Halos and voids in a multifractal model of cosmic structure

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PLAN OF THE TALK

1. Motivation
2. Multifractals
3. Mass concetrations $\longrightarrow$ halos
4. Mass depletions $\longrightarrow$ voids
5. Multifractal clustering as fractal distributions of haloes.
6. Conclusions.

MULTIFRACTALS

- Scale invariance $\Rightarrow$ fractals
- Mass distributed according to a highly irregular pattern + scale invariance $\Rightarrow$ multifractals
- Mass concentrations

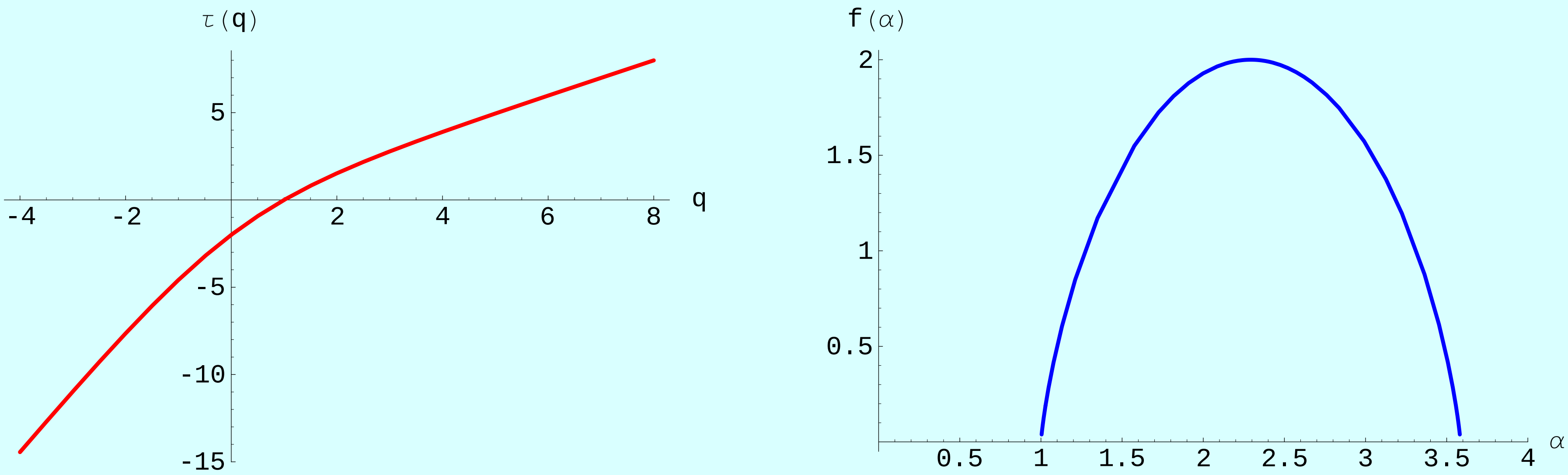
$$\alpha(\boldsymbol{x}) = \lim_{r\to 0} \frac{\log m[B(\boldsymbol{x},r)]}{\log r} \iff m[B(\boldsymbol{x},r)] \sim r^{\alpha(\boldsymbol{x})}$$

- *Multifractal spectrum* $f(\alpha)$ is the function that gives the fractal dimension of the set of points with exponent α.

  Monofractal: constant $\alpha = f(\alpha)$.

Correlation moments

- Coarse multifractal analysis: put an ℓ-mesh of cubes and define $M_q(\ell) = \sum_i (m_i/\ell^3)^q/\ell^{-3} = \sum_i m_i^q/\ell^{3(q-1)} = \langle \rho^q \rangle$.
  Scaling: $M_q(\ell) \sim \ell^{\gamma(q)}, \quad \gamma(q) = \tau(q) - 3(q-1).$
- Multifractal spectrum: $f(\alpha) = \min_q[q\,\alpha - \tau(q)]$, namely,
  $\alpha(q) = \tau'(q), \quad f(\alpha) = q(\alpha)\,\alpha - \tau[q(\alpha)].$



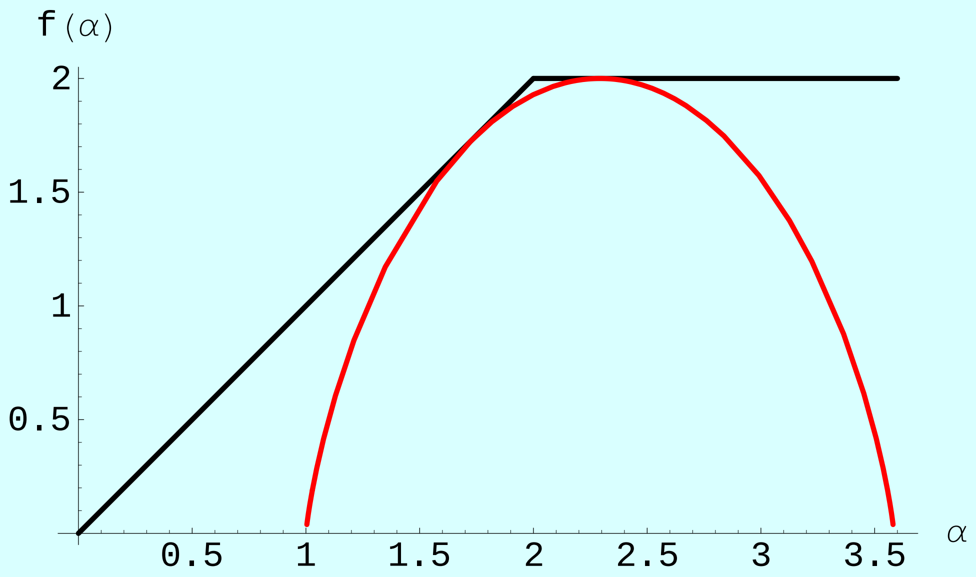
τ-function and spectrum of a multinomial multifractal

Correlation moments

- $D(q) = \tau(q)/(q-1)$ decreases with q. For a monofractal: $\alpha = f(\alpha) = D(q) =$ constant. In general:

  $q = 0$: $f(\alpha) = D(0) = -\tau(0)$, $f'(\alpha) = q = 0 \Rightarrow$ largest fractal dimension $\rightarrow$ measure's support.

  $q = 1$: $\alpha = f(\alpha) = D(1)$, $f'(\alpha) = q = 1$ and convex $\Rightarrow$ $f(\alpha) \leq \alpha \rightarrow$ measure's concentrate.

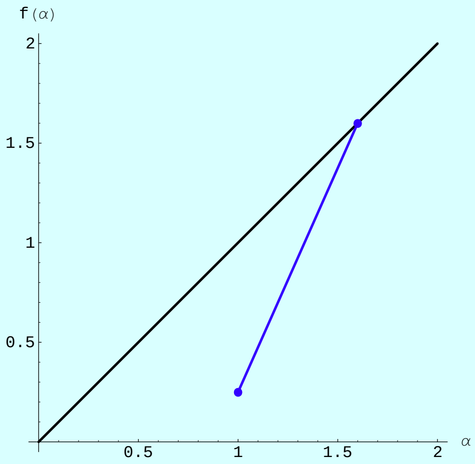


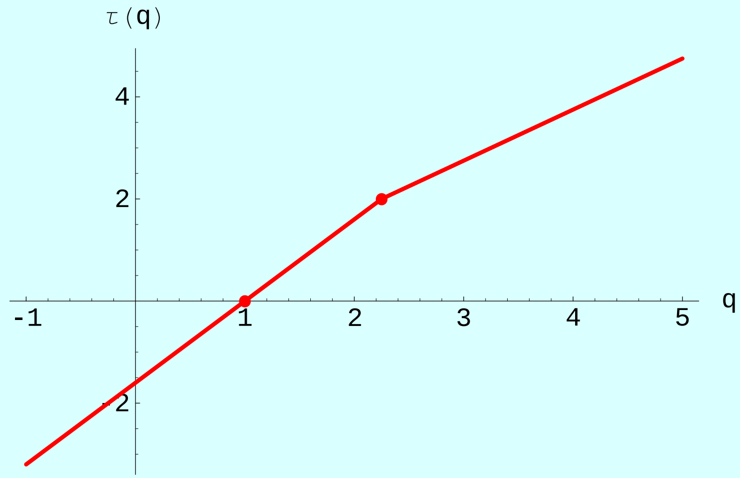
Spectrum of multinomial multifractal, showing measure's support and concentrate

Linear MF spectrum

MF spectrum can be linear $\Rightarrow$ bifractal:

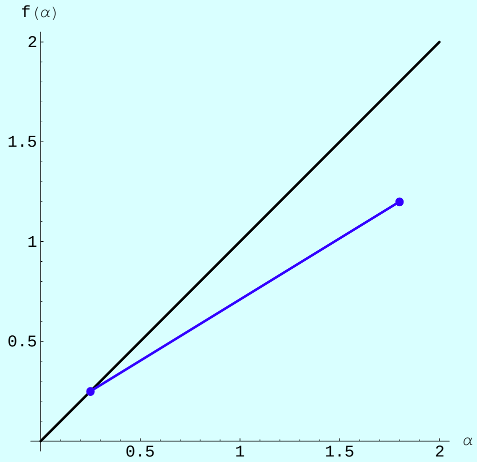
- $\alpha_1 < \alpha_2 = f(\alpha_2)$

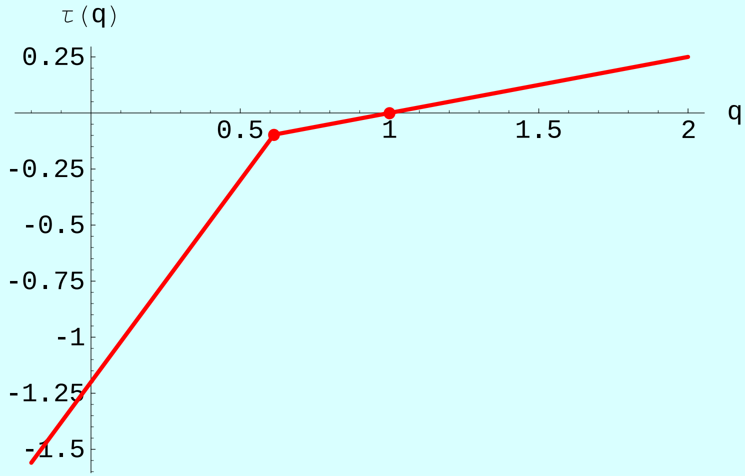




Linear spectrum and τ-function. Crossover at $q = f'(\alpha)$

- $f(\alpha_1) = \alpha_1 < \alpha_2$



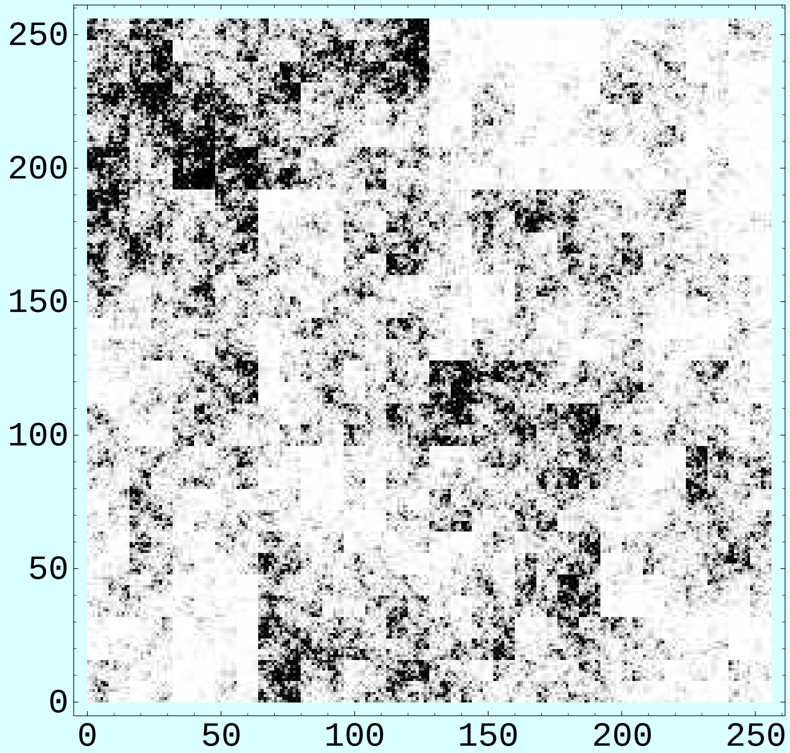


Example: multinomial MF

Multinomial multifractals are self-similar multifractals: the unit square is divided into (4) cells, the unit mass distributed among cells ($\{p_i\}$), and the process iterated.

The MF spectrum can be obtained:

$$\tau(q) = -\log_2 \sum_i p_i^q \,.$$



Random multinomial measure with distribution $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}\}$.

MASS CONCENTRATIONS: HALOS

Fix the coarse-graining length $\ell \rightarrow$ mass concentrations of size ℓ with singular power-law profile $\rho(r) \propto r^{-\beta}$ ($\beta = 3 - \alpha$).

Cosmology: natural value for ℓ is the lower limit to scaling (lower cutoff). In N-body simulations, the largest of:

(i) The linear size of the volume per particle.

(ii) The gravitational softening length.

We identify mass concentrations with equal-size halos $\rightarrow$ virialization affects scales below the lower cutoff.

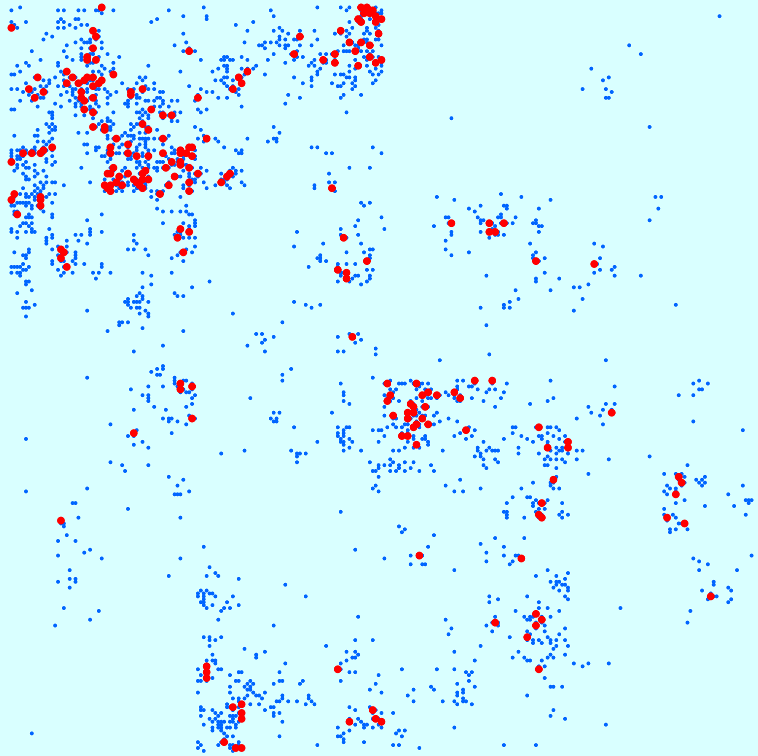
Halo *mass-function*: $N(m) \sim \ell^{-f(\alpha)}, \; \alpha = \log m / \log \ell.$

Multinomial bifractal

A bifractal can be extracted: select $\{\alpha_1, \alpha_2\} \Leftrightarrow \{m_1, m_2\}$.

Multifractal models support halo populations with different levels of clustering.

Note voids.

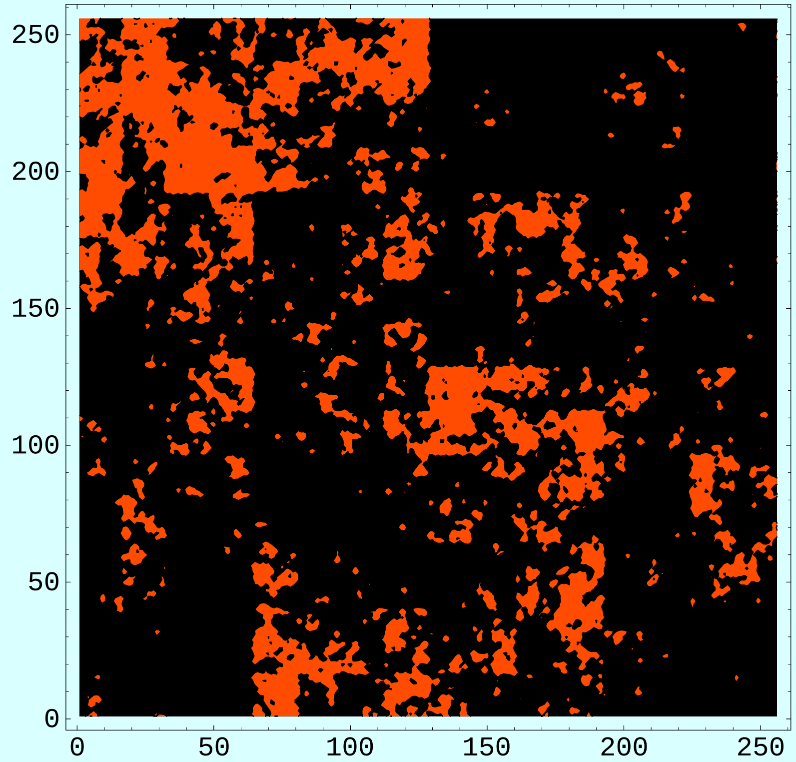


Two populations in a multinomial multifractal.

MASS DEPLETIONS: VOIDS

Halos have singular power-law profile $\rho(r) \propto r^{-\beta}$ ($\beta = 3-\alpha$). If $\alpha > 3 \Rightarrow \rho(0) = 0$ $\rightarrow$ voids.

Boundaries of voids: points with $\alpha = 3 \Rightarrow \rho(0) > 0$ and finite. They may not be regular surfaces but fractal surfaces with $D = f(3) > 2$.



Fractal boundary of voids in multinomial MF: $f(2) = 1.93$.

BIASING AND VOIDS

Biasing: peculiar distribution of certain set of objects (galaxies $\leftrightarrow$ halos) with respect to the total matter distribution.

Bias from linear theory:

$$\frac{\delta\rho_g}{\rho_g} = b\,\frac{\delta\rho}{\rho} \Rightarrow \xi_{gg}(r) = b^2\xi(r),\ b > 1.$$

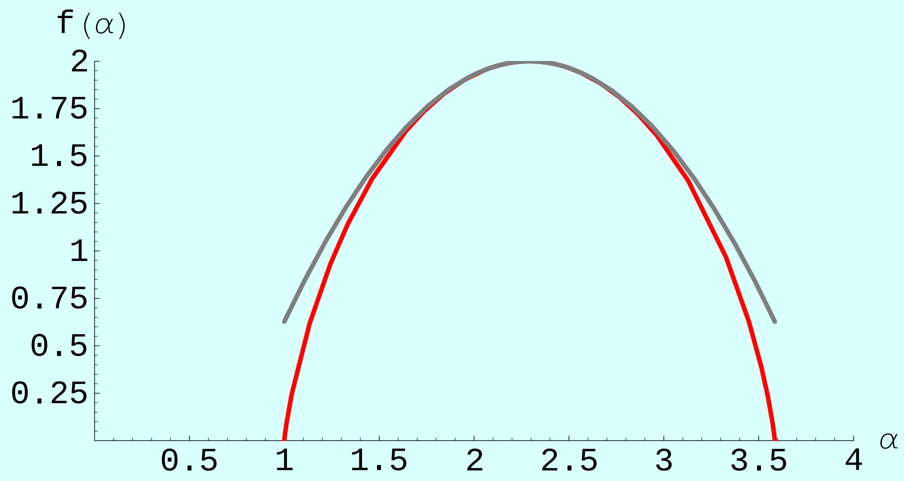
Constant b bias in the nonlinear regime $\Rightarrow$ similar voids for every population $\rightarrow$ false in MF.

- Voids not empty but harbor faint galaxies $\leftrightarrow$ galaxy formation (Peebles, 2001).
- Distribution of dark matter inside voids (Gottlöber et al, 2003).

LOGNORMAL PDF

Lognormal model: extension of the Gaussian linear theory into the linear regime (Coles and Jones, 1991).

Lognormal pdf is the basic approximation to MF spectrum $\Rightarrow$ also to $N(m) \sim \ell^{-f(\alpha)} \sim \ell^{c(\alpha-\alpha_0)^2} \sim \exp(-c\,\frac{[\ln(m/m_0)]^2}{|\ln \ell|})$ (theory of *large deviations*).



Lognormal approximation to multinomial multifractal

Bad approximation to bifractal.

PRESS-SCHECHTER MASS FUNCTION

Press-Schechter spherical collapse formalism + power-law spectrum of initial (Gaussian) fluctuations $\Rightarrow$ power law $N(m)$ (exponential cutoff for large mass) $\Leftrightarrow$ bifractal.

Spherical collapse $\Rightarrow$ large m. In fact, collapse along the three axes in only 8% of regions (Doroshkevich, 1970), and they are very overdense: $P(\lambda_3 > 0|\delta)$ grows with δ and reaches 0.5 when $\delta = 1.5\sigma$ (Lee & Shandarin, 1998).

Lognormal for $m \gg m_0$ becomes power law (in a small range).

Can the real MF spectrum be more linear than its lognormal approximation?

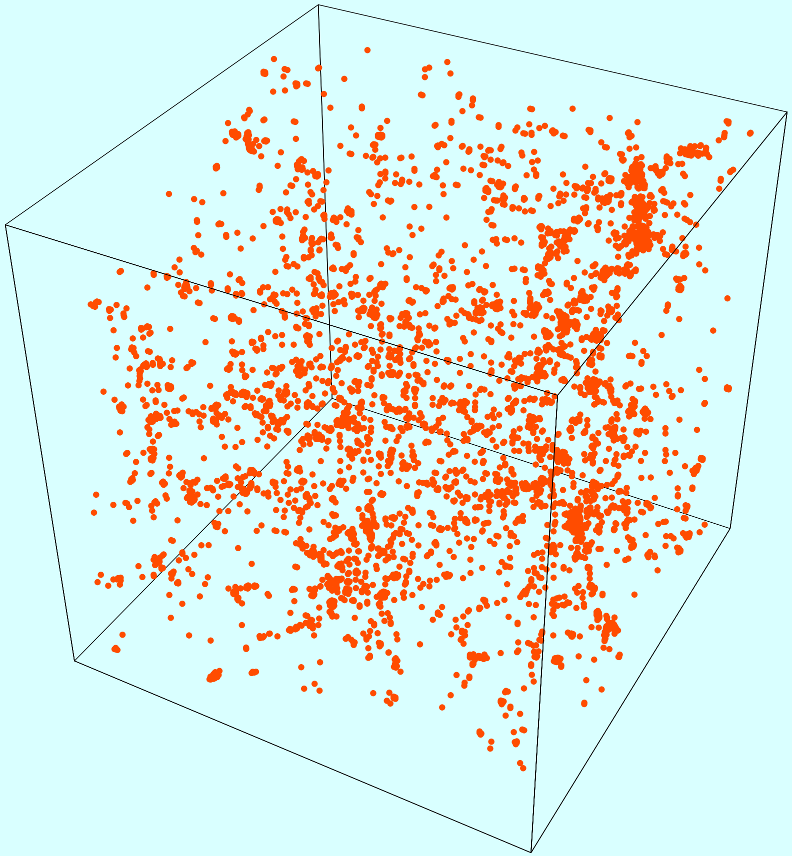
MULTIFRACTALITY IN N-BODY SIMULATIONS

$z = 0$ positions in ΛCDM GIF2 simulation (Virgo Consortium): AP3M code; 400^3 particles in a volume of $(110\ h^{-1}$ Mpc$)^3 \Rightarrow$ particle mass is $0.173\ 10^{10}\, h^{-1} M_\odot$.

Statistics by *counts-in-cells*:

$$M_q(\ell) = \sum_{m=1}^{\infty} N(m)\, m^q.$$

Halos: $\ell_H = 256^{-1} > 400^{-1}$ ($\ell_H = 0.43\ h^{-1}$ Mpc).

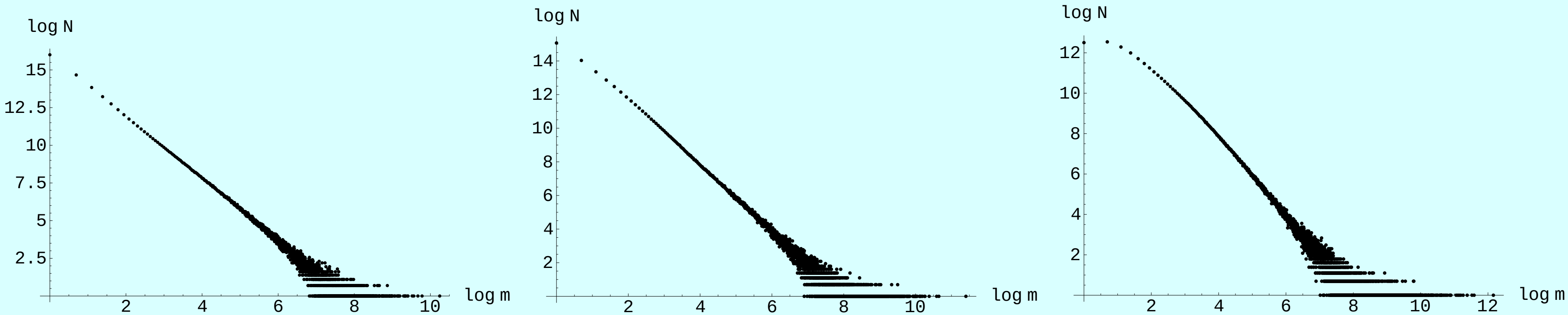


Distribution of 5515 haloes (cutoff 1000)

Mass-function evolution with ℓ

If MF scaling holds $\Rightarrow$ stable MF spectrum.

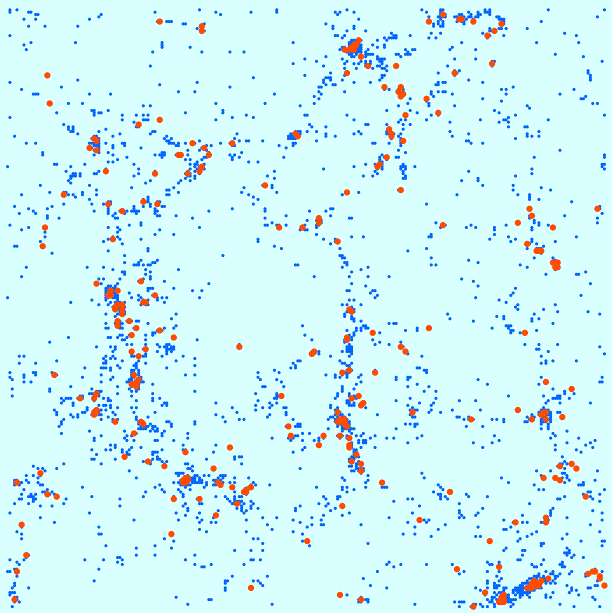
For $\ell > \ell_H$ we expect a stable MF spectrum. How does it change for $\ell < \ell_H$?

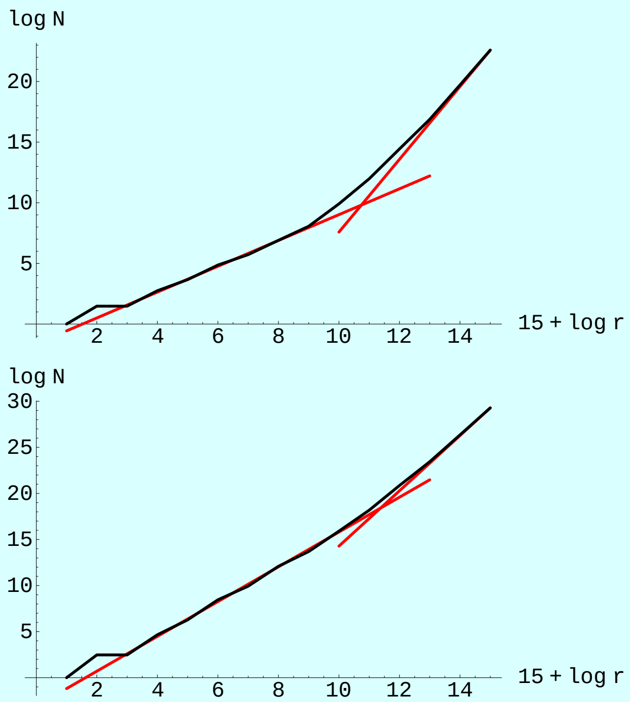


Log-log plots of number of halos N versus their mass m (number of particles) at coarse-graining scales 512^{-1} (left), 256^{-1} (middle), and 128^{-1} (right). The rightmost plot shows a maximum for $m = 2$.

Power-law (Press-Schechter) for $\ell < \ell_H \leftrightarrow$ virialization.

MF AS FRACTAL DISTRIBUTIONS OF HALOS + VOIDS



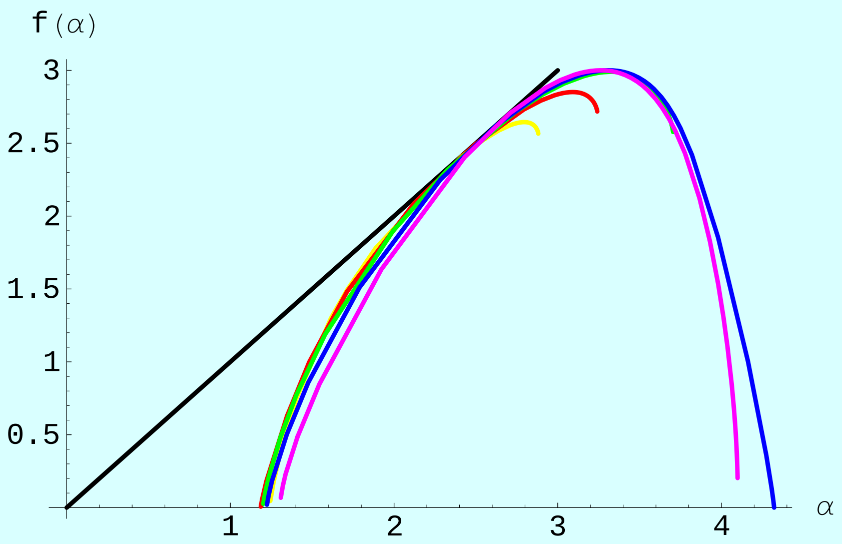


- GIF2 heavy haloes with 750 to 1000 particles (red);
- GIF2 light haloes with 100 to 150 particles (blue).

Number function $N[B(\boldsymbol{x}, r)] \sim r^D$: fractal dimensions $D = 1.1$ and $D = 1.9$. Transition to homogeneity starts at $\simeq 14\ h^{-1}$ Mpc.

GIF2 MF spectrum

Stable MF spectrum for $\ell > \ell_H$, but *full* spectrum only for larger scales.



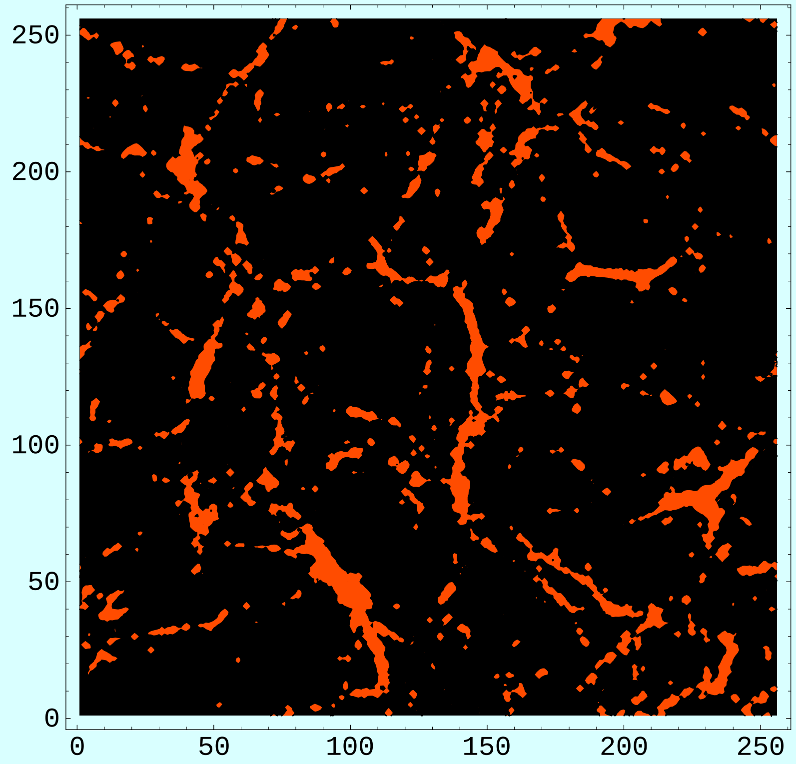
Full multifractal spectrum $f(\alpha)$ for $\ell = 2^{\{-9,-8,-7,-6,-5\}}$ (yellow, red, green, blue, magenta).

- Entropy dimension = 2.5.
- Dimension of the support = 3.
- $\alpha(q=0) \simeq 3.3 > 3 \Rightarrow$ support dominated by voids.

Voids in the GIF2 simulation

MF spectrum shows that $f(3) \simeq 2.9 \Rightarrow$ boundary is a fractal surface with large dimension.

It is best to represent a slice to see the morphology $\rightarrow$ no symmetry between halos and voids.



Fractal boundary of voids in a GIF2 slice

CONCLUSIONS

- Multifractals are the most general scaling mass distributions supporting halos and voids.
- Natural definition of halos ($\alpha < 3$), voids ($\alpha > 3$), and their boundary ($\alpha = 3$), which is a fractal surface.
- Linear MF spectrum $\leftrightarrow$ Press-Schechter power-law halo mass function (bifractal).
- Halos and, therefore, galaxies have *nonlinear bias* w.r.t. the full dark matter distribution $\longrightarrow$ fractal debate (?)

CONCLUSIONS

- Multifractal analysis in terms of intermingled fractal populations of haloes $\rightarrow$ good scaling.
- *Non-symmetric* MF spectrum $\rightarrow$ bifractal.
- Stable MF spectrum for $q > 0$. Compatible with linear spectrum with *large-mass cutoff* (Press-Schechter).