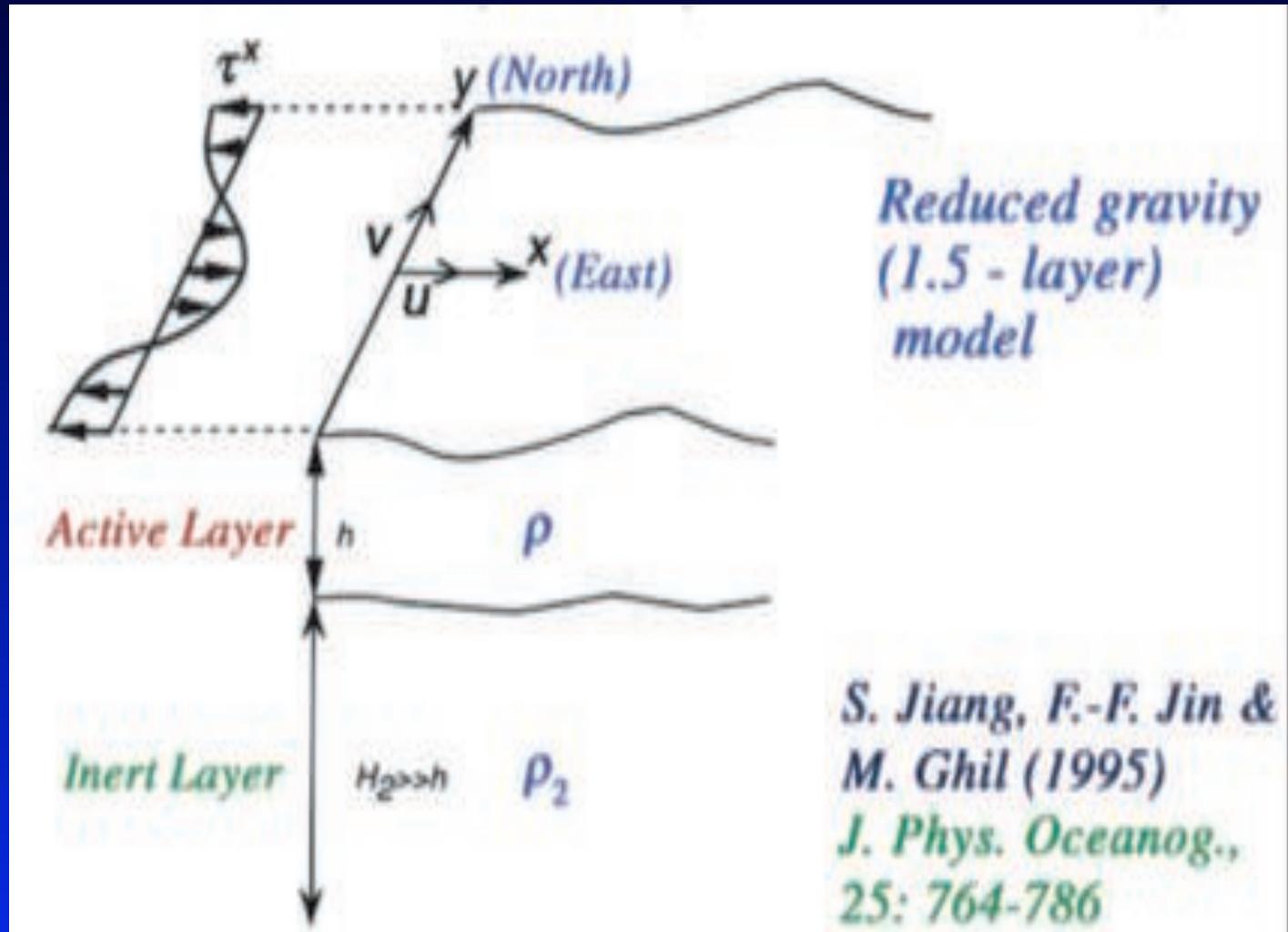
g' : reduced gravity, $g' = g(\rho_2 - \rho)/\rho$

A : viscosity coefficient ($= 300 \text{ m}^2\text{s}^{-1}$)

R : Rayleigh coefficient ($= 1/200 \text{ day}^{-1}$)

τ^x : wind stress $= \tau_0 \cos(2\pi/L)$ ($\tau_0 = 1 \text{ dyn cm}^{-2}$ & $L = 2000 \text{ km}$)

Shallow-water model (continued)

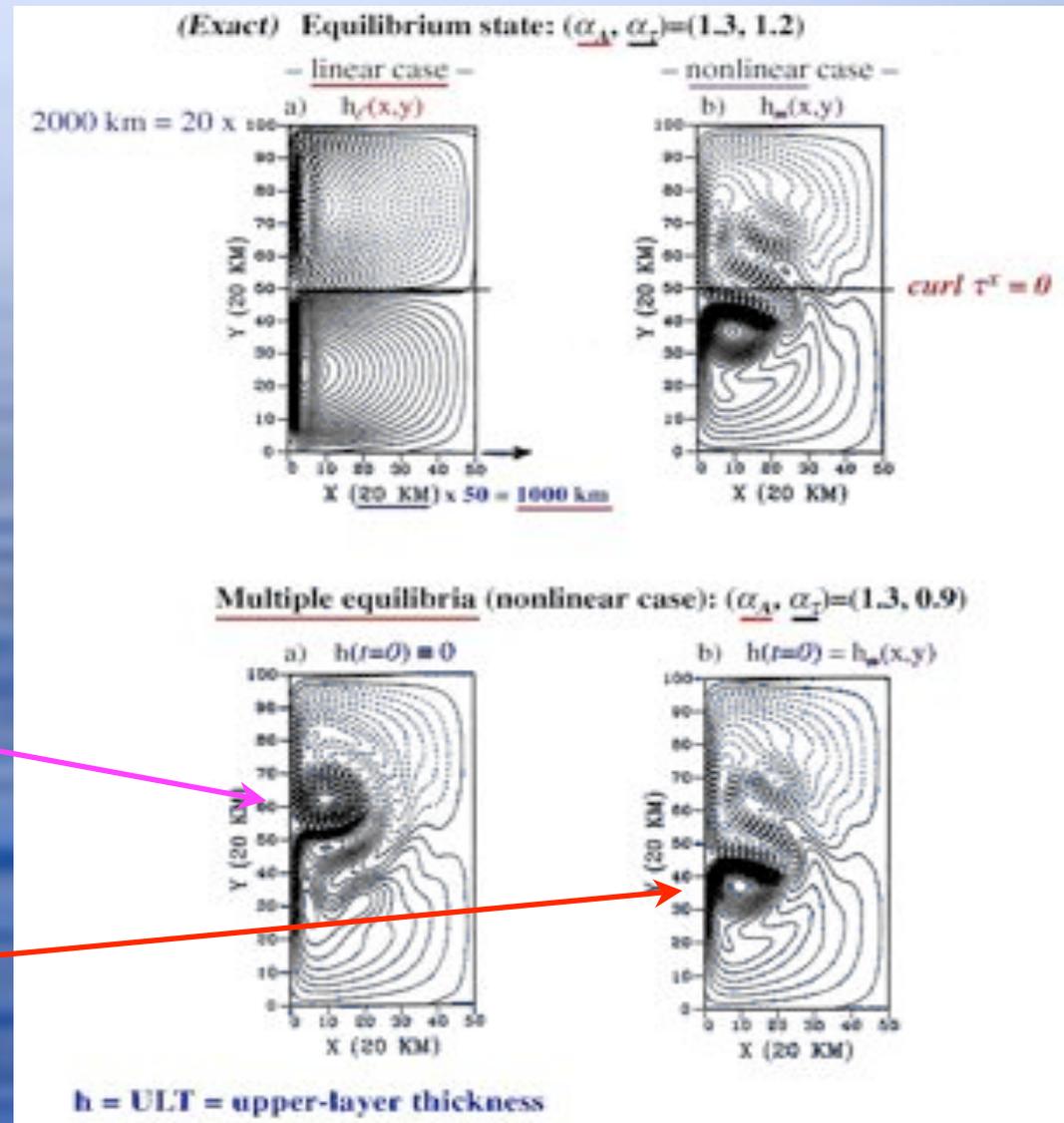


The JJG model's equilibria

Nonlinear (advection) effects break the (near) symmetry: (perturbed) pitchfork bifurcation?

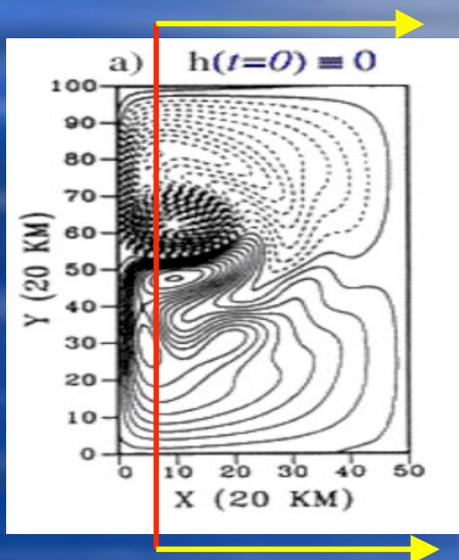
Subpolar gyre dominates

Subtropical gyre dominates



Time-dependent solutions: periodic and chaotic

To capture space-time dependence, meteorologists and oceanographers often use Hovmöller diagrams

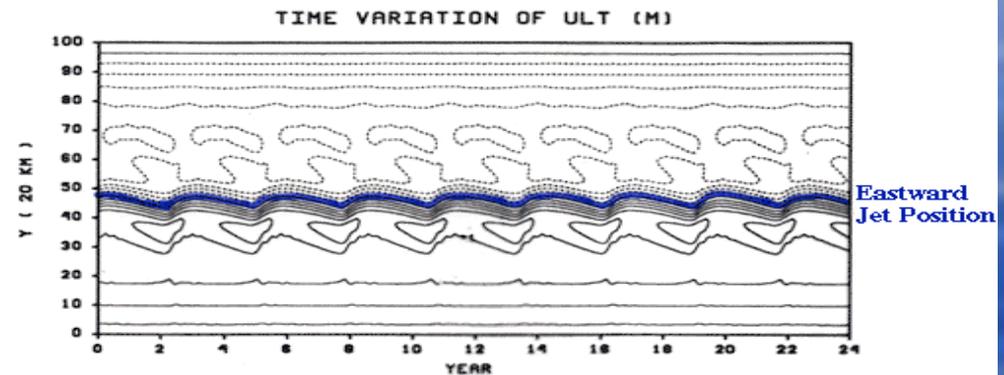


Time-dependent solutions

1. Periodic, w/ interannual period (2.8 years)

$$\alpha_A = 1.0$$

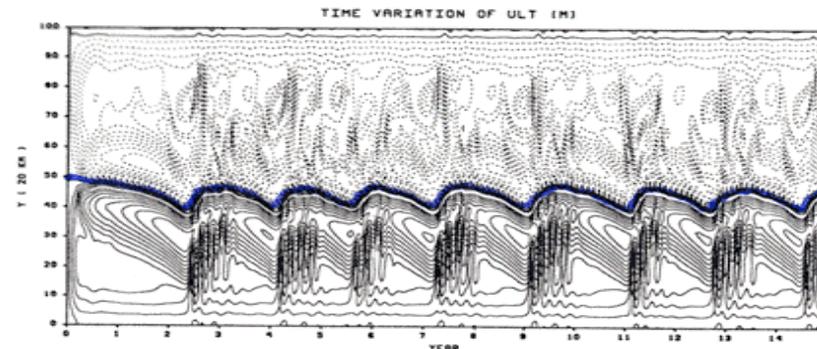
$$\alpha_\tau = 0.8$$



2. Aperiodic (weakly chaotic)

$$\alpha_A = 1.0$$

$$\alpha_\tau = 1.6$$

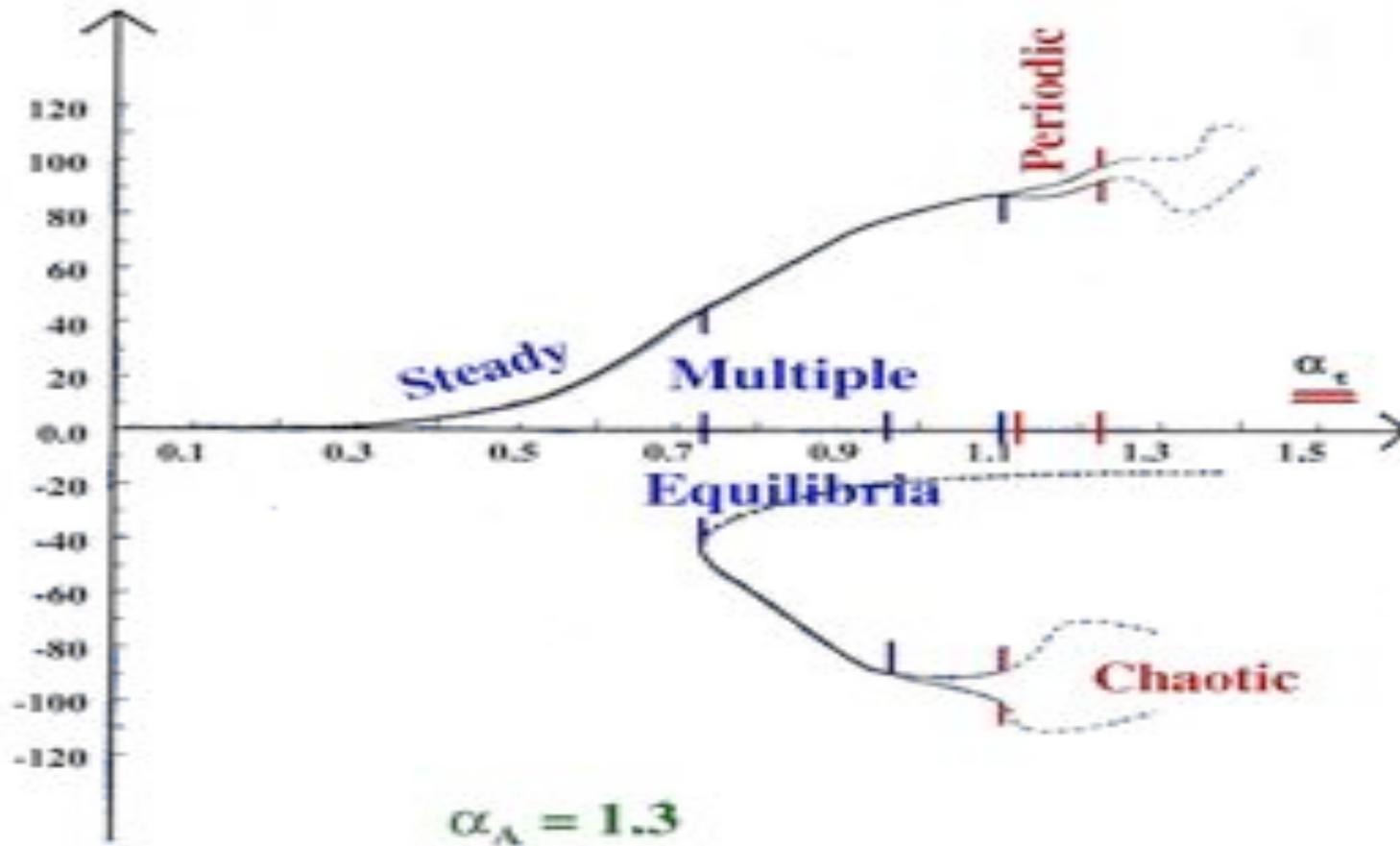


Poor man's continuation method

Bifurcation diagram

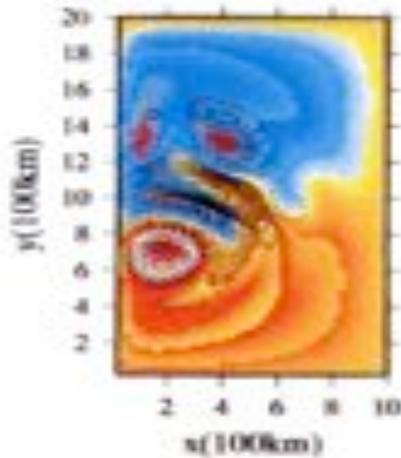
Perturbed pitchfork + Hopf + transition to chaos

Position of Merging Point (km)

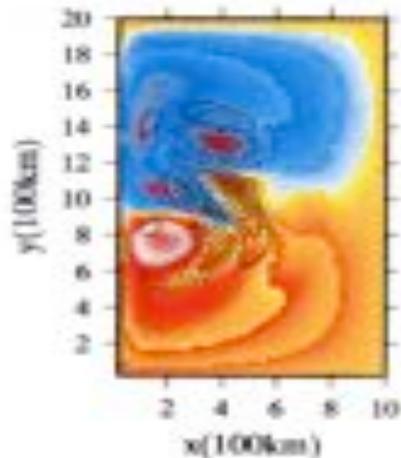


Interannual variability: relaxation oscillation

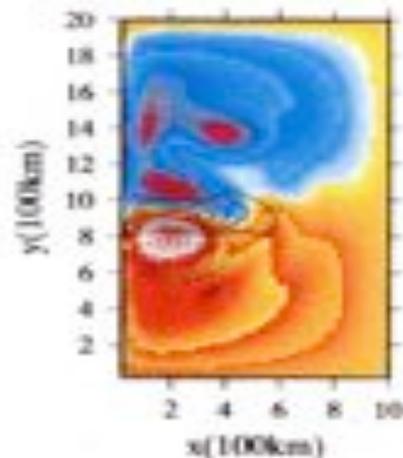
0 years



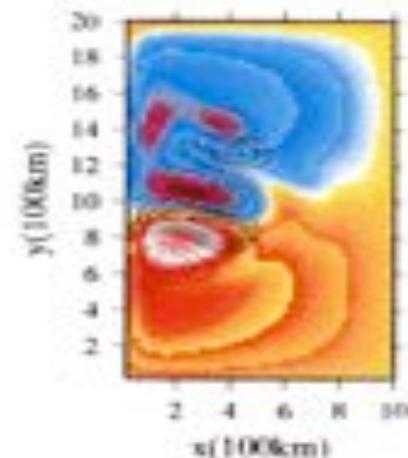
0.4 years



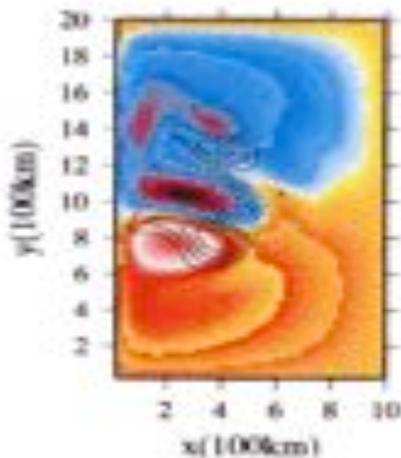
0.8 years



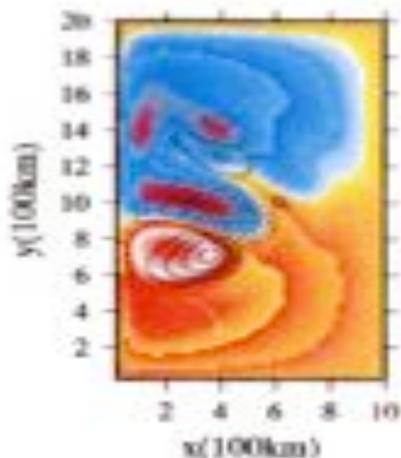
1.2 years



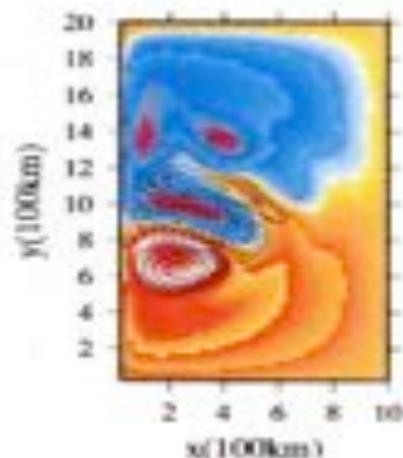
1.6 years



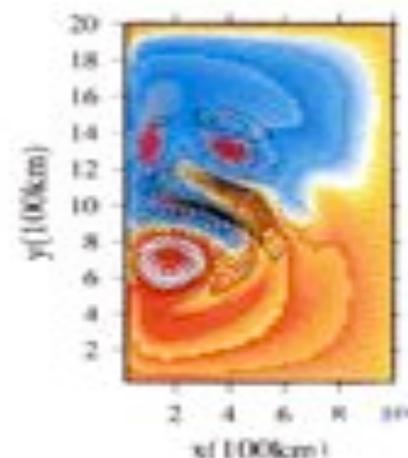
2.0 years



2.4 years



2.8 years



The double-gyre circulation:

A different rung of the hierarchy

Another "intermediate" model of the double-gyre circulation: slightly different physics, higher resolution – down to 10-km in the horizontal and more layers in the vertical, much larger domain, ...

Quasi-geostrophic, 2.5-layer model:

$$\begin{aligned} \frac{\partial}{\partial t} (\nabla^2 h_1 - \lambda_1^2 (h_1 - h_2)) + \beta \frac{\partial h_1}{\partial x} &= -\frac{g'}{f_0} J[h_1, \nabla^2 h_1 - F_1^2 (h_1 - h_2)] \\ &+ A_h \nabla^4 h_1 - C \nabla^2 (h_1 - h_2) + \frac{f_0}{\rho_0 g' H_1} \text{curl}(\vec{\tau}) \\ \frac{\partial}{\partial t} (\nabla^2 h_2 - \lambda_2^2 (h_2 - h_1)) + \beta \frac{\partial h_2}{\partial x} &= -\frac{g'}{f_0} J[h_2, \nabla^2 h_2 - F_2^2 (h_2 - h_1)] \\ &+ A_h \nabla^4 h_2 - C \nabla^2 (h_2 - h_1) - R \nabla^2 h_2 \end{aligned}$$

where h_1, h_2 : height anomalies for upper and lower layer

H_1, H_2 : mean heights for upper and lower layer

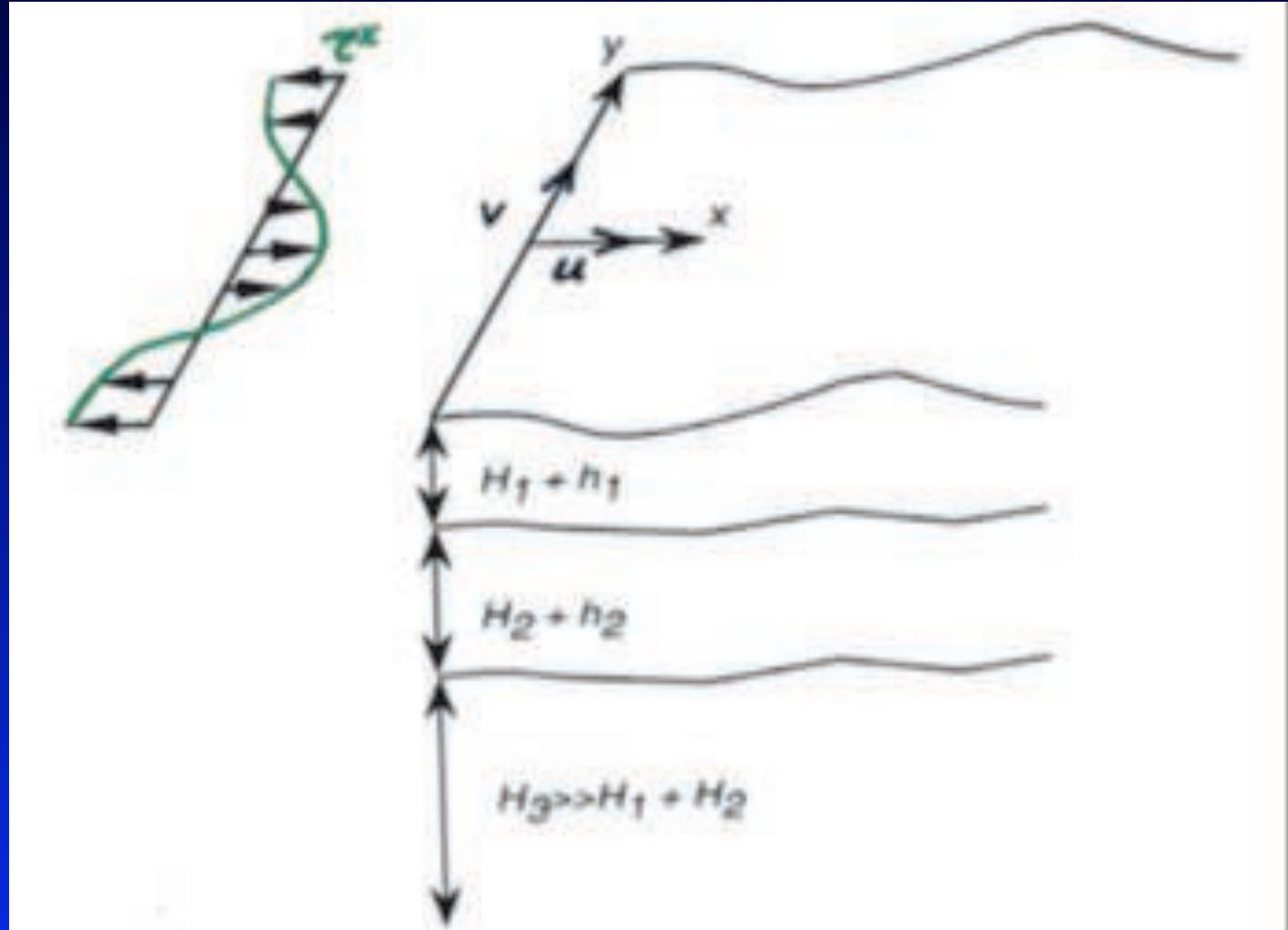
λ_1, λ_2 : Rossby radii of deformation; $\lambda_1 = \sqrt{h' H_1 / f_0^2}$, $\lambda_2 = \sqrt{h' H_2 / f_0^2}$

f_0, β : Coriolis and beta parameters

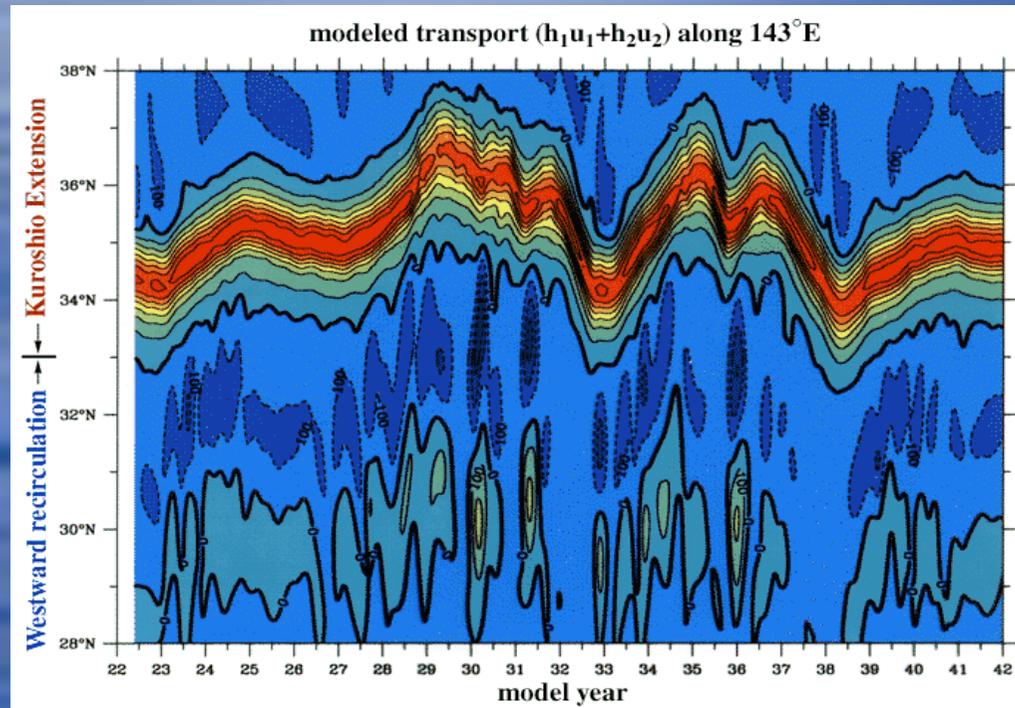
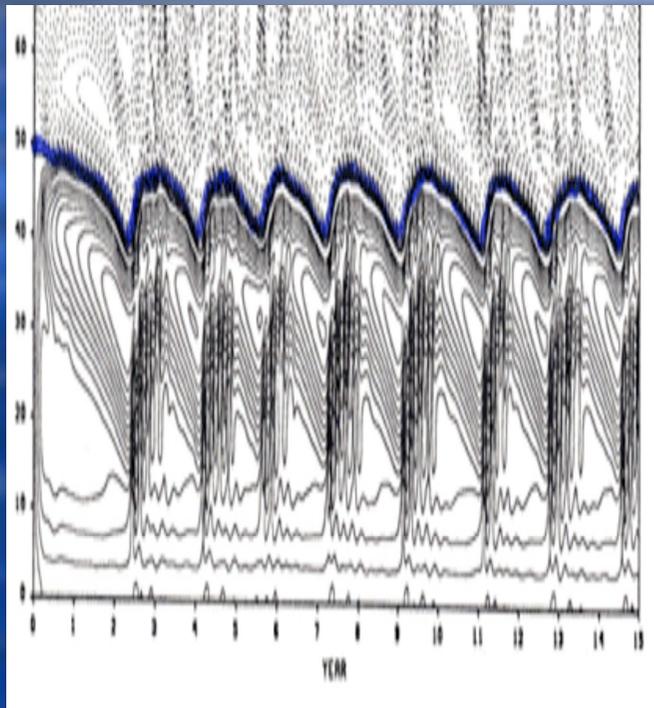
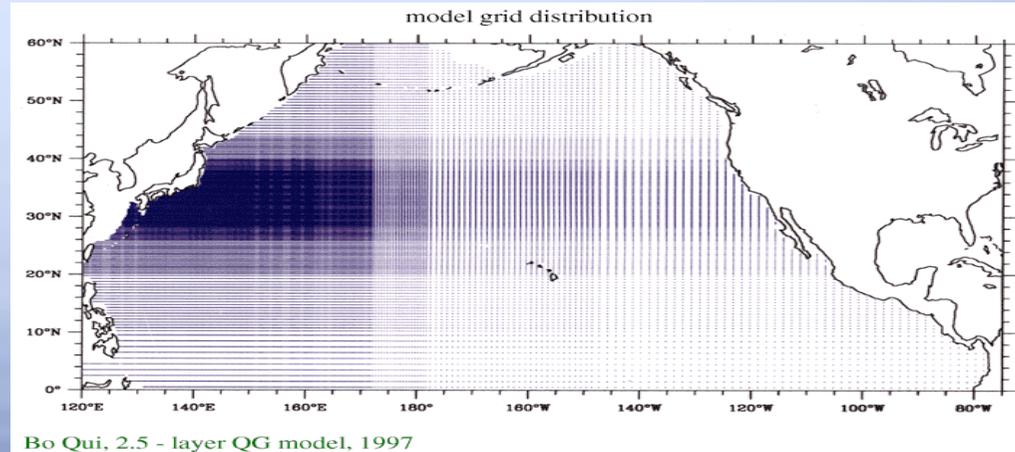
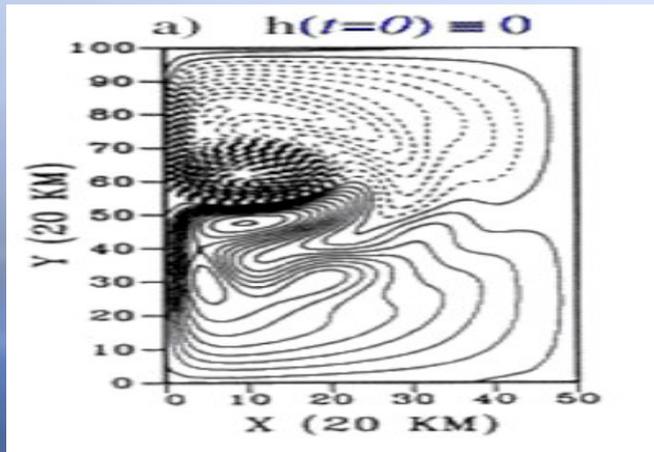
ρ_0, g' : mean density and reduced gravity

C, R : Rayleigh coefficient for interface and lower layer, and $\vec{\tau}$: wind stress

Quasi-geostrophic model (continued)

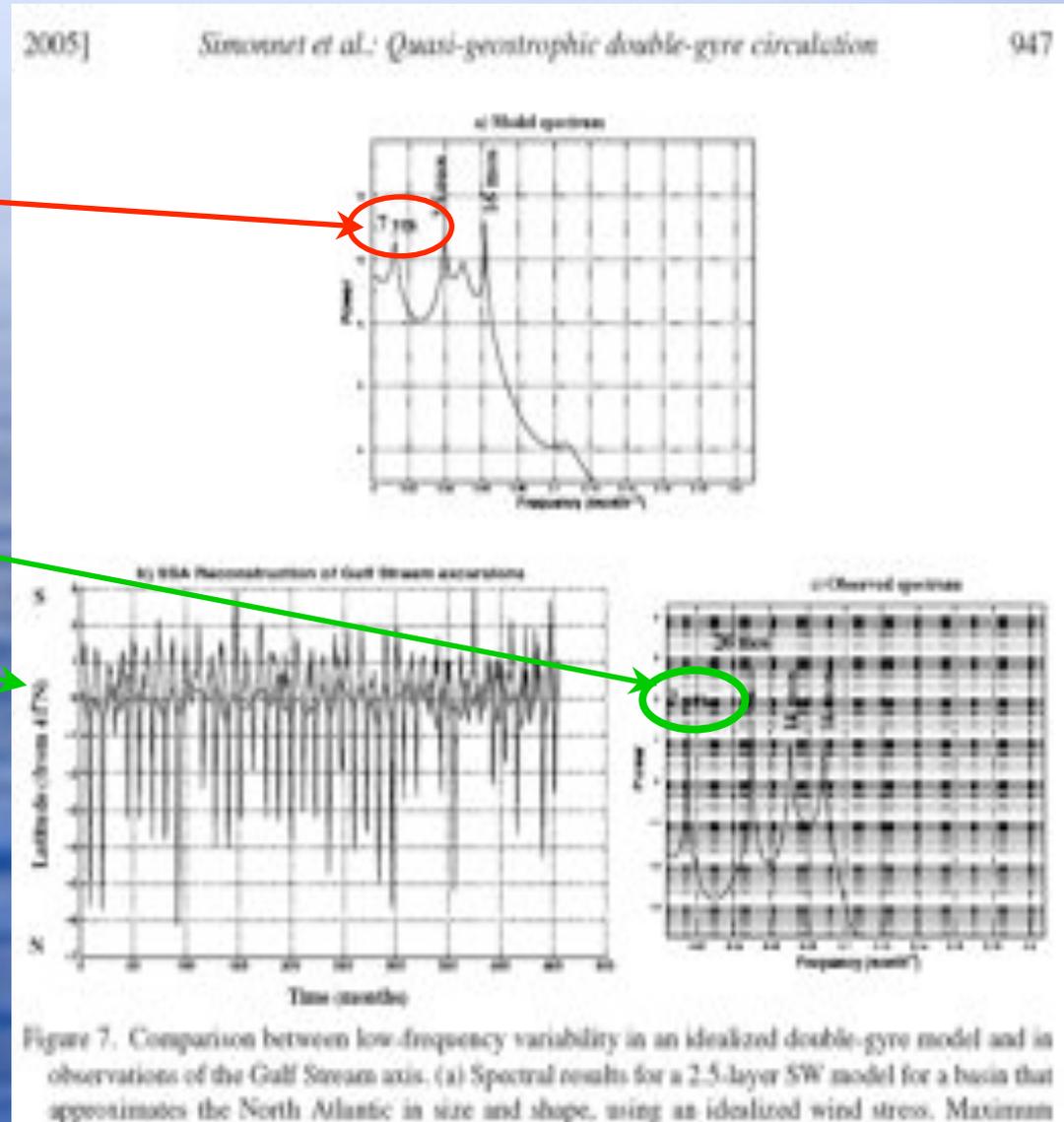


Model-to-model, qualitative comparison



Model-and-observations, quantitative comparison

Spectra of
(a) kinetic energy of
2.5-layer shallow-water
model in North-Atlantic-
shaped basin; and
(b) Cooperative Ocean-
Atmosphere Data Set
(COADS) Gulf-Stream
axis data



More spatio-temporal data

Multi-channel SSA analysis of the UK Met Office monthly mean SSTs for the century-long 1895–1994 interval

Marked similarity with the 7–8-year “gyre mode” of a full hierarchy of ocean models, on the one hand, and with the North Atlantic Oscillation (NAO), on the other: explanation?

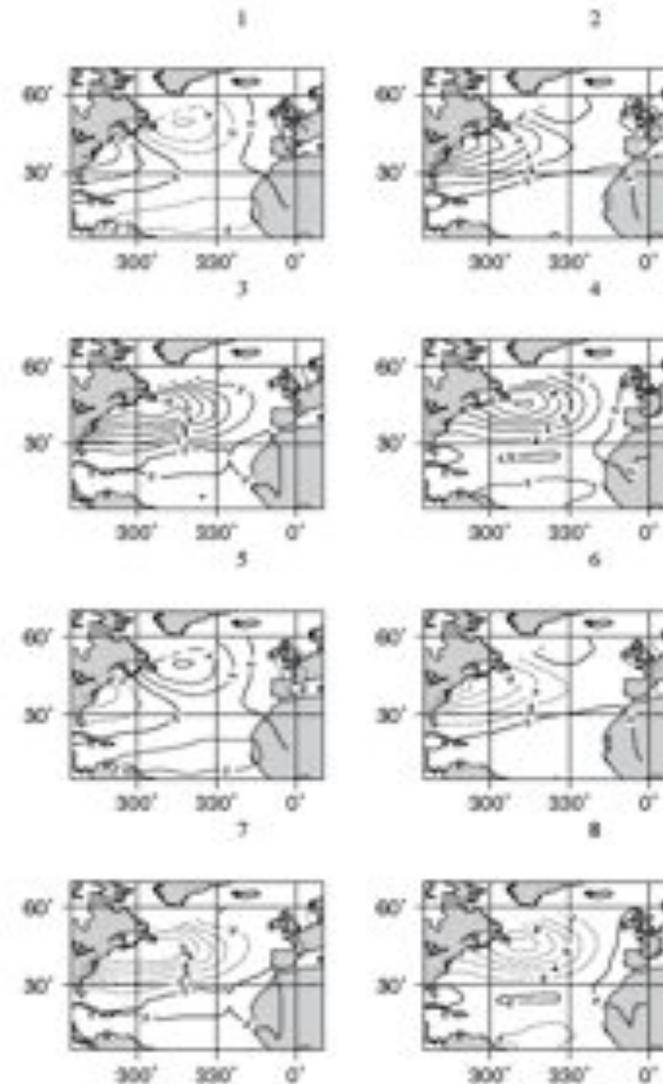
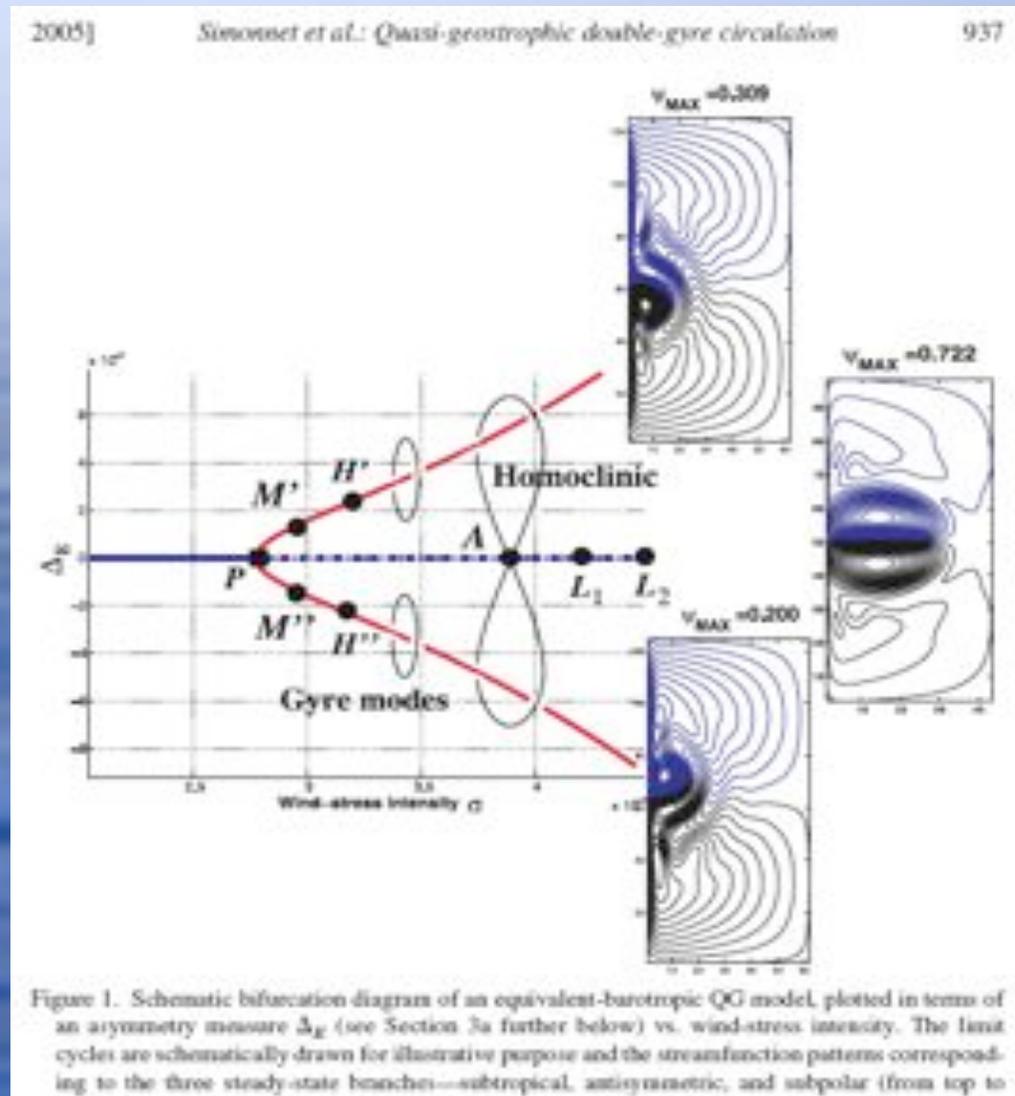


Figure 8. Phase composites of the reconstructed 7–8-year SST oscillation. The MSSA window length is 40 year and the contour interval is 0.02°C.

Global bifurcations in “intermediate” models

Bifurcation tree in a QG, equivalent-barotropic, high-resolution (10 km) model: pitchfork, mode-merging, Hopf, and homoclinic



Homoclinic orbit: numerical and analytical

2005]

Simonnet et al.: Quasi-geostrophic double-gyre circulation

939

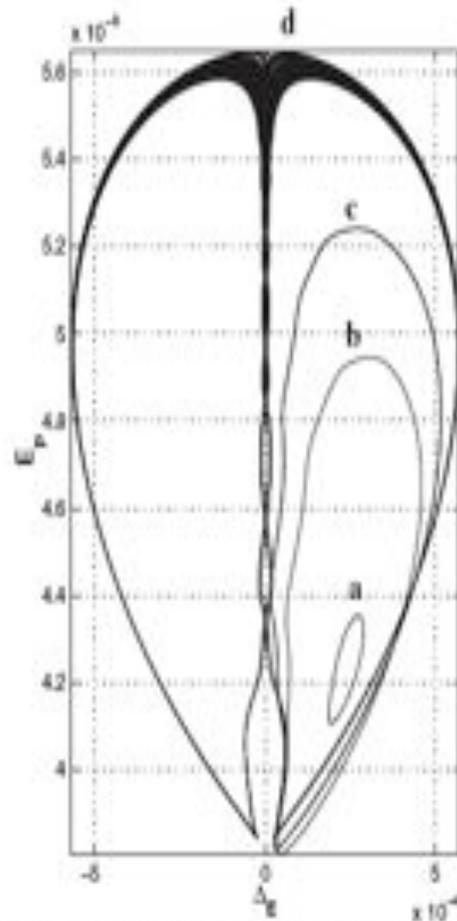


Figure 2. Unfolding of the relaxation oscillations induced by the gyre modes, shown in the plane spanned by the total potential energy of the solution E_p and the difference Δ_E between the subpolar potential energy and the subtropical one (see text for details). The orbits of several limit cycles are

2005]

Simonnet et al.: Quasi-geostrophic double-gyre circulation

941

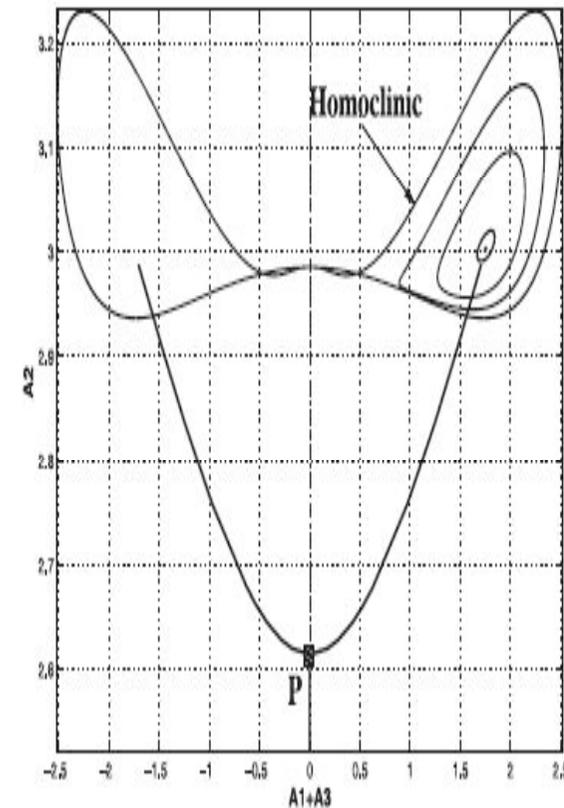


Figure 3. Bifurcation diagram of the highly truncated, four-mode model (5), projected onto the $(A_1 + A_3, A_2)$ plane for $\mu = 1$ and $s = 2$; P stands for pitchfork bifurcation at $\sigma = \sigma_p = 7.61$, while $\sigma = \sigma_{hc} \approx 10.4299$ at the homoclinic bifurcation. The branches of periodic orbits are replaced by several explicitly computed limit cycles.

Uncertainties in the forcing

Contributions to the forcing, natural and **anthropogenic**, also have substantial uncertainties

Source : IPCC (2001),
TAR, WGI, SPM

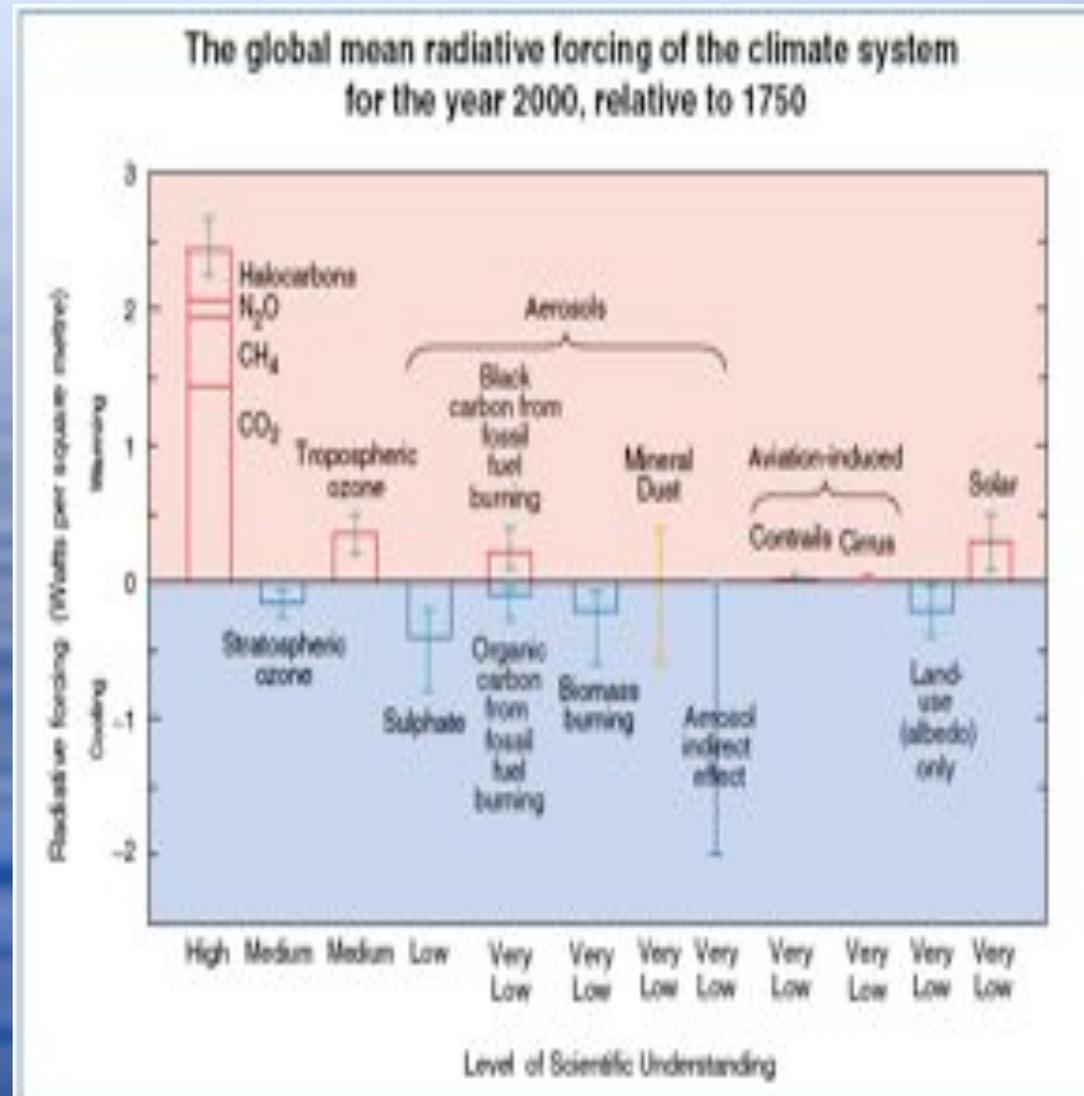


Figure 3: Many external factors force climate change.

So what's it gonna be like, by 2100?

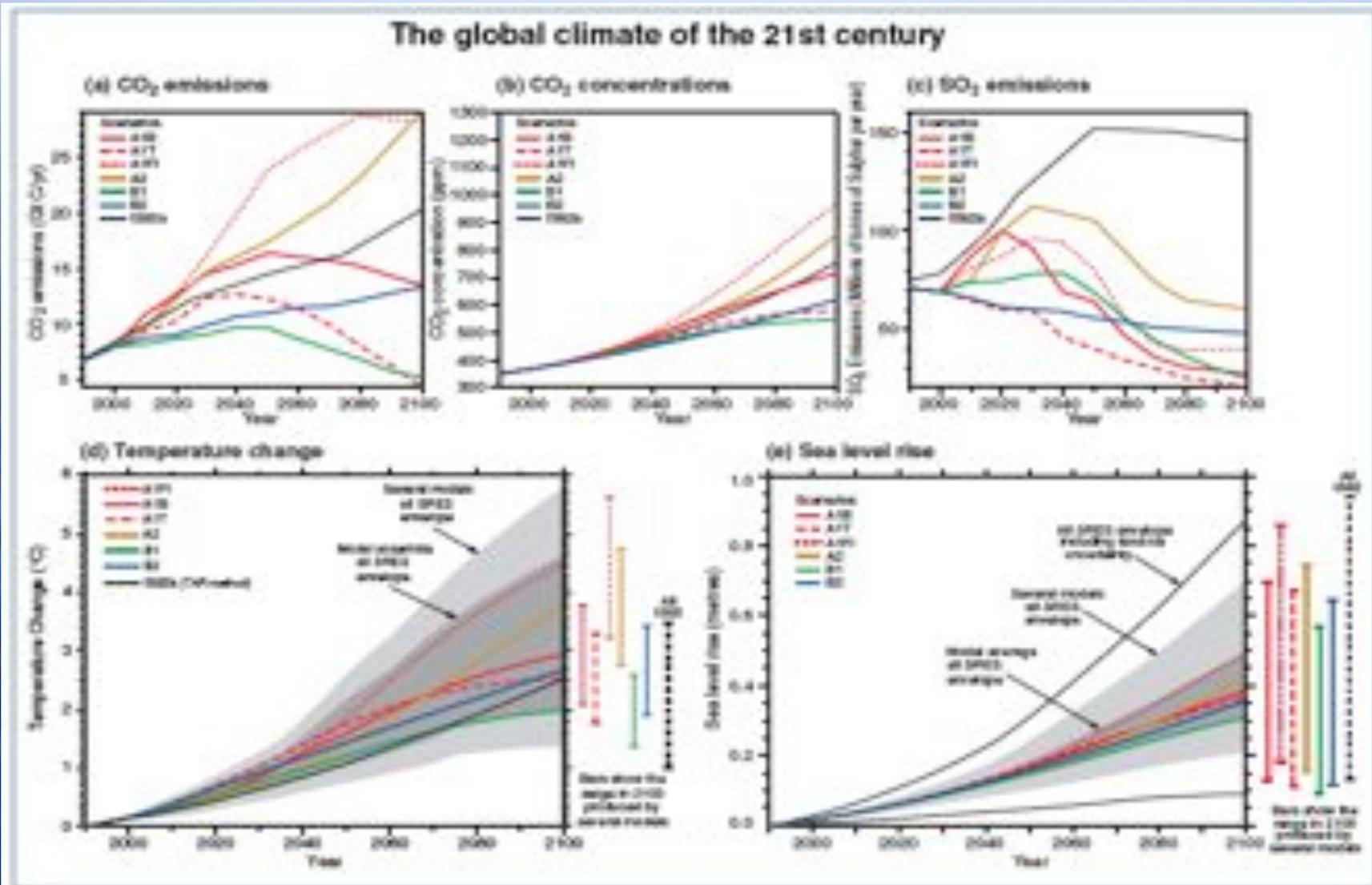


Figure 5: The global climate of the 21st century will depend on natural changes and the response of the climate system to human activities.

Can we, nonlinear people, help?

The uncertainties
might be *intrinsic*,
rather than mere
“tuning problems”

If so, maybe
*stochastic structural
stability* could help!

Might fit in nicely with
recent taste for
“stochastic
parameterizations”

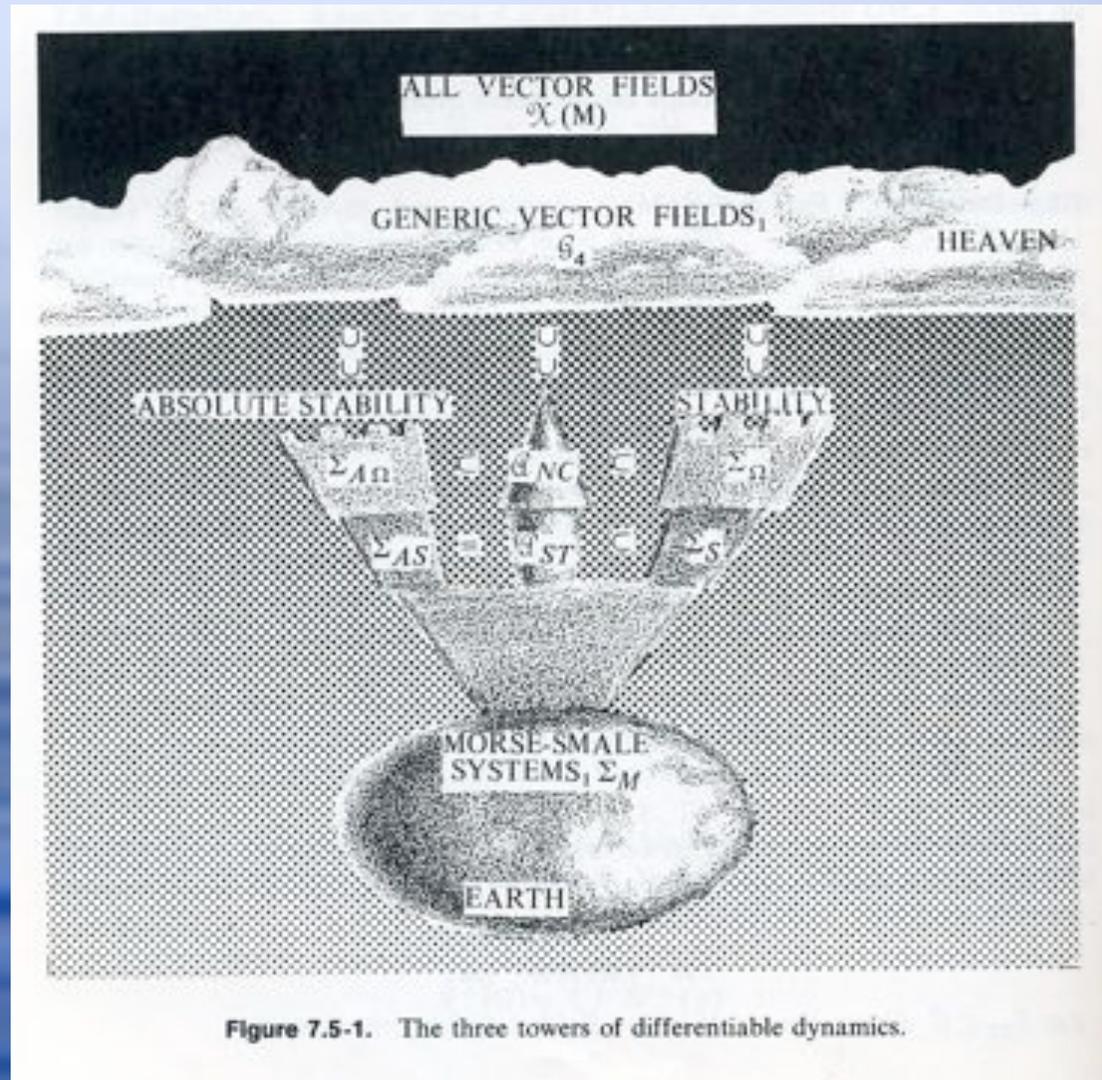


Figure 7.5-1. The three towers of differentiable dynamics.

The DDS dream of structural stability (from Abraham & Marsden, 1978)

Random Dynamical Systems – RDS

- This theory is a combination of measure (probability) theory and dynamical systems, initiated by the "Bremen group" (L. Arnold, 1998).

Random Dynamical Systems – RDS

- This theory is a combination of measure (probability) theory and dynamical systems, initiated by the "Bremen group" (L. Arnold, 1998).
- It allows one to treat stochastic differential equations (SDEs), and more general systems driven by some "noise", as flows.

The setting of RDS theory

- A phase space X . **Example:** \mathbb{R}^n .

The setting of RDS theory

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- A probability space $(\Omega, \mathcal{F}, \mathbb{P})$.
Example: The Wiener space $\Omega = C_0(\mathbb{R}; \mathbb{R}^n)$ with Wiener measure $\mathbb{P} = \gamma$.

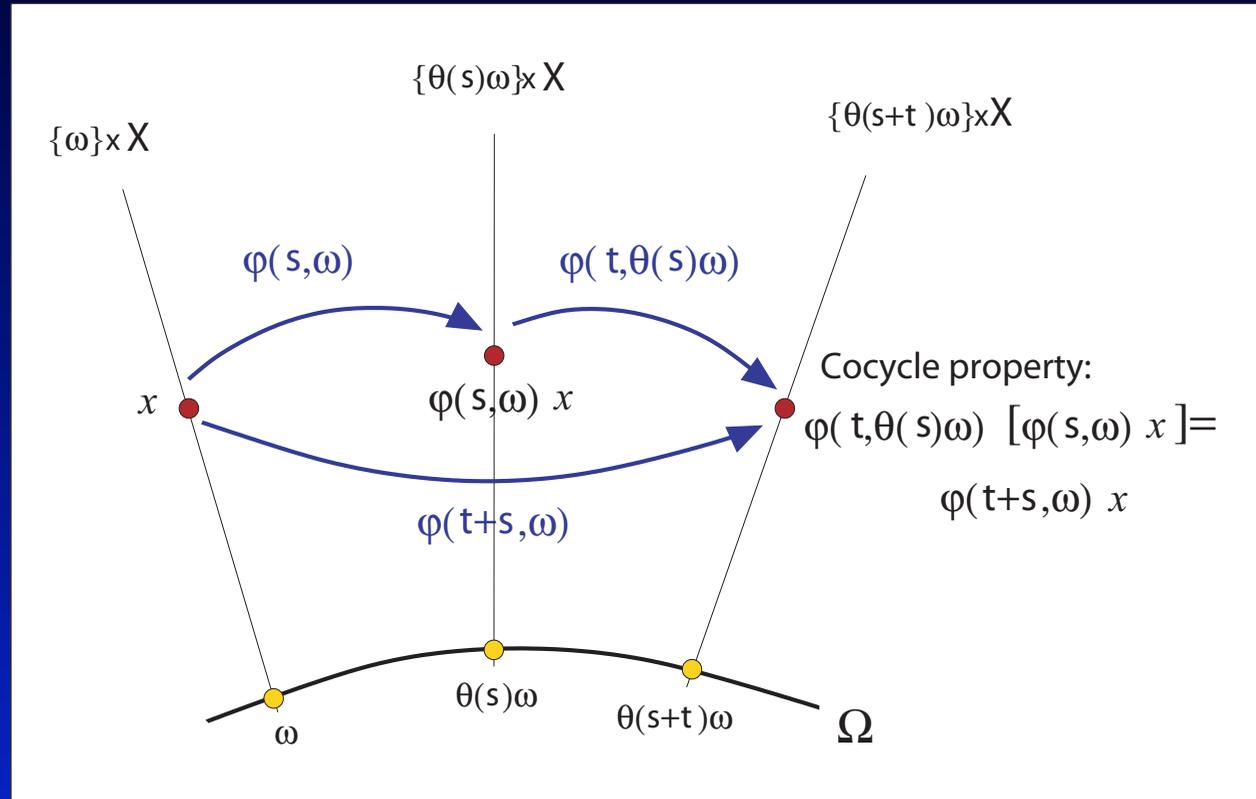
The setting of RDS theory

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Example: The Wiener space $\Omega = C_0(\mathbb{R}; \mathbb{R}^n)$ with Wiener measure $\mathbb{P} = \gamma$.
- A model of the noise $\theta(t) : \Omega \rightarrow \Omega$ that preserves the measure \mathbb{P} , i.e. $\theta(t)\mathbb{P} = \mathbb{P}$; θ is called the driving system.
Example: $W(t, \theta(s)\omega) = W(t + s, \omega) - W(s, \omega)$; it starts the noise at s instead of $t = 0$.

The setting of RDS theory

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Example: $W(t, \theta(s)\omega) = W(t+s, \omega) - W(s, \omega)$; it starts the noise at s instead of $t = 0$.
- A mapping $\varphi : \mathbb{R} \times \Omega \times X \rightarrow X$ with the cocycle property.
Example: The solution of an SDE.

RDS – A geometric view of SDEs



- φ is a random dynamical system (RDS)
- $\Theta(t)(x, \omega) = (\theta(t)\omega, \varphi(t, \omega)x)$ is a flow on the bundle

Stochastic equivalence

Toward a robust classification

A tool for classification: stochastic conjugacy

- **Stochastic conjugacy:** two cocycles $\varphi_1(t, \omega)$ and $\varphi_2(t, \omega)$ are conjugated iff there exists a *random homeomorphism* $h \in \text{Homeo}(X)$ and an invariant set $\tilde{\Omega}$ of full \mathbb{P} -measure (w.r.t. θ) such that $h(\omega)(0) = 0$ and

$$\varphi_1(t, \omega) = h(\theta(t)\omega)^{-1} \circ \varphi_2(t, \omega) \circ h(\omega);$$

h is also called a **cohomology** of φ_1 and φ_2 :
it is a *random change of variables*!

Stochastic equivalence (continued)

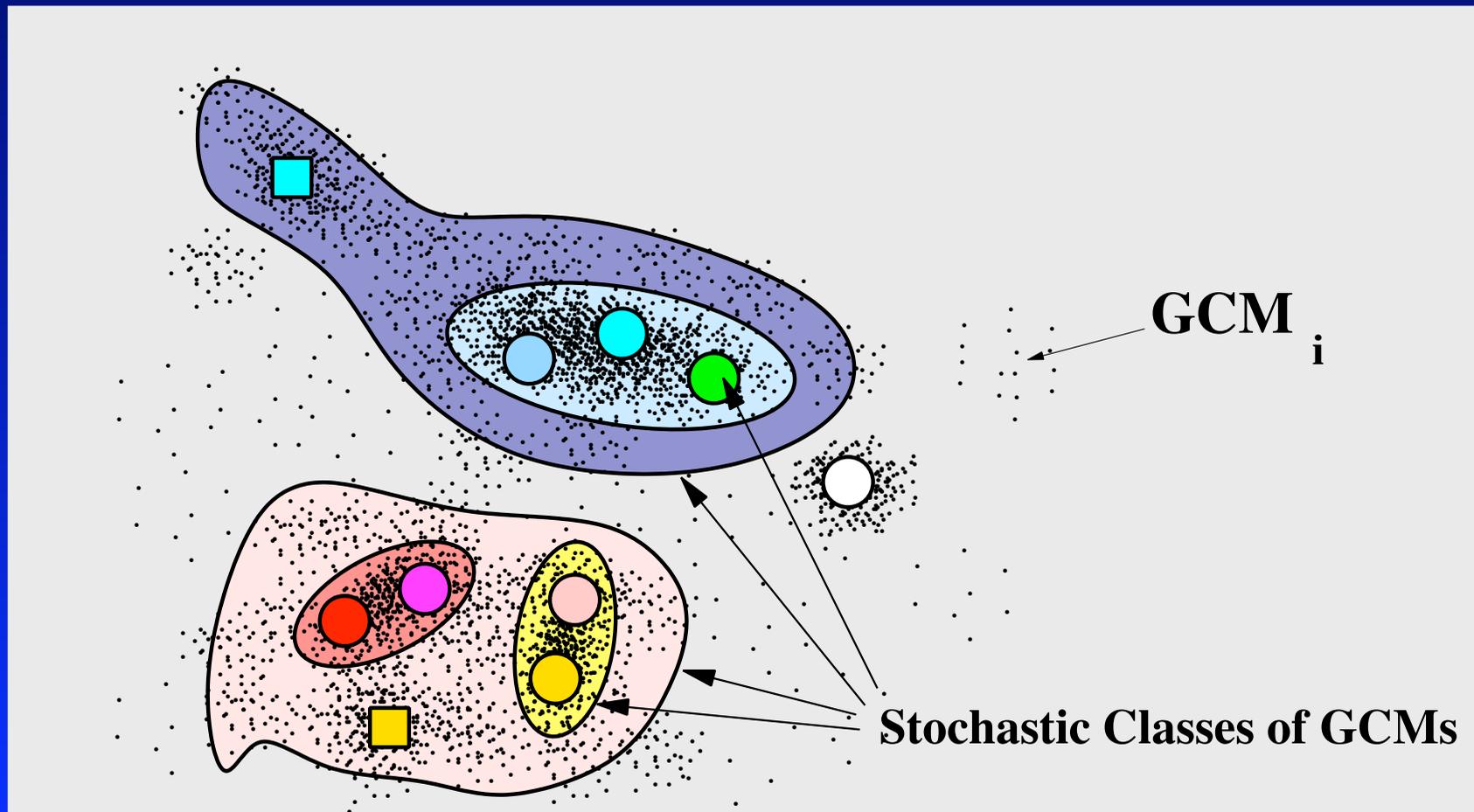
- *Motivation:* We would like to measure quantitatively the difference between **climate models**.

As the noise variance tends to zero and/or the parametrizations are switched off, one recovers the structural instability, as a “**granularity**” of model space.

For **nonzero variance**, the random attractor $\mathcal{A}(\omega)$ associated with several GCMs might fall into **larger** and **larger** classes as the **noise level increases**.

Stochastic equivalence (continued)

Could noise help the classification?



RDS – Concluding remarks

- Difference between models

$$(\text{GCM} - \text{team})_1 : dU = f_1(U)dt + \sigma_1(x, U)dW_t$$

$$(\text{GCM} - \text{team})_2 : dU = f_2(U)dt + \sigma_2(x, U)dW_t$$

Under which conditions on $f_1 - f_2$ and $\sigma_1 - \sigma_2$ will $\mathcal{A}_1(\omega) \approx \mathcal{A}_2(\omega)$ hold?

- Increase in resolution

Let k denote the GCM resolution $dU = f(U, \theta(t)\omega, k)dt$.
One would like to study the behavior of $\mathcal{A}_k(\omega)$ as $k \rightarrow 0$.

- Model validation with data

- Joint analysis of model simulations and observational data sets.

- Parameter estimation, based on data assimilation methods (sequential, variational).

Some conclusions &/or questions

What do we know?

- It's getting warmer.
- We do contribute to it.
- So, we should act as best we know and can!

What do we know less well?

- How does the fluid dynamics of the climate system really work?
- How does natural variability interact with anthropogenic forcing?

What to do?

- Better understand the system and its forcings.
- Better understand the effects on economy and society, and vice-versa.
- Explore the models', and system's, stochastic structural stability.

Some general references

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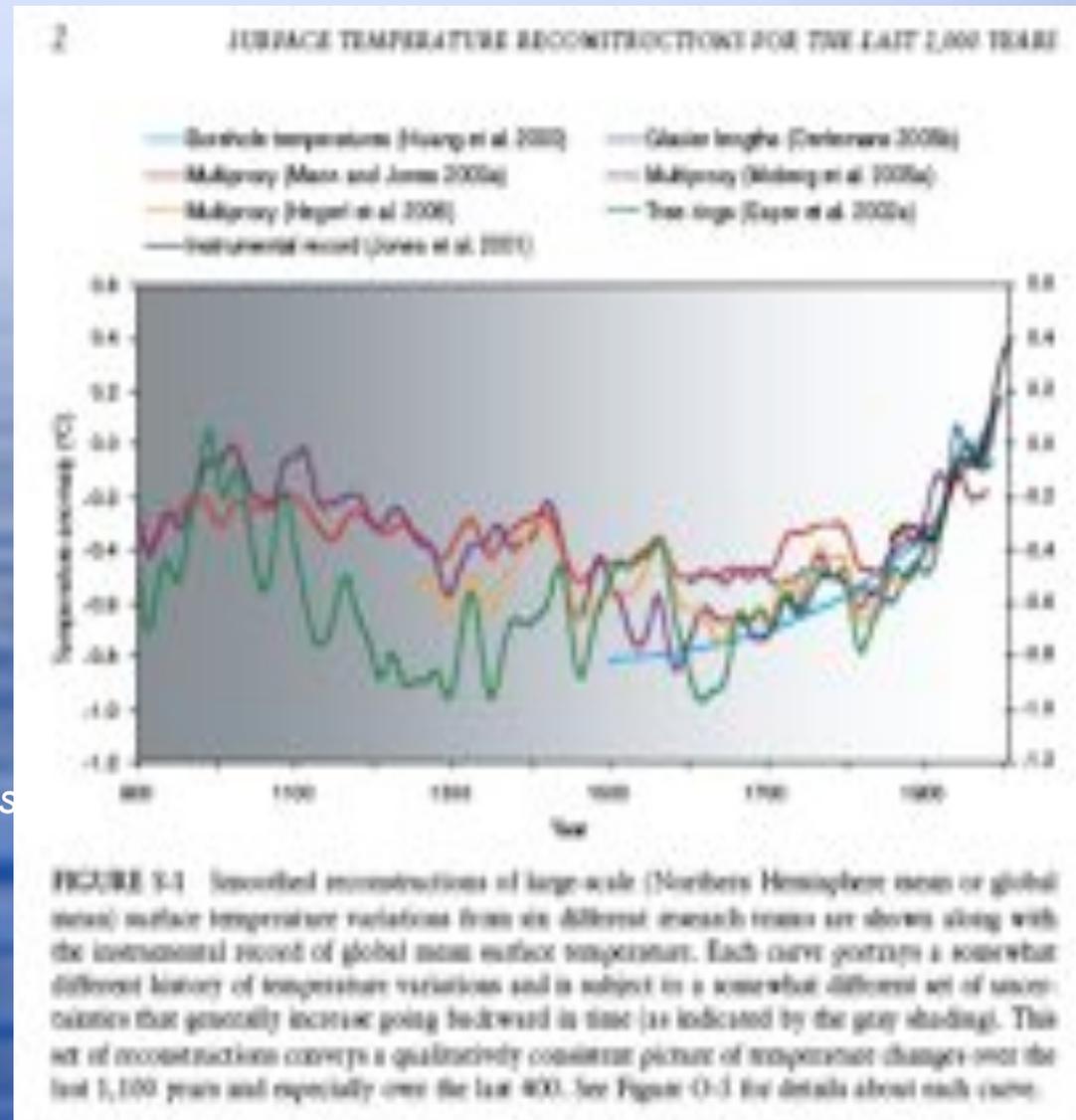
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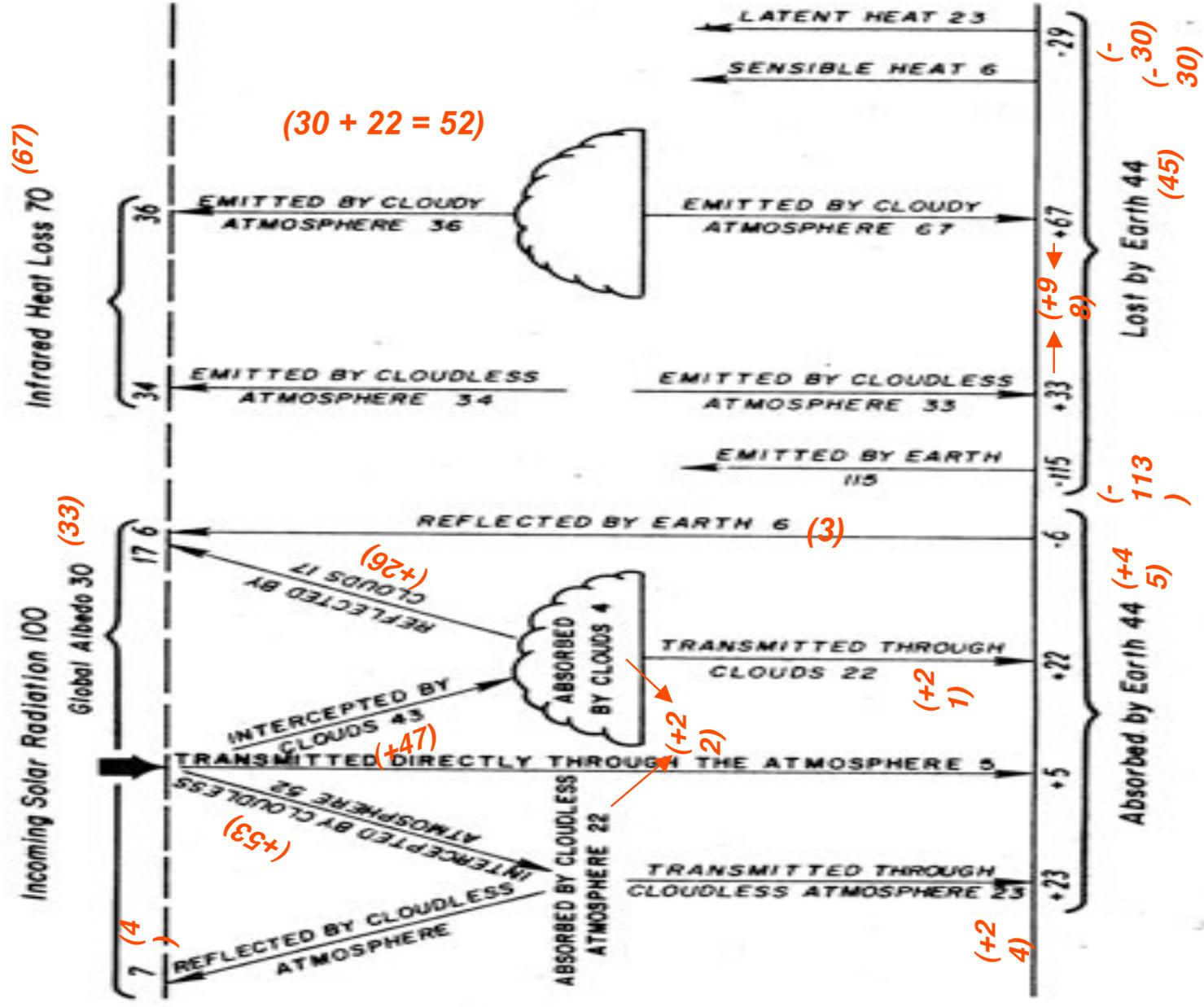
Reserve slides

The “hockey stick” & beyond

The “hockey stick” of TAR (3rd Assessment Report) is a typically (over)simplified version of much more detailed and reliable knowledge.

National Research Council, 2006:
Surface Temperature Reconstructions For the Last 2000 Years.
National Academies Press,
Washington, DC, 144 pp.
http://www.nap.edu/openbook.php?record_id=11676&page=2



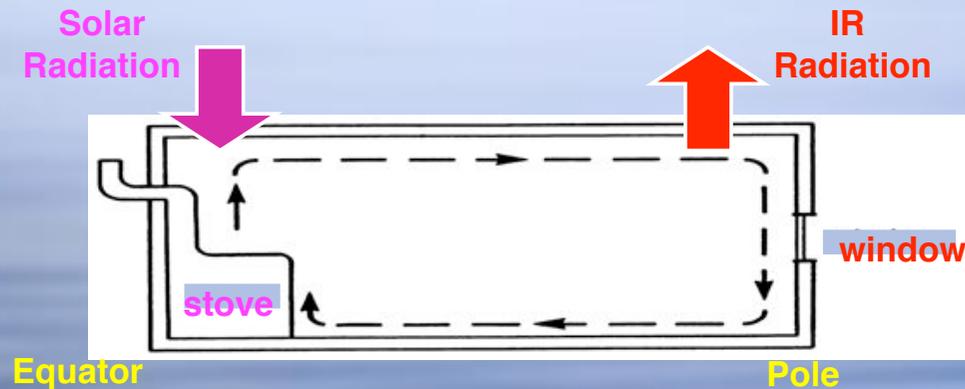


Valeurs en rouge: cf. figure précédente.

D'après Kuo-Nan Liou, 1980: *An Introduction to Atmospheric Radiation* (fig. 8.19)

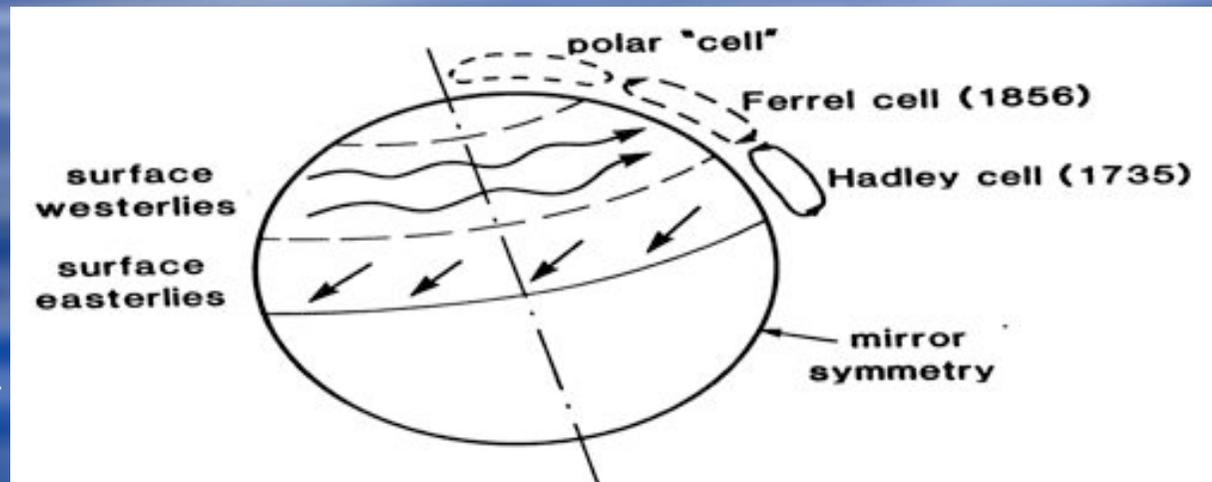
THE GENERAL ATMOSPHERIC CIRCULATION – Schematic Diagrams

Hadley Circulation (direct)



The general circulation of the atmosphere, cross-section. *

The observed mean circulation is made up of the Hadley, Ferrel and polar cells; these are complemented by many other structures (monsoons, semi-permanent “centers of action, etc.).



The general circulation of the atmosphere, perspective view. *

*after Ghil & Childress (1987), Ch. 4

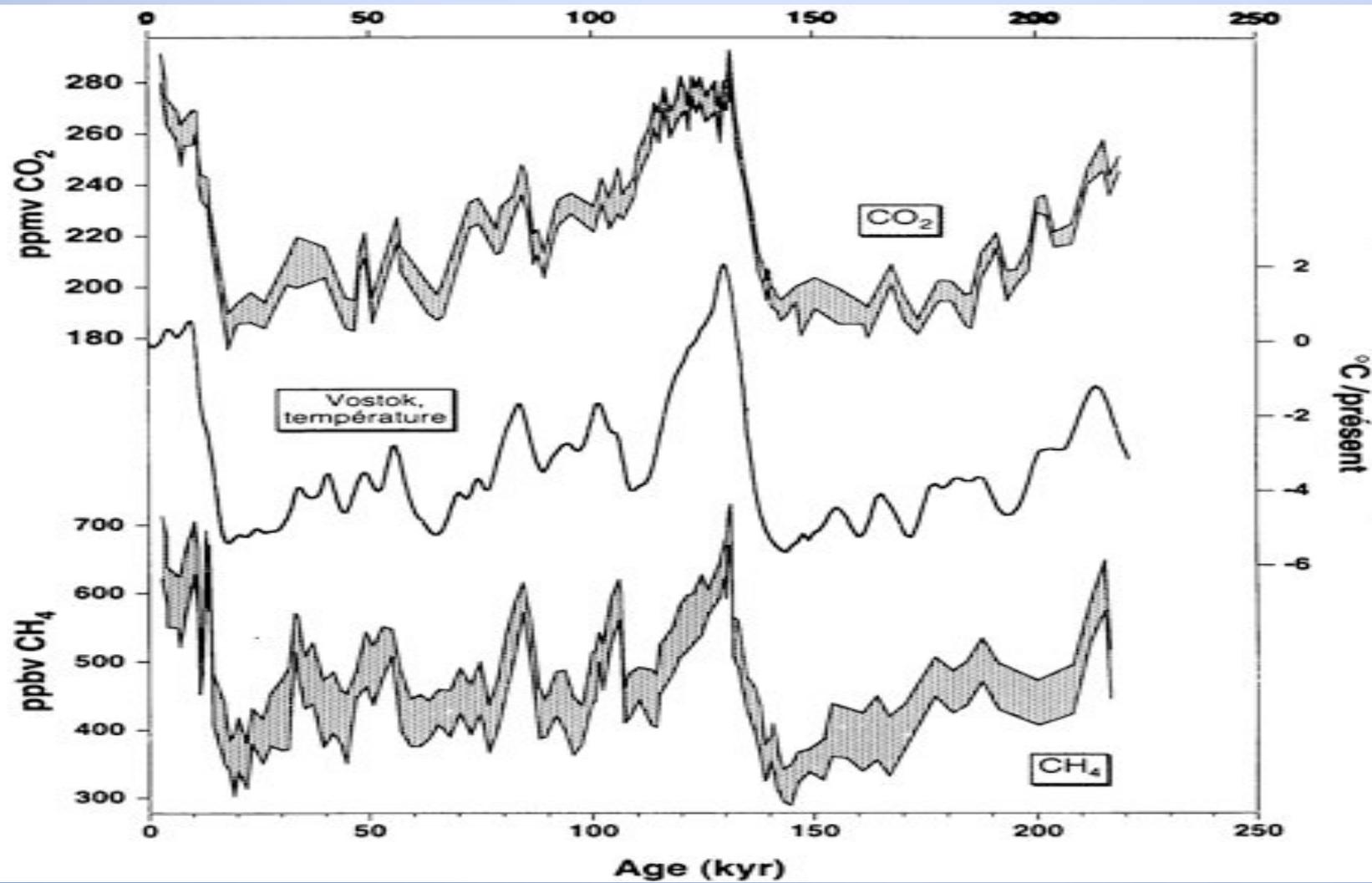
Modeling Hierarchy for the Oceans

Ocean models

- ◆ 0-D: box models – chemistry (BGC), paleo
- ◆ 1-D: vertical (mixed layer, thermocline)
- ◆ 2-D – meridional plane – THC
 - also 2.5-D: a little longitude dependence
 - horizontal – wind-driven
 - also 2.5-D: reduced-gravity models (n.5)
- ◆ 3-D: OGCMs - simplified
 - with bells & whistles (“kitchen sink”)

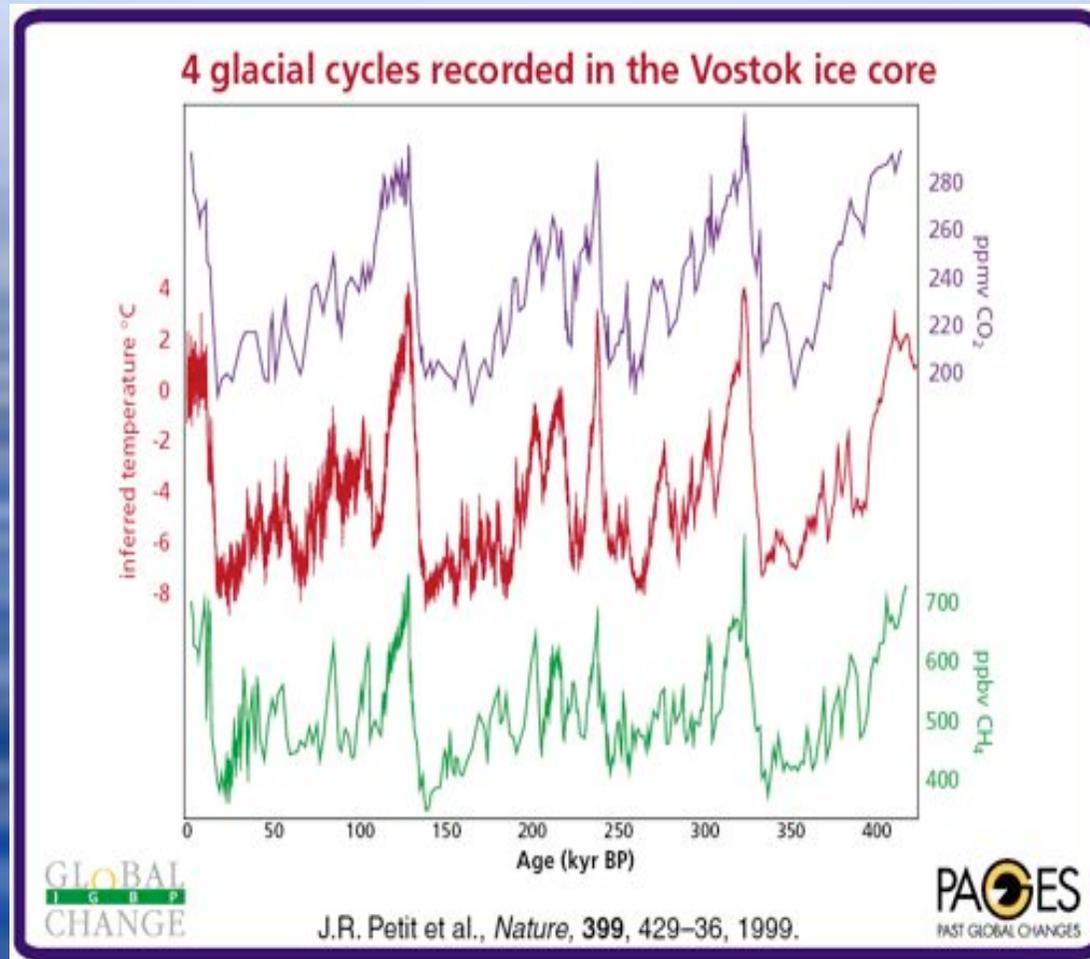
Coupled 0-A models

- ◆ Idealized (0-D & 1-D): intermediate couple models (ICM)
- ◆ Hybrid (HCM) - diagnostic/statistical atmosphere
 - highly resolved ocean
- ◆ Coupled GCM (3-D): CGCM



Isotopic (proxy) temperatures and GHGs at Vostok, over the last glacial cycle; courtesy of P. Yiou

T_s and GHGs over 400 kyr



The same lead-lag relations are apparent over these 4 glacial cycles ...



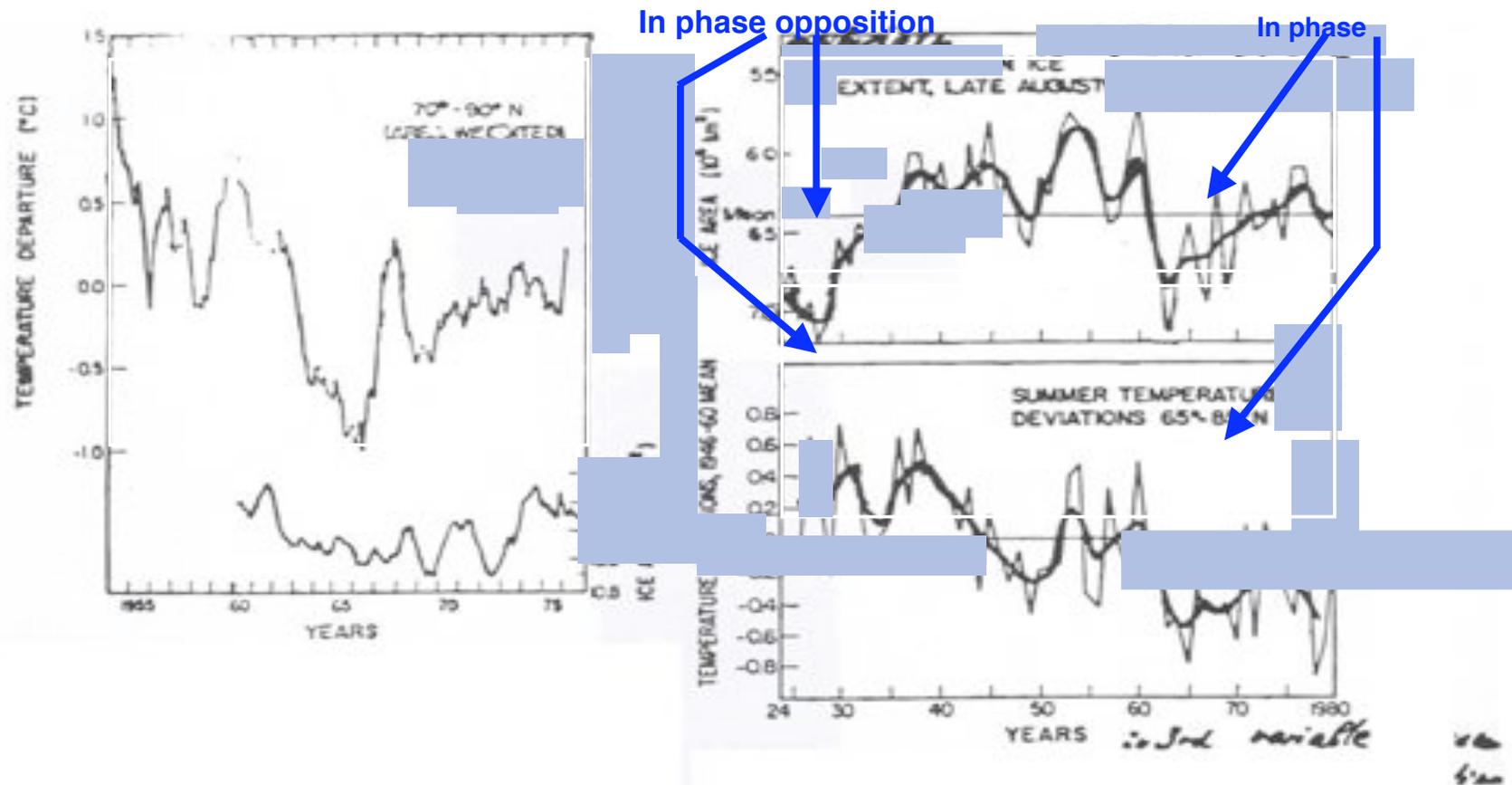
The Glacier des Bossons, under the Mont Blanc

Temperate valley glaciers obey complex dynamics, due to their hydrologic budget and nonlinear flow rheology.

This is true, a fortiori,
of polar ice sheets!

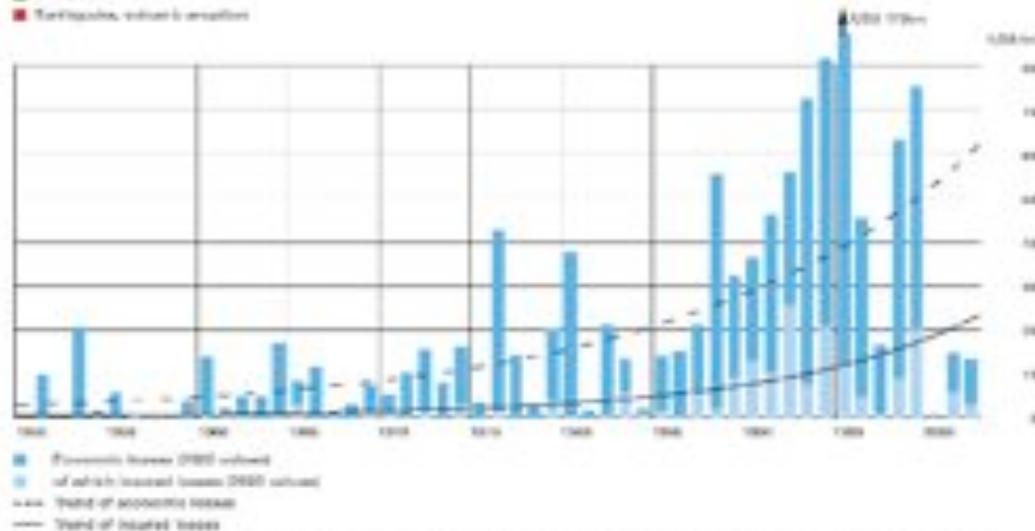
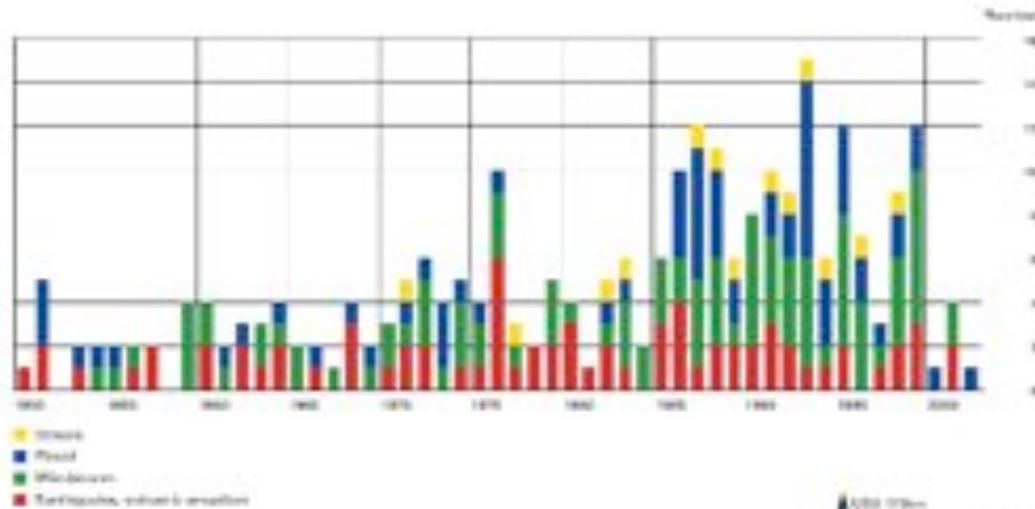


Ice cover of the Arctic Ocean and subpolar seas



Ice cover of the Arctic Ocean at the end of August (above) and summer temperature deviations w.r. to the 1940–1960 mean (below). The heavy curve is a 5-year running mean; after Barry (1983).

Great Natural Catastrophes 1950–2003



Number and cost of major natural catastrophes, by year and type of event (from Munich Re, Topics geo 2003)

Extreme Events: Causes and Consequences (E2-C2)

- **EC-funded project** bringing together researchers in **mathematics, physics, environmental and socio-economic sciences**.
- **€1.5M** over three years (March 2005–Feb. 2008).
- **Coordinating institute:** Ecole Normale Supérieure.
- **17 'partners' in 9 countries.**
- **72 scientists + 17 postdocs/postgrads.**
- **PEB:** M. Ghil (ENS, Paris, P.I.), S. Hallegatte (CIRED), B. Malamud (KCL, London), A. Soloviev (MITPAN, Moscow), P. Yiou (LSCE, Gif s/Yvette, Co-P.I.)



Belgium

France

Germany

Italy

Luxembourg

Romania

Russia

UK

USA

Sun-Climate Relations

- It ain't new:
v. ~1000
papers (in
1978!), as well
as Marcus *et al.*
(1998, *GRL*).
- “Corrélation
n'est pas
raison.”
- Requires
serious study of
solar physics.

Climatology Supplement

Nature 278, 348–352 (23 November 1978), doi:10.1038/278348a0

Solar–terrestrial influences on weather and climate

GEORGE L. SIOGGE

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During the past century over 1,000 articles have been published claiming or refuting a correlation between some aspect of solar activity and some feature of terrestrial weather or climate. Nevertheless, the sense of progress that should attend such an outpouring of 'results' has been absent for most of this period. The problem all along has been to separate a suspected Sun–weather signal from the characteristically noisy background of both systems. The present decade may be witnessing the first evidence of progress in this field. Three independent investigations have revealed what seem to be well resolved Sun–weather signals, although it is still too early to have unreserved confidence in all cases. The three correlations are between terrestrial climate and Maunder Minimum-type solar activity variations, a regional drought cycle and the 22-yr solar magnetic cycle, and winter hemisphere atmospheric circulation and passages by the Earth of solar sector boundaries in the solar wind. The apparent emergence of clear Sun–weather signals stimulated numerous searches for underlying physical causal links.

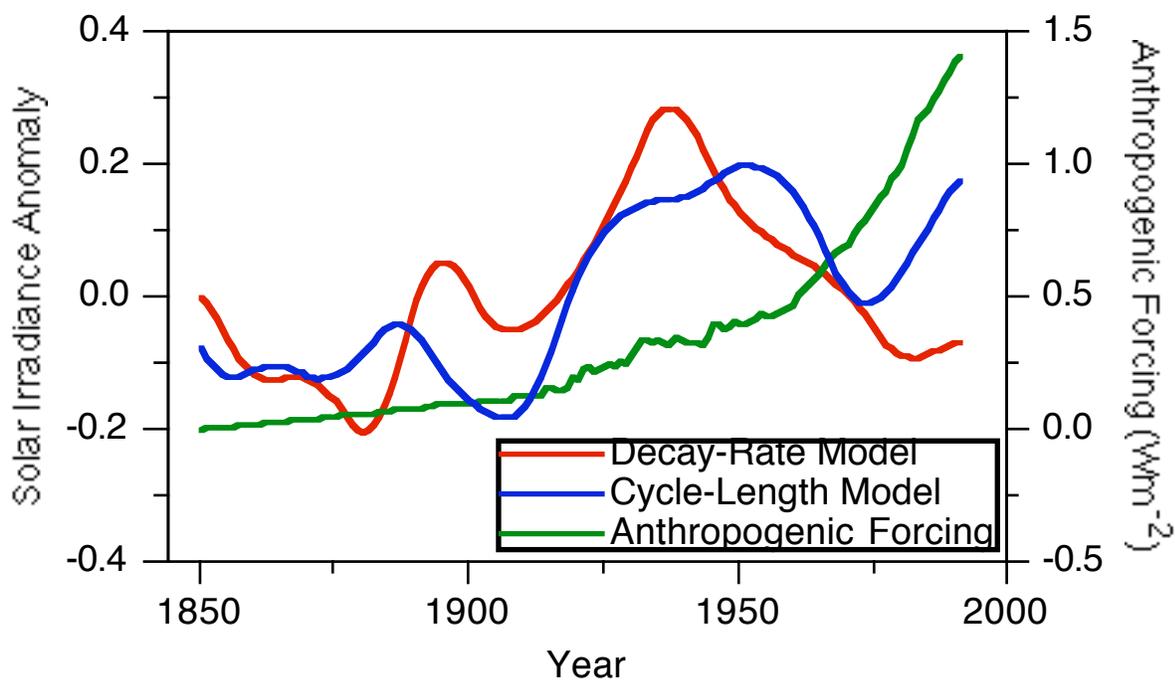
Solar Effects on Interdecadal Climate Variability

Irradiance Anomaly: $dw/dt + w/\tau = k F(t)$

($t_m \sim$ time of solar min.; $t_M \sim$ solar max.; $\tau \sim L^2/\nu$)

1) Cycle-Length (CL) Model: $F_2(t_{m+1/2}) = k_2 / (t_{m+1} - t_m)$

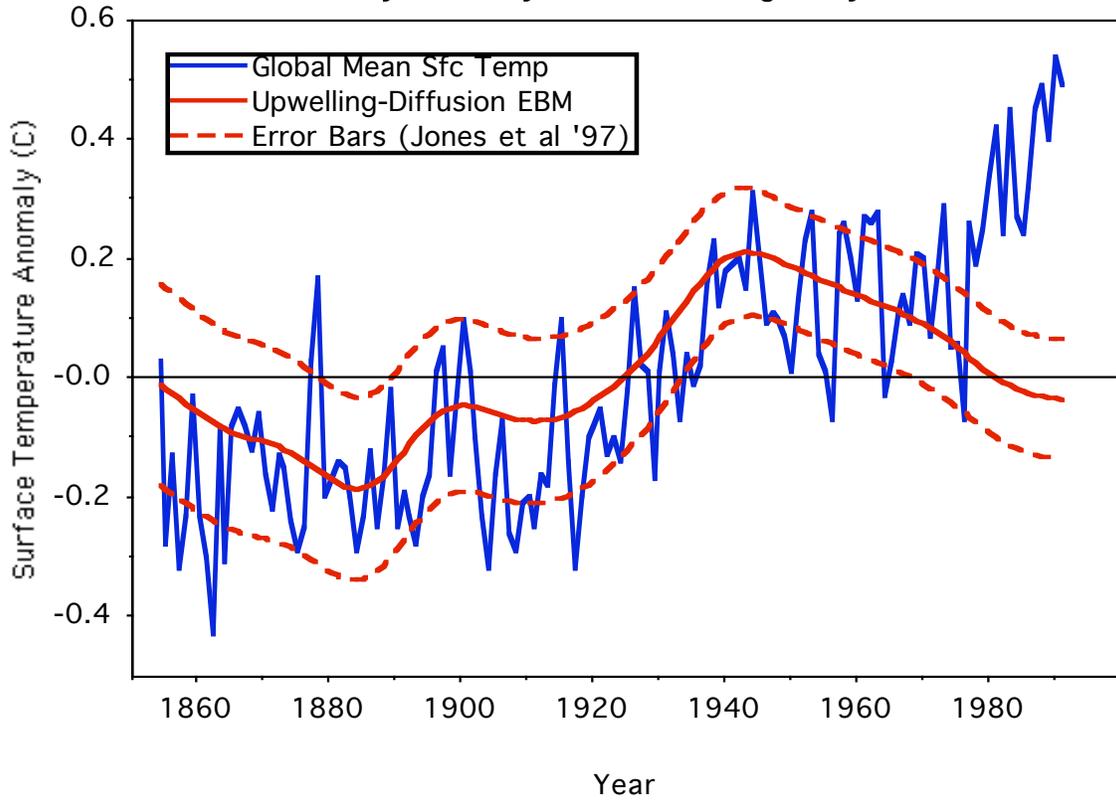
2) Cycle Decay-Rate (CD) Model: $F_1(t_m) = k_1 / (t_m - t_M)$



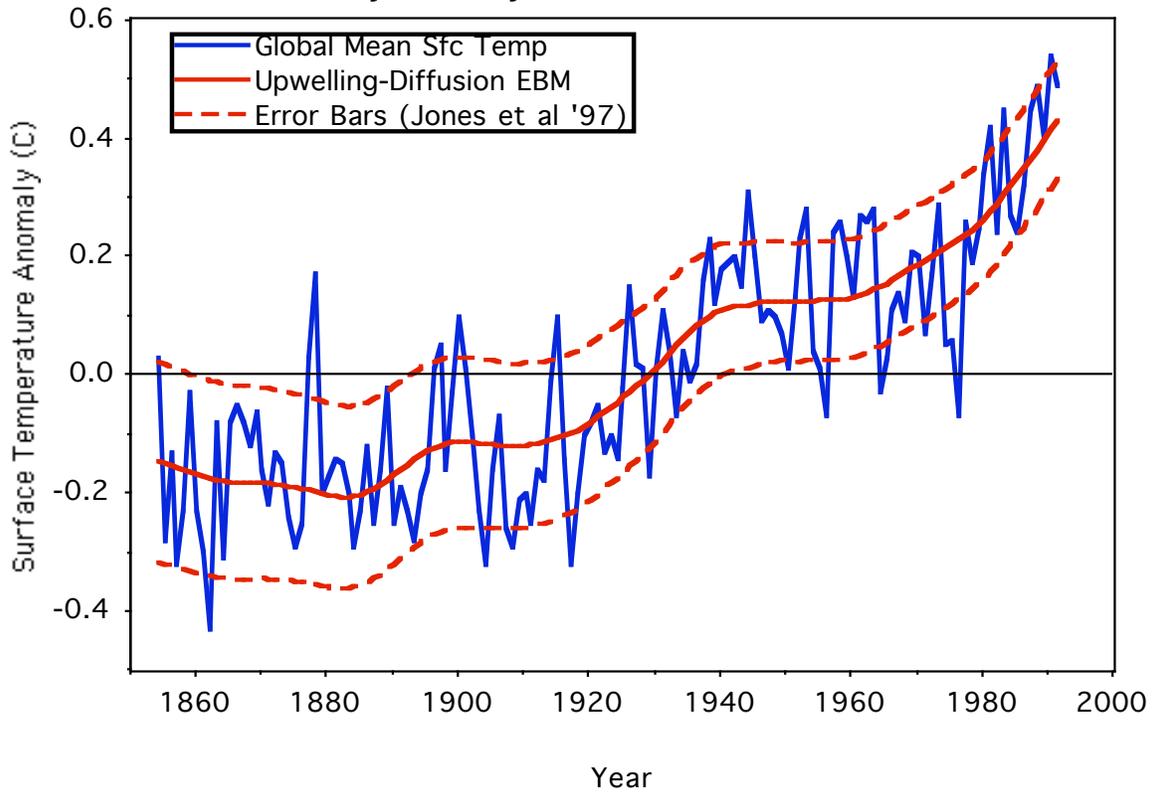
Climate Model: Global energy-balance model, with upwelling-diffusion ocean model underneath (cf. IPCC)

S. L. Marcus, M. Ghil, and K. Ide, *Geophys. Res. Lett.*, **26** 1449-1452, 1999

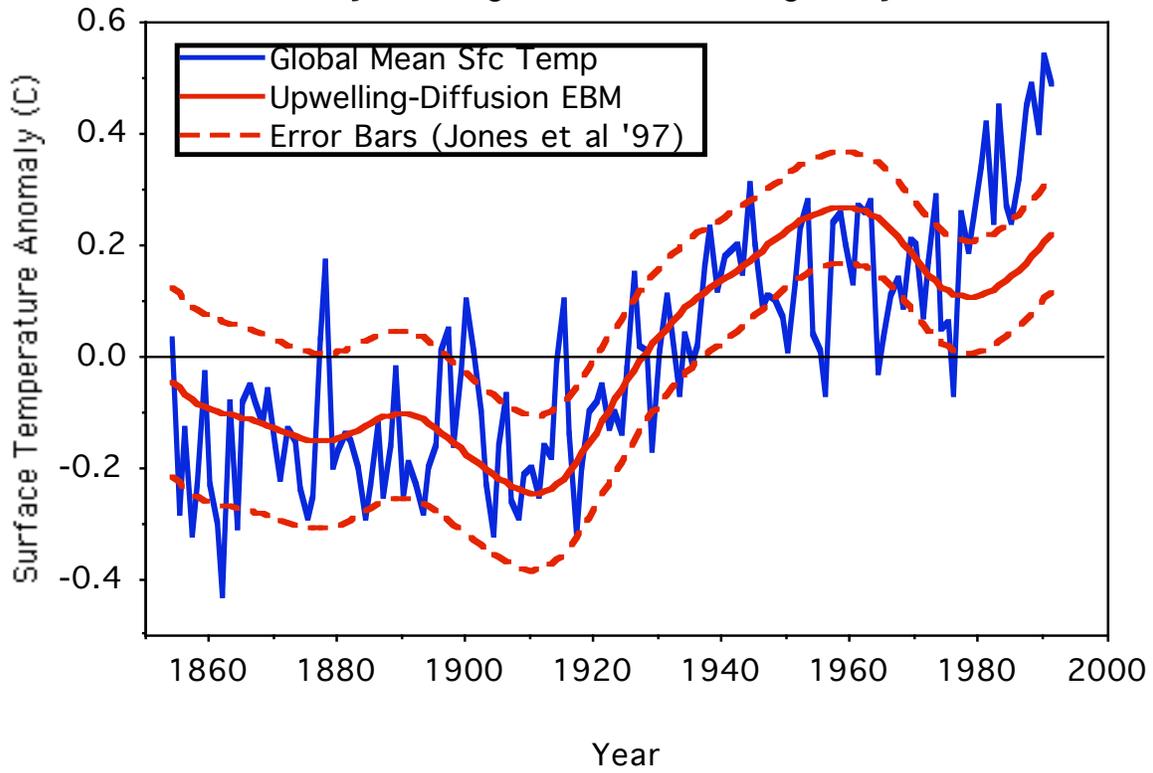
Cycle-Decay: Solar Forcing Only



Cycle-Decay: Solar + GHG/Sulfate



Cycle-Length: Solar Forcing Only



Cycle-Length: Solar + GHG/Sulfate

