MODELLING SUBRESOLUTION SCALES IN N–BODY SIMULATIONS

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- Motivation
- Theoretical background: Small-Size Expansion
- N-body simulations:
  - Density and velocity fields
  - Dark–Matter Halos
- Conclusions

MOTIVATION

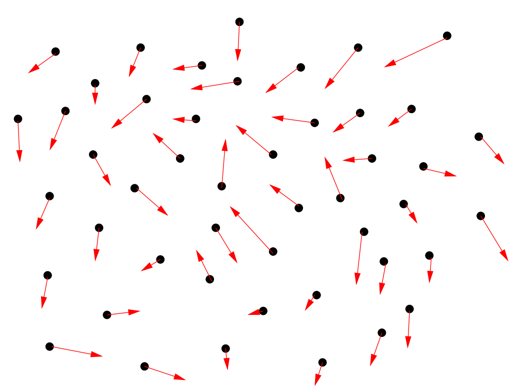
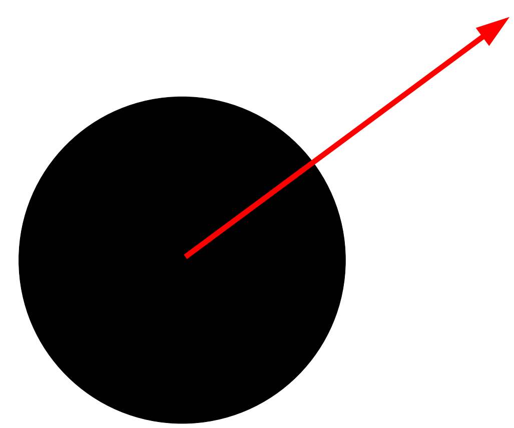
Structure formation in Cold Dark Matter by gravitational instability:
self–gravitating, collisionless gas $\Rightarrow$ Vlasov–Poisson eq. for $f(\mathbf{x}, \mathbf{u}, t)$

DESCRIPTION IS TOO DETAILED

1. Observational access to density and velocity fields only with a finite resolution L $\rightarrow$ hydrodynamic–like eqs. for $\varrho(\mathbf{x}, t; L)$ and $\mathbf{u}(\mathbf{x}, t; L)$?

2. Limited resolution of numerical simulations

   N-body simulations purport to integrate Vlasov–Poisson numerically

   however:  $\rightsquigarrow$ 

   N-body particles = coarse, Lagrangian particles

   $\rightarrow$ what are the evolution eqs.? (influence of subresolution scales)

THEORETICAL BACKGROUND

Newtonian evolution eqs. in an expanding background

for the fields ϱ, $\mathbf{u}$	for the particles $\alpha = 1, \ldots, N$
$\frac{\partial \varrho}{\partial t} + \frac{1}{a}\nabla \cdot \mathbf{u} + 3H\varrho = 0$	$\dot{\mathbf{x}}_\alpha = \frac{1}{a}\mathbf{u}_\alpha$
$\frac{\partial \mathbf{u}}{\partial t} + \frac{1}{a}(\mathbf{u}\cdot\nabla)\mathbf{u} + H\mathbf{u} = \mathbf{w} + \mathbf{C}(\mathbf{x}, t)$	$\dot{\mathbf{u}}_\alpha + H\mathbf{u}_\alpha = \mathbf{w}(\mathbf{x}_\alpha, t) + \mathbf{C}(\mathbf{x}_\alpha, t)$
$\nabla \cdot \mathbf{w} = -4\pi G a\,(\varrho - \varrho_b)$	$\nabla \cdot \mathbf{w} = -4\pi G a\,(\varrho - \varrho_b)$
$\nabla \times \mathbf{w} = \mathbf{0}$	$\nabla \times \mathbf{w} = \mathbf{0}$

$\mathbf{w}(\mathbf{x}, t)$ = gravity by monopolar moment of coarsening cells of size L

$\mathbf{C}(\mathbf{x}, t)$ = dynamical coupling to scales below L

A model of $\mathbf{C}(\mathbf{x}, t)$: Small–Size Expansion (SSE)

A. Domínguez, PRD **62** (2000) 103501

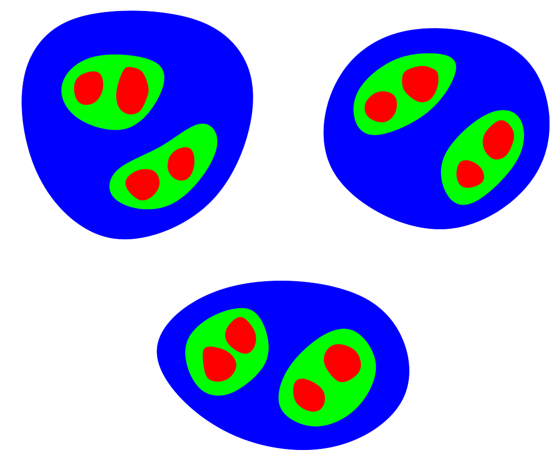
A. Domínguez, MNRAS **334** (2002) 435

T. Buchert & A. Domínguez, A&A **438** (2005) 443

SMALL-SIZE EXPANSION

Bottom-up structure formation $\Rightarrow$ nested matter distribution

Large scales ($\gg L$)
weakly coupled
to small scales ($\ll L$)



Mode–mode coupling estimated
by an expansion in ($L\,\nabla$)
(akin to Large–Eddy Simulations)

$\Downarrow$

$$\mathbf{C} = \frac{B\,L^2}{\varrho}\left\{(\nabla\varrho\cdot\nabla)\mathbf{w} - \frac{1}{a}\nabla\cdot[\varrho(\partial_i\mathbf{u})(\partial_i\mathbf{u})]\right\} + O(L^4) \qquad B = \tfrac{1}{3}\int d\mathbf{y}\; y^2\, W(\mathbf{y})$$

$\Uparrow$ higher than monopole (local tidal forces)

$\Uparrow$ velocity dispersion (local strains)

$W(\cdot)$ = coarsening window

CLOSURE ANSATZ

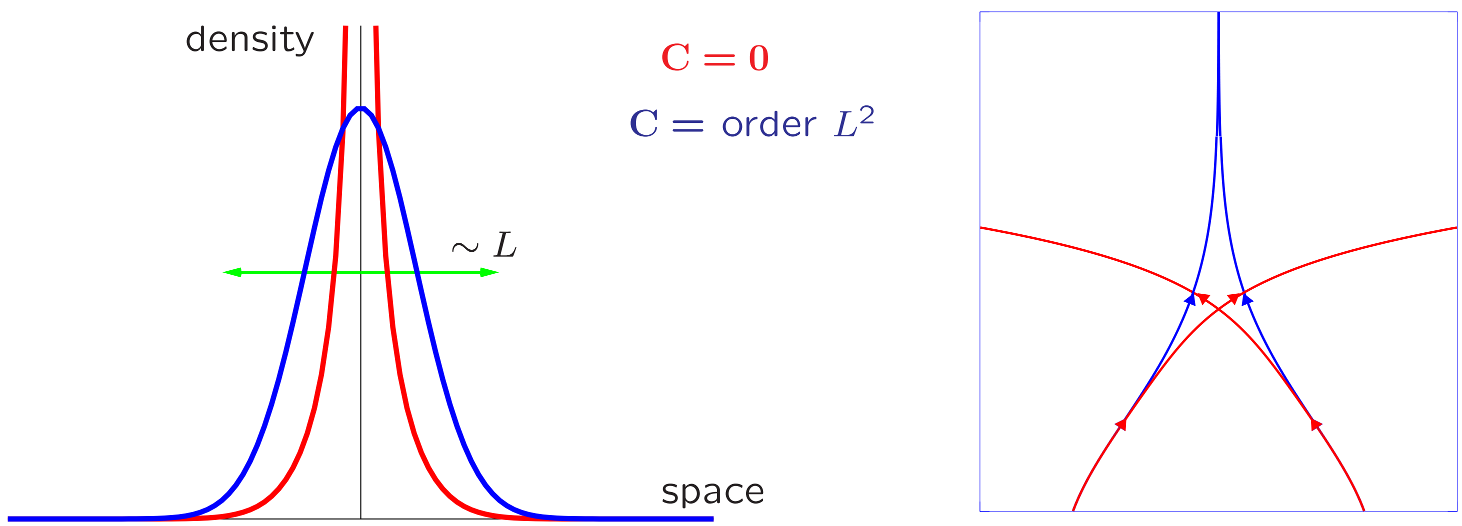
high–orders terms do not add into a relevant contribution so that the expansion can be truncated

SMALL–SIZE EXPANSION

- To order L^0: $\mathbf{C}(\mathbf{x},t) = \mathbf{0} \rightarrow$ dust model, usual N–body simulations

- To order L^2:

$\rightsquigarrow$ adhesion model (in Zel'dovich + locally plane–parallel collapse)



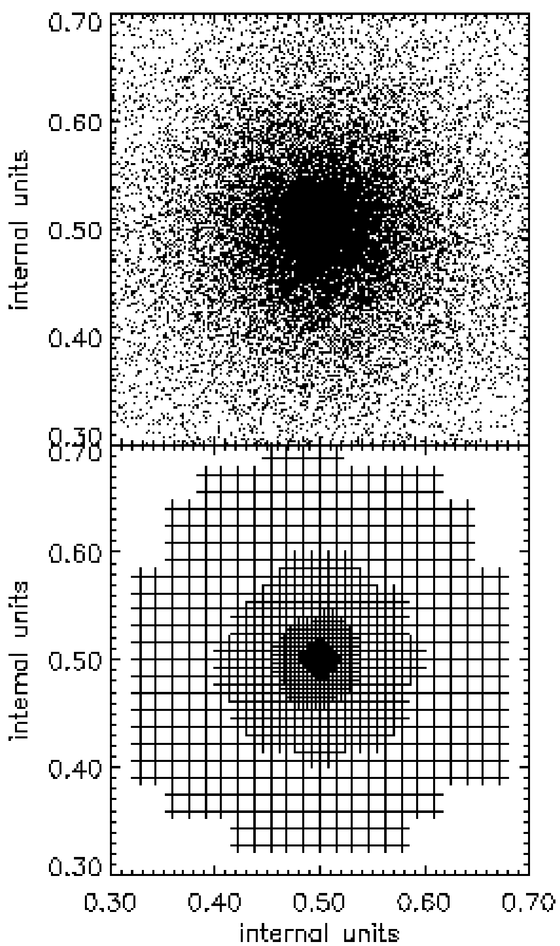
$\rightsquigarrow$ exact result: velocity dispersion acts like a sink of kinetic energy for volume elements collapsing along the three axes

$\rightsquigarrow$ generation of vorticity by tidal torques and shear stretching

MULTILEVEL ADAPTIVE PARTICLE–MESH (MLAPM) code

Knebe, Green & Binney, MNRAS **325** (2001) 845

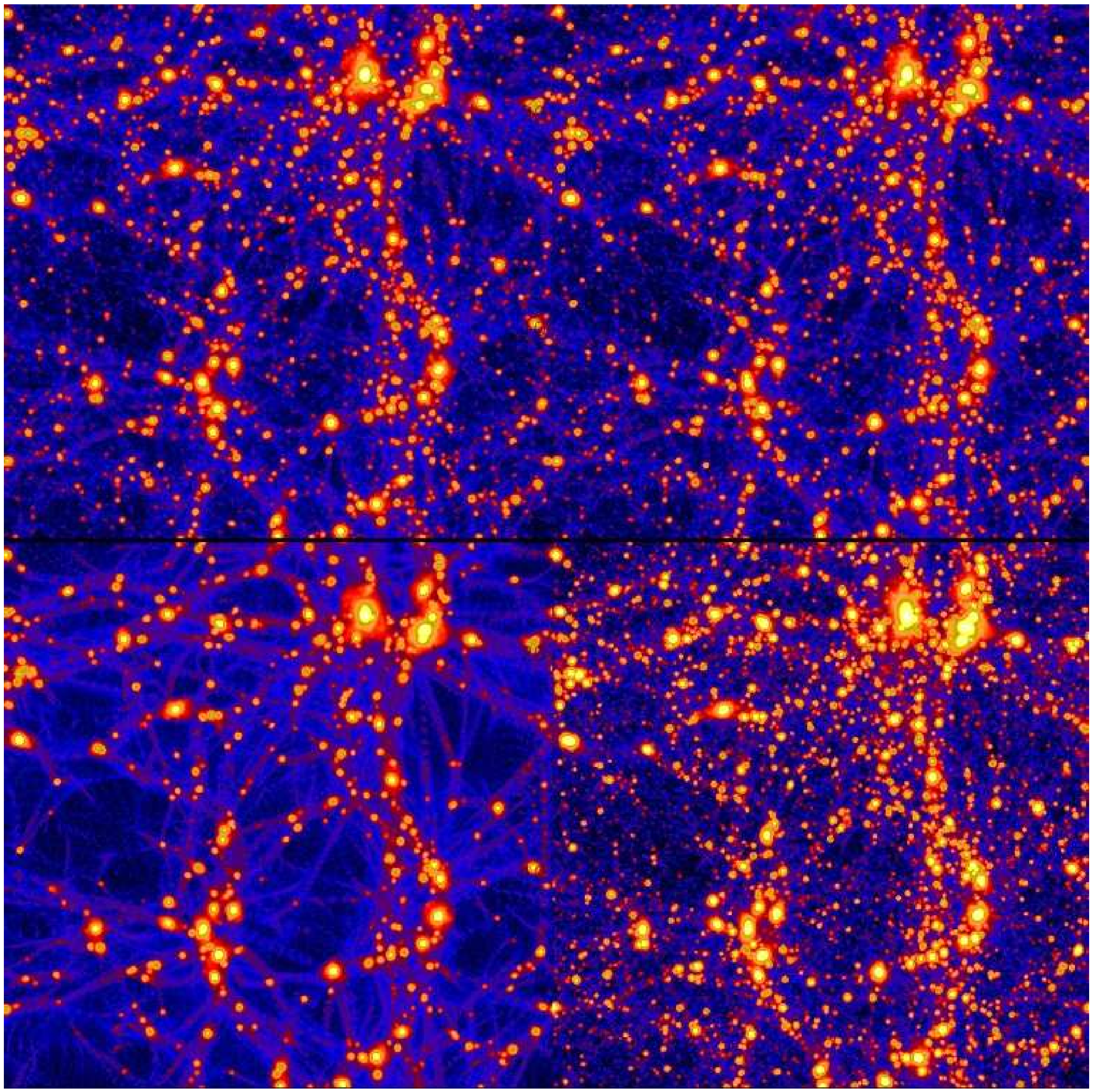
$$\{\mathbf{x}_\alpha, \mathbf{u}_\alpha\} \longrightarrow \varrho(\mathbf{x}), \mathbf{u}(\mathbf{x}) \text{ in a grid} \longrightarrow \mathbf{w}(\mathbf{x}), \mathbf{C}(\mathbf{x}) \text{ in a grid} \longrightarrow \{\mathbf{w}_\alpha, \mathbf{C}_\alpha\}$$



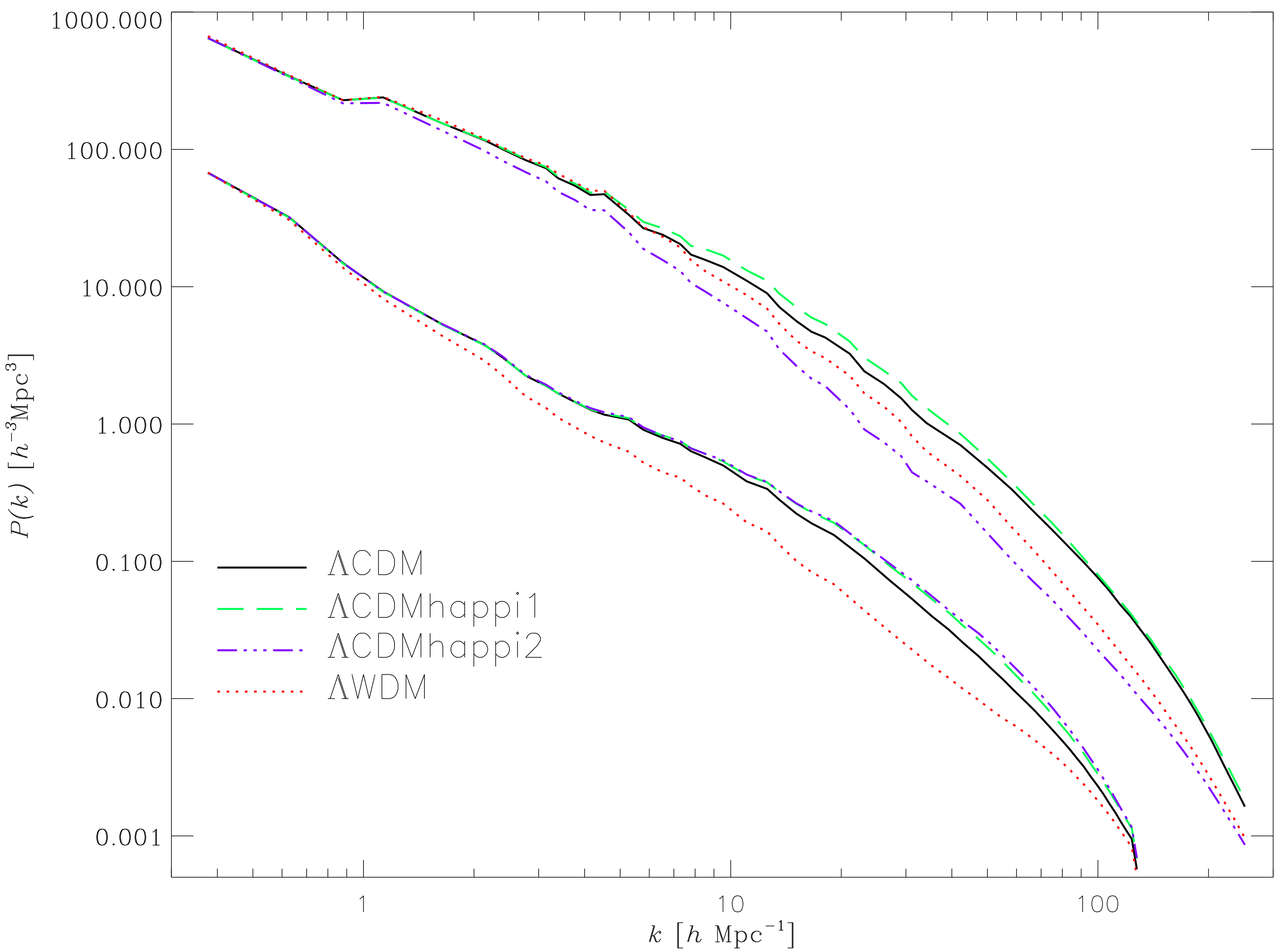
- purely grid–based algorithm
- automatic grid (de–)refinement according to the local density
- force resolution $\sim$ spatial resolution
- best suited for the hydrodynamic approach

Results: Knebe, Domínguez & Domínguez–Tenreiro, submitted

$N = 128^3$, box sidelength= 25 Mpc/h, concordance model



POWER SPECTRUM of the density field

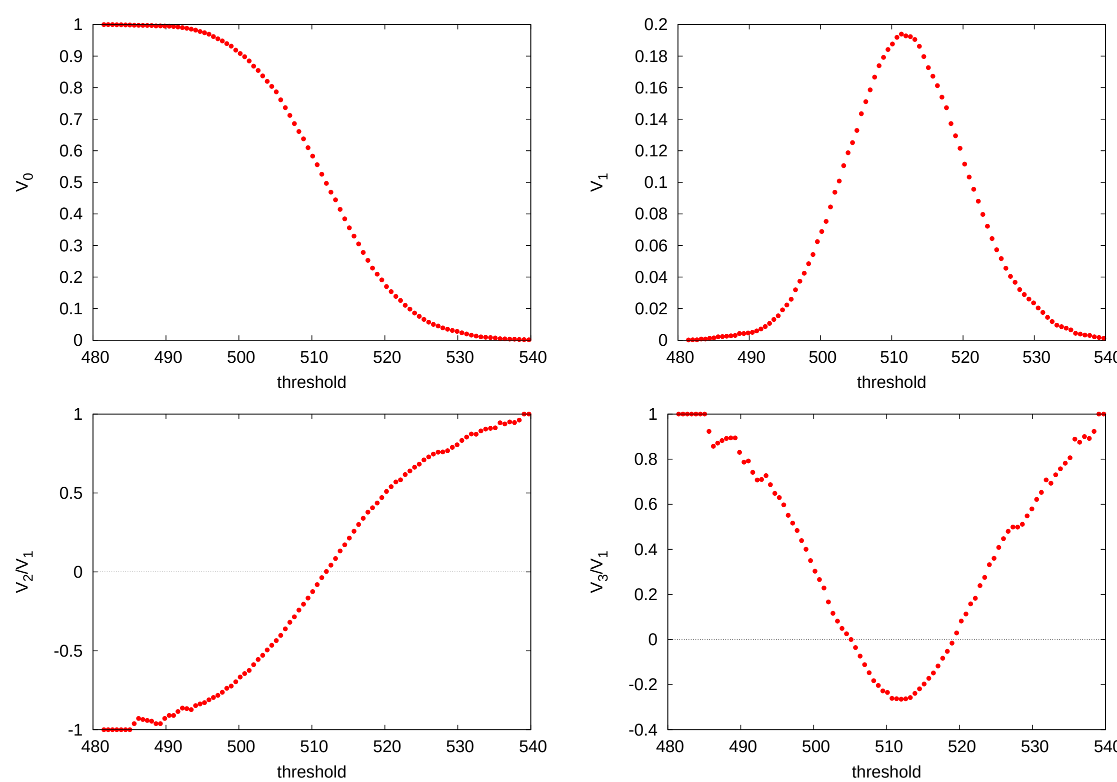


- $B = 1$ or WDM: clusters are smoother

MINKOWSKI FUNCTIONALS of random fields

Statistics of higher order than 2–points

Excursion set $\longrightarrow$ isodensity surface $\mathcal{S} := \{\mathbf{x}|\varrho(\mathbf{x}) = \text{threshold}\}$



$\longrightarrow$ Enclosed volume

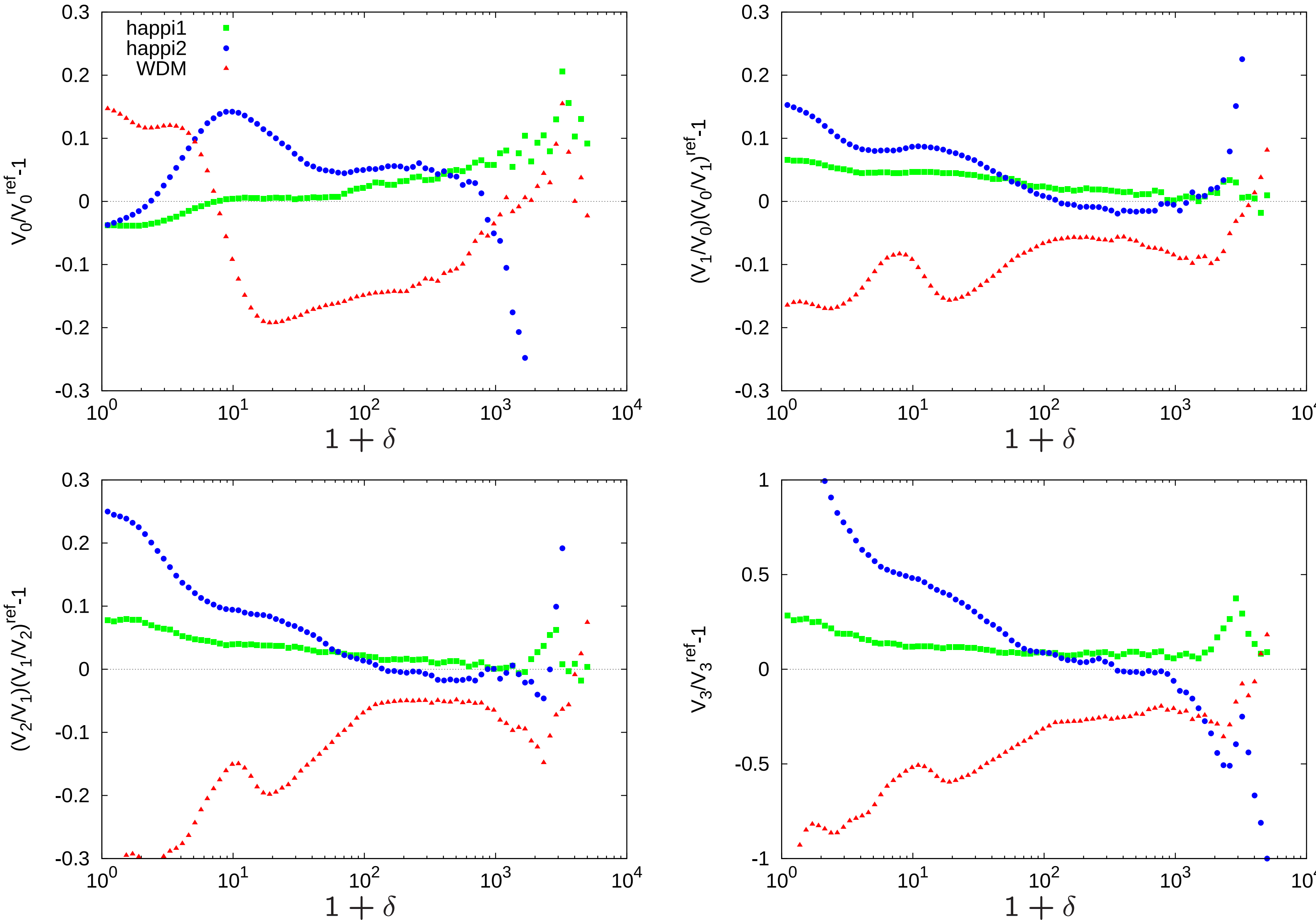
$\longrightarrow$ Area of $\mathcal{S}$

$\longrightarrow$ Mean curvature

$\longrightarrow$ Gaussian curvature (Euler characteristic)

Minkowski functionals of the density field (relative to LCDM)

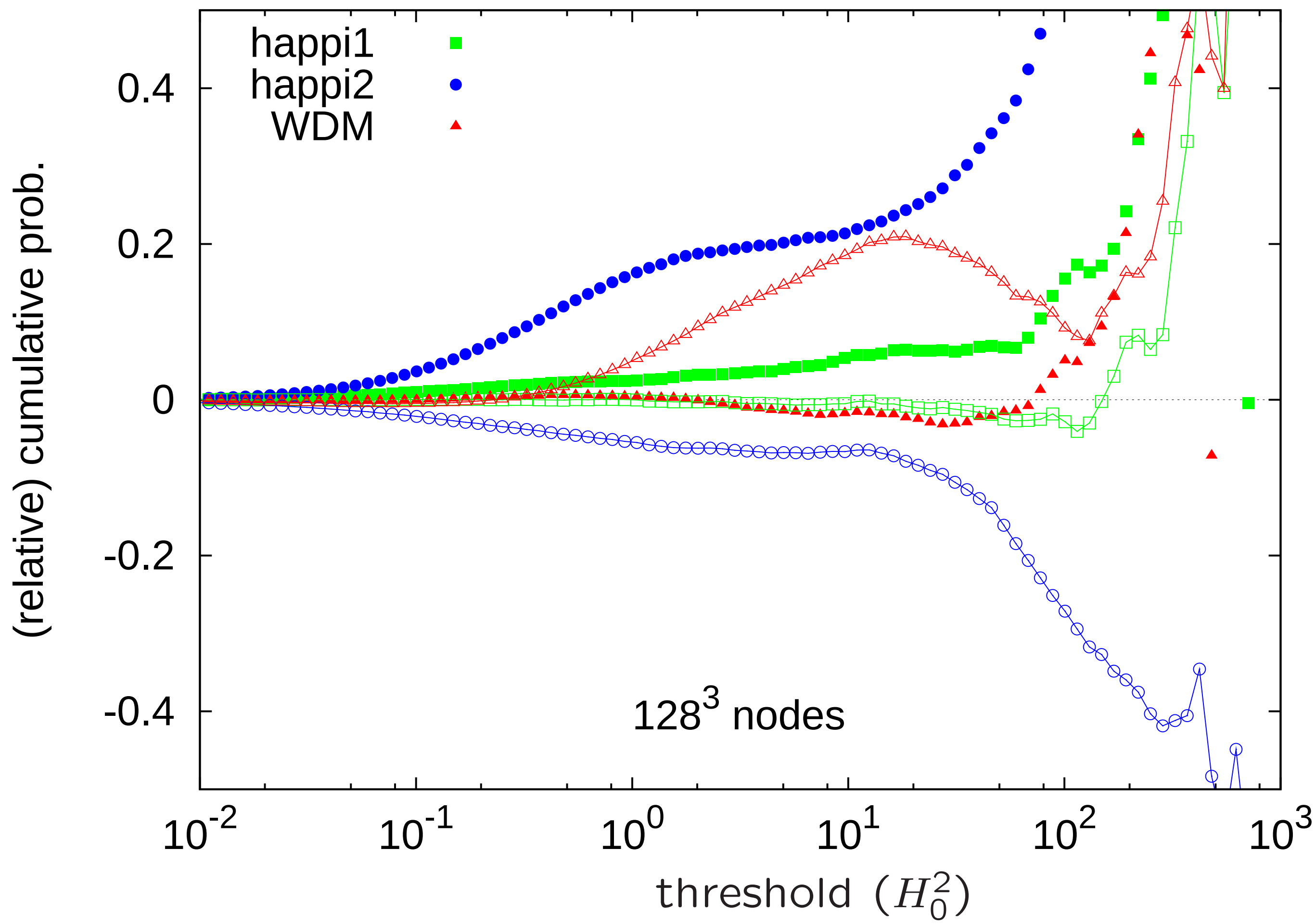
Density field $\varrho(\mathbf{x})$ at $z = 0$ in a cubic grid of 128^3 nodes



- **If C $\neq$ 0**: more small, rounder clusters
- WDM: less voids, more filamentary–like structures

VELOCITY FIELD

Vorticity, $|\nabla \times \mathbf{u}|^2$, and divergence, $|\nabla \cdot \mathbf{u}|^2$, in a cubic grid of 128^3 nodes



When $\mathbf{C} \neq \mathbf{0}$

- Vorticity tends to be larger
- Divergence tends to be smaller

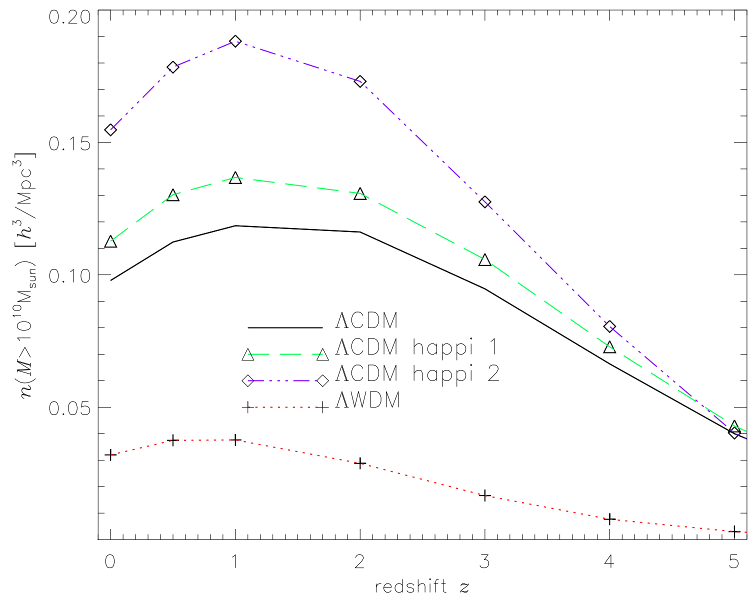
DARK MATTER HALOS

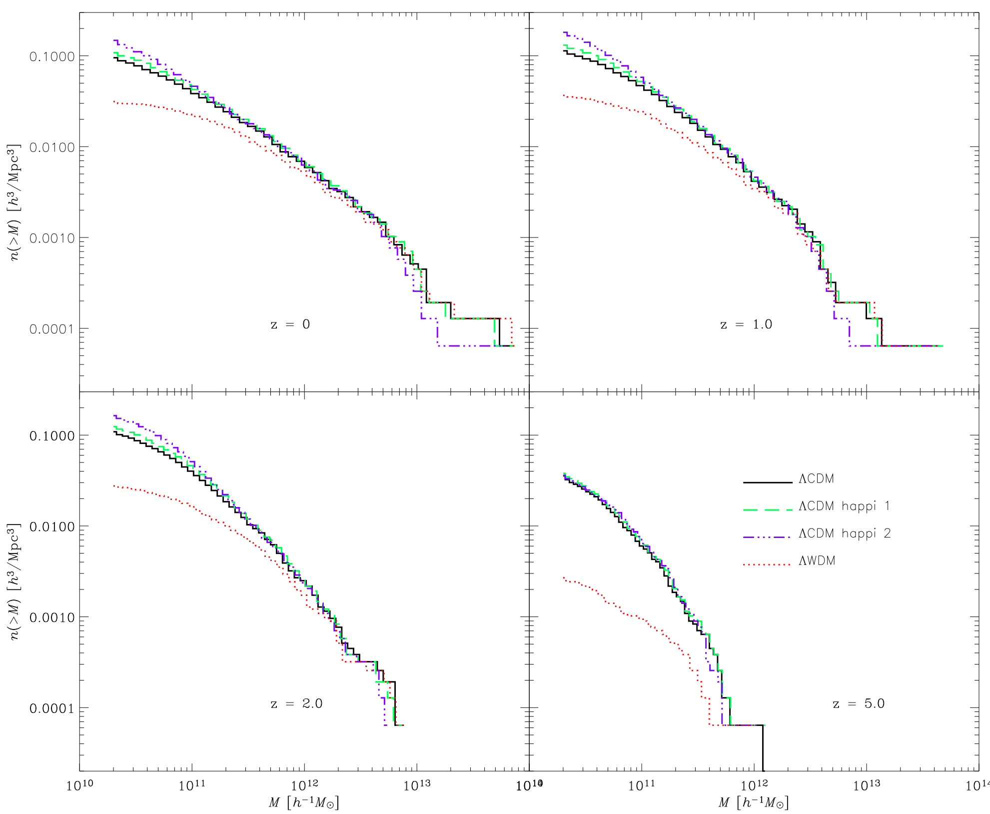
DM halos identified with the MLAPM halo finder
Gill, Knebe & Gibson, MNRAS **351** (2004) 399

- The algorithm relies on the refined grid of the simulation to select prospective halo centers (local maxima of the density)
- Only gravitationally bound particles are collected
- The algorithm is parameter–free

Abundance of halos

Number density of halos more massive than $10^{10}\,M_\odot/h$



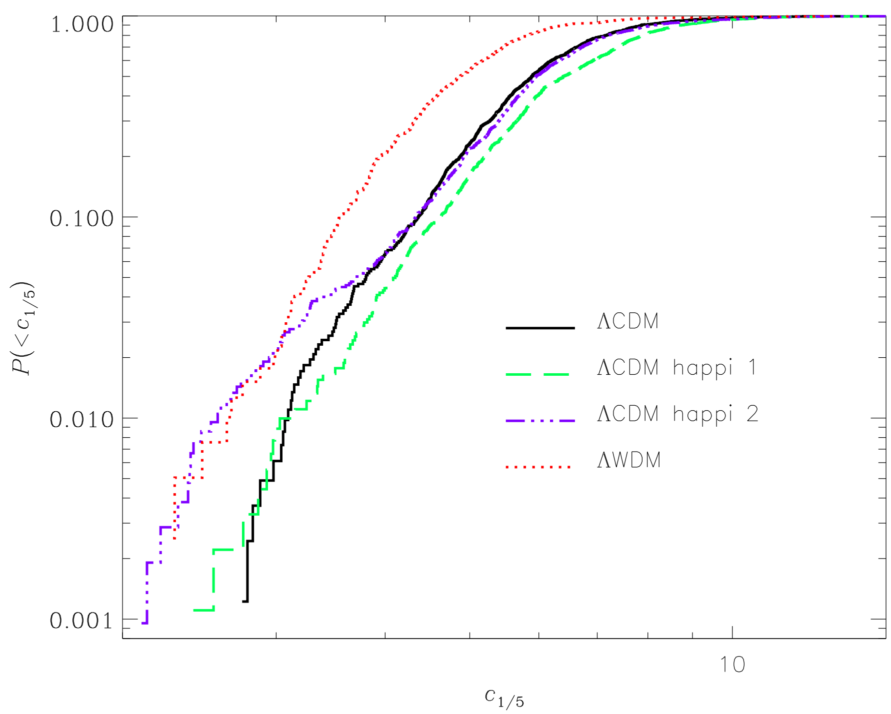


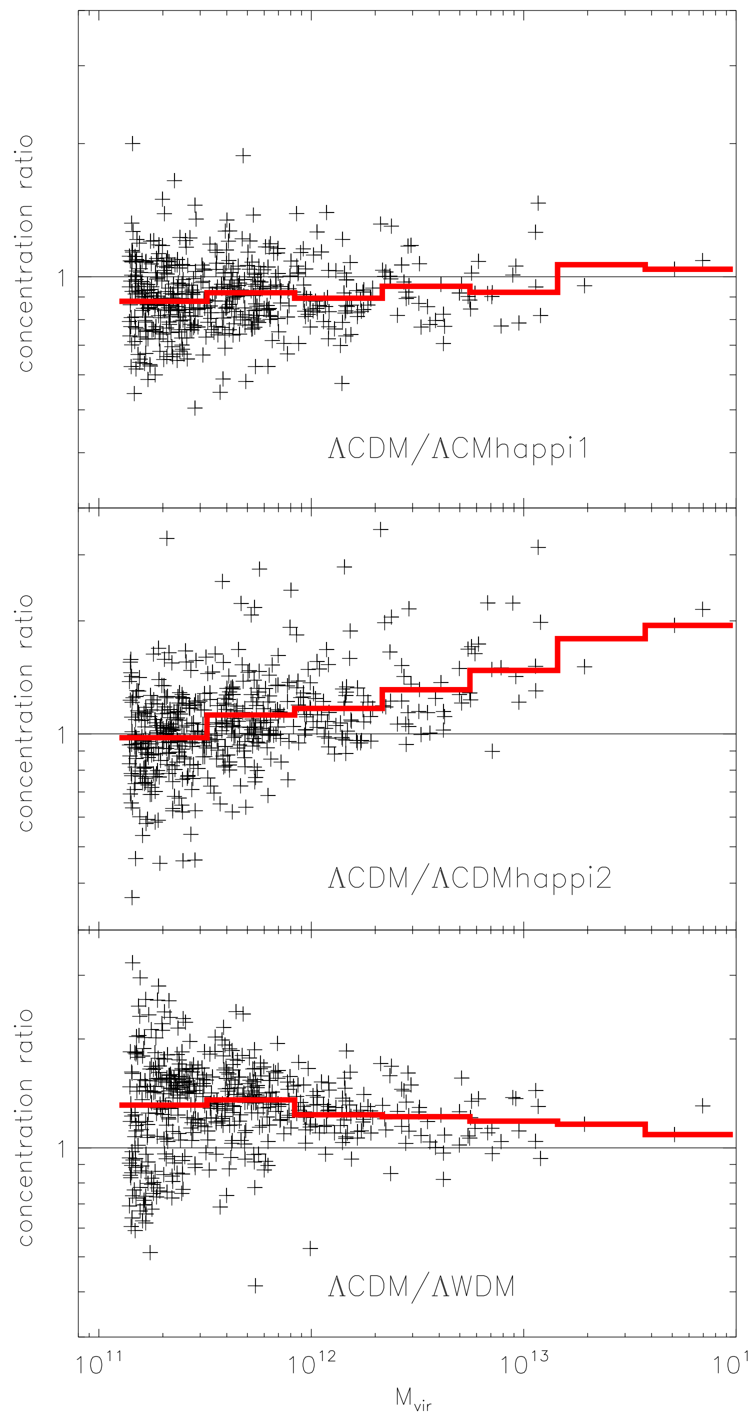
If $\mathbf{C} \neq \mathbf{0}$:

Proliferation of small–mass halos

Mass concentration in halos

$$c_{1/5} = \frac{r_{\rm vir}}{r_{1/5}}, \qquad M(r_{1/5}) = \frac{1}{5} M_{\rm vir}$$



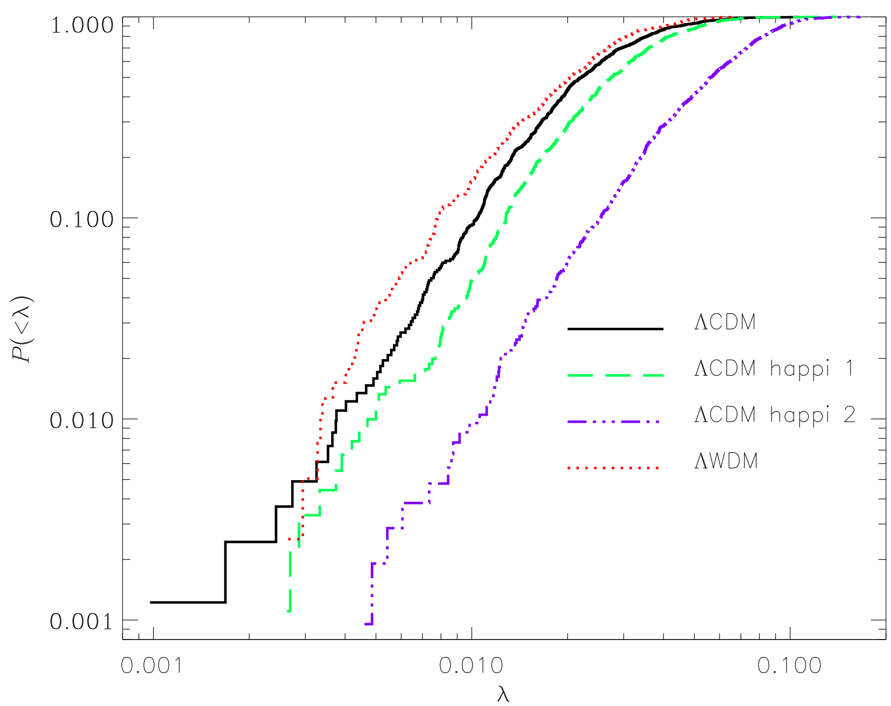


If $B = 1/4$: larger concentration

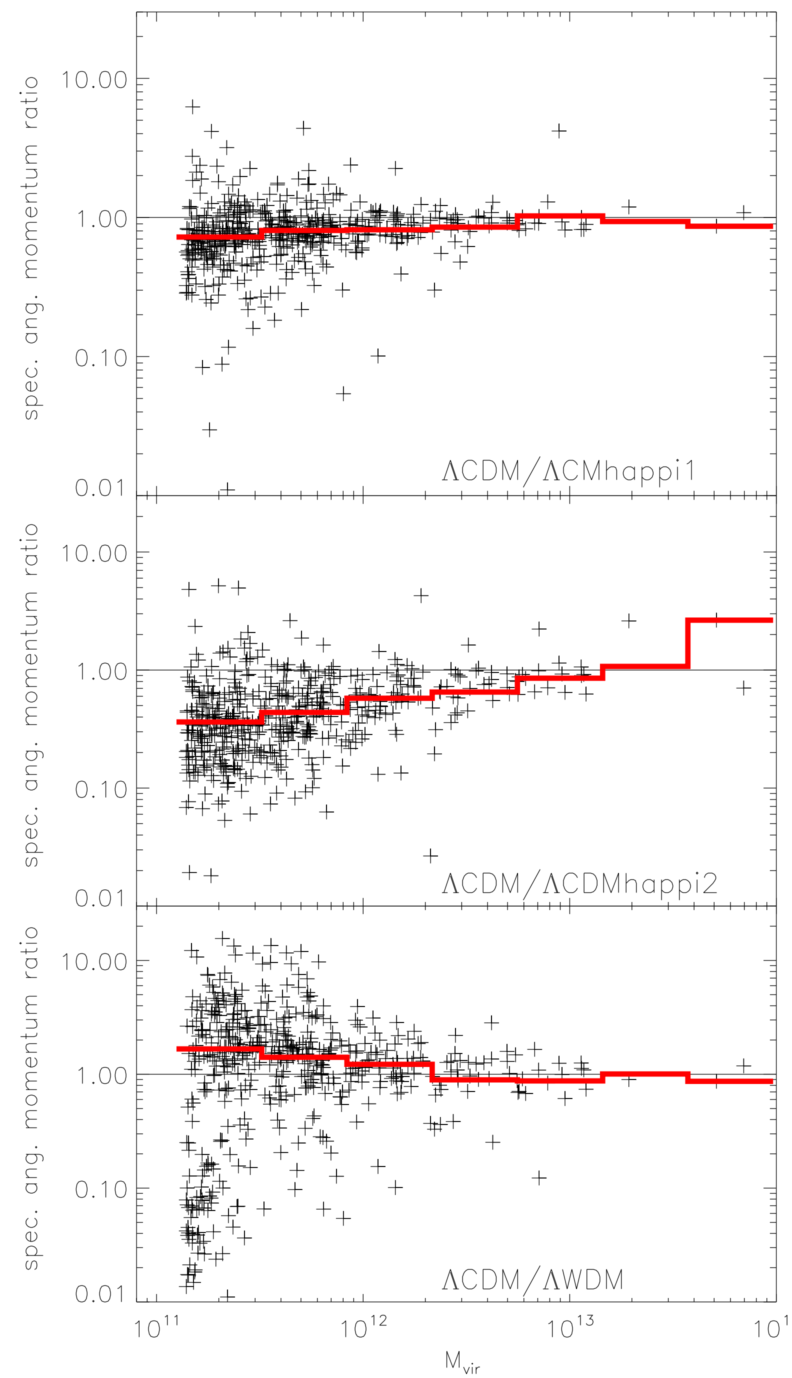
If $B = 1$: smaller concentration

Spin parameters of halos

$$\lambda = \frac{|\mathbf{L}|}{\sqrt{2}\, M_{\rm vir}\, v_{\rm vir}\, r_{\rm vir}}$$



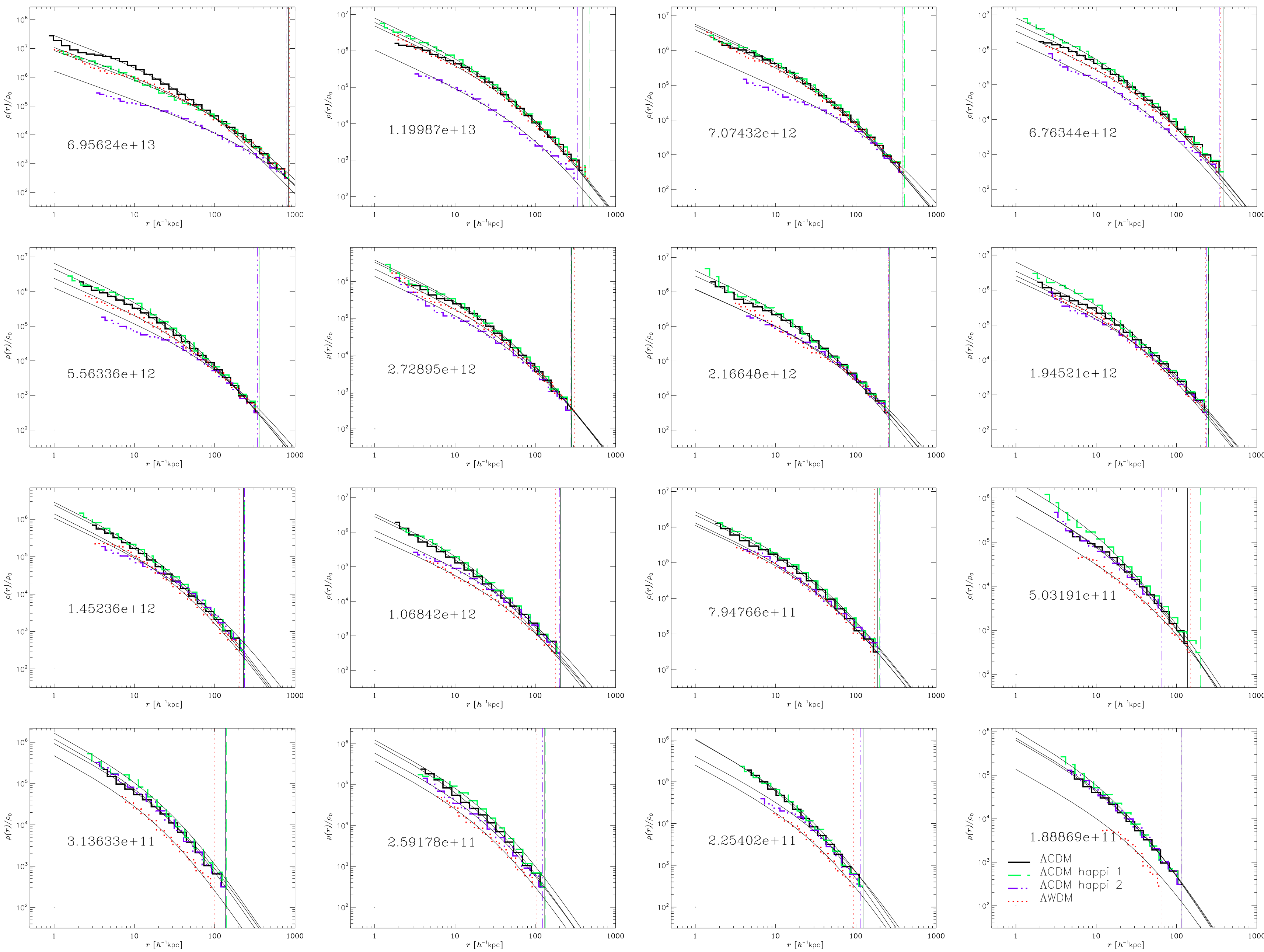
$|\mathbf{L}|/M_{\rm vir}$



If $\mathbf{C} \neq \mathbf{0}$:
larger angular momentum in halos, especially in low–mass halos

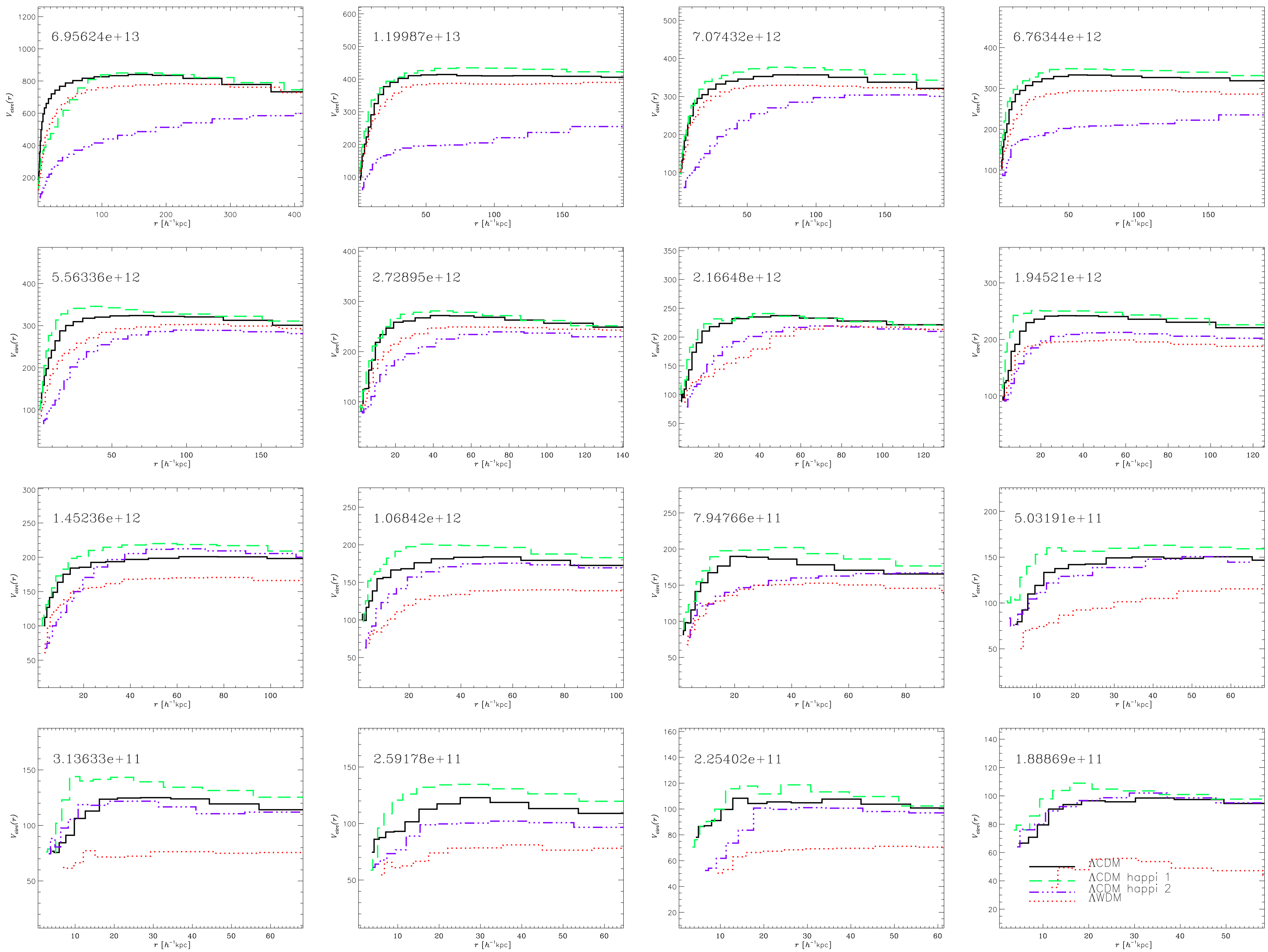
Density profiles of halos

Fit to NFW profile: $\rho(r) = \dfrac{\varrho_s \, r_s^3}{r\,(r_s + r)^2}$



Rotation curves of halos

$$v_{\text{circ}}(r) = \sqrt{\frac{GM(<r)}{r}}$$



CONCLUSIONS

- The effect of $\mathbf{C} \neq \mathbf{0}$:
  - Proliferation of low–mass halos (also field halos)
  - Higher vorticity of the velocity field
    Larger angular momentum of the halos
  - However, the halo concentration is smaller if $B = 1$
    larger if $B = 1/4$
- The results are consistent with high–resolution simulations
- The simulations and the theoretical predictions agree qualitatively