The effects of rotation

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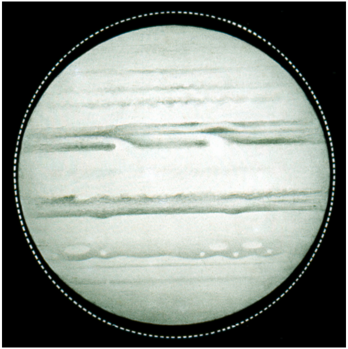
Jupiter's rotation

- Jupiter is a rapidly rotating planet:
  - $P = 9.925\ \mathrm{hr}$ (Seidelmann et al. 2007)
  - $\varepsilon = 1 - R_{\mathrm{pol}}/R_{\mathrm{eq}} = 6.49\%$
- rotation needs to be properly taken into account when studying its structure and pulsations



(Royal Astronomical Society of Canada)

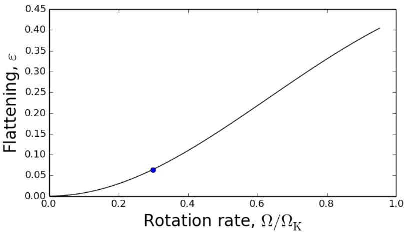
Effects of rotation

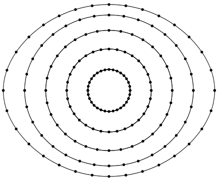
- rotation introduces 2 inertial forces
  - the centrifugal force
  - the Coriolis force
- neither respects spherical symmetry
  - 2D non-separable problem (structure, oscillations)
  - oscillation modes are no longer described by a single spherical harmonic

Effects of rotation

Centrifugal force

- flattening $= \varepsilon \propto \dfrac{\Omega^2 R_{\mathrm{eq}}^3}{GM}$
- the outer layers are the most deformed
- effect on acoustic modes $\propto \frac{\varepsilon}{\lambda} \propto \omega\Omega^2$
  - $\lambda =$ mode's wavelength, $\omega =$ mode's frequency
- smaller effect on gravito-inertial modes which tend to be deeper inside





(Roxburgh 2004)

Effects of rotation

Coriolis force

- conservation of angular momentum
- intervenes in dynamic phenomena (oscillations, convection ...)
- scales as $2\Omega/\omega$ in oscillations
  - strongest effect on low frequency modes $\Rightarrow$ gravito-inertial modes
  - inertial modes owe their existence to the Coriolis force (e.g. Papaloizou & Pringle, 1978, Rieutord et al. 2001)

(Heimpel, Gastine, & Wicht, 2016, Nat. Geo.)

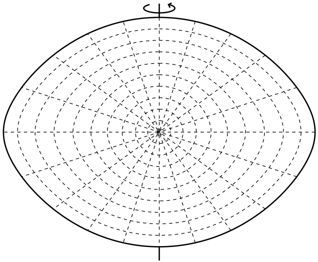
How to include rotation

Different methods

- **perturbative**: power series in Ω
- **traditional approximation**: neglects horizontal component of the Coriolis force and deformation
- **two-dimensional approach**: the most accurate and costly
- **ray dynamics**: insights into asymptotic behaviour

Our method

- a 2D approach
- surface fitting coordinates
- spectral methods (very accurate):
  - r: Chebyshev polynomials
  - θ, ϕ: spherical harmonics



Model for Jupiter

- we used an $N = 1$ polytrope at $\Omega/\Omega_{\mathrm{K}} = 0.298622$ (Kong et al. 2013)
- uniform rotation

Quantity	**Jupiter**	**Our model**	$\varepsilon_{\mathrm{rel}}$
R_{eq} (km)	$71\,492 \pm 4$	$71\,492$	$0\,\%$
R_{pol} (km)	$66\,854 \pm 10$	$66\,931$	$0.116\,\%$
$\varepsilon = 1 - \frac{R_{\mathrm{eq}}}{R_{\mathrm{pol}}}$	0.06487(15)	0.06379	$-1.67\,\%$
Mass (kg)	1.8986×10^{27}	1.8986×10^{27}	$0\,\%$
Rot. period (hr)	9.925	9.925	$-0.00036\,\%$
Ω/Ω_K	0.298621(41)	0.298622	$0.00036\,\%$

A few equations

Polytropic model, $N = 1$

$$
\begin{aligned}
P_o &= K\rho_o^{1+\frac{1}{N}} \\
\vec{0} &= -\vec{\nabla} P_o - \rho_o \vec{\nabla}\left(\Psi_o + \Omega^2 s^2\right) \\
\Delta\Psi_o &= 4\pi G \rho_o
\end{aligned}
$$

Oscillations

$$
\begin{aligned}
i\omega\rho &= -\vec{\nabla}\cdot\left(\rho_o \vec{v}\right) \\
i\omega\rho_o\vec{v} &= -\vec{\nabla} p + \rho \vec{g}_{\mathrm{eff}} - \rho_o \vec{\nabla}\Psi - 2\rho_o \vec{\Omega}\times\vec{v} \\
i\omega\left(p - c_o^2 \rho\right) &= \frac{\rho_o c_o^2 N_o^2}{\|\vec{g_{\mathrm{eff}}}\|^2}\vec{v}\cdot\vec{g}_{\mathrm{eff}} \\
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A few equations

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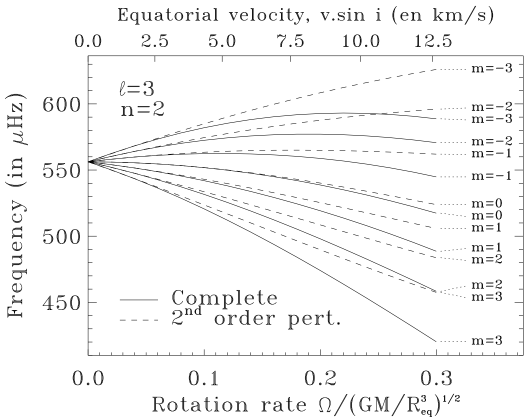
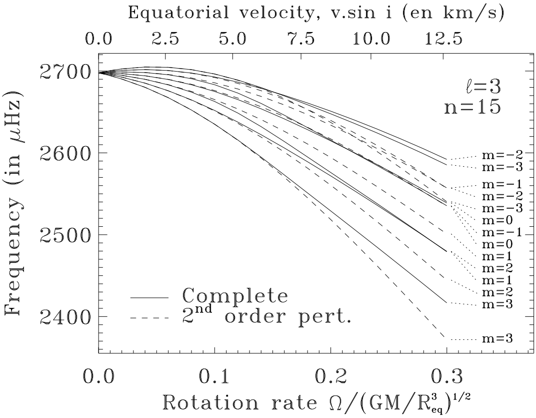
The multiplet



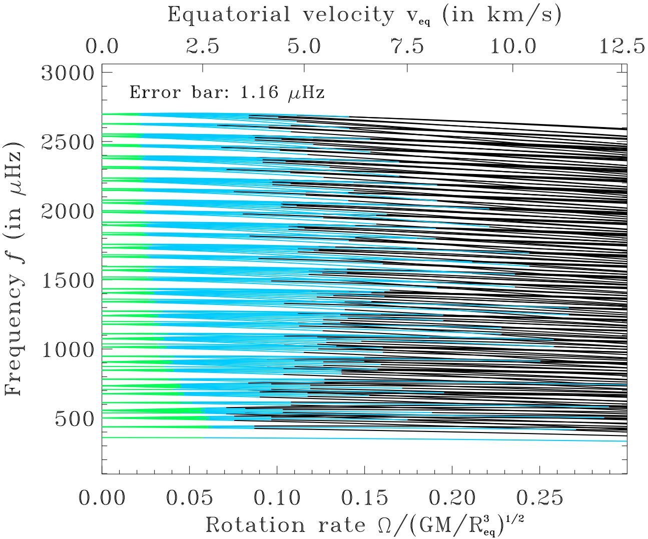
$$\omega = \omega_0 - m(1-\mathcal{C})\Omega + \mathcal{O}(\Omega^2) \qquad \mathcal{C} = \text{Ledoux's constant}$$

The multiplet

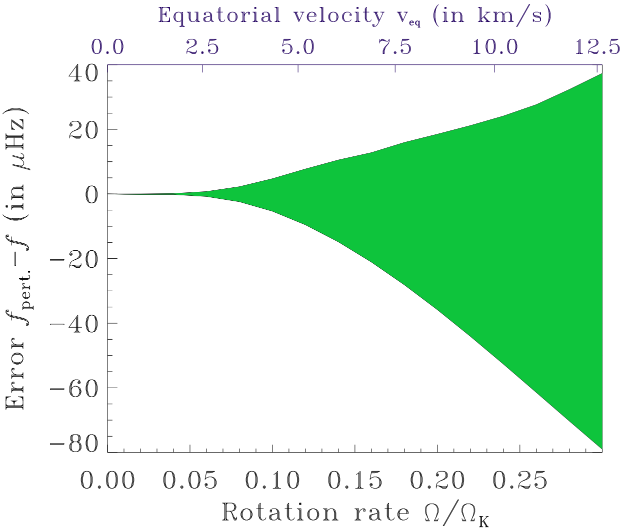


$\omega = \omega_0 - m(1 - \mathcal{C})\Omega + \mathcal{O}(\Omega^2)$ $\mathcal{C}$ = Ledoux's constant

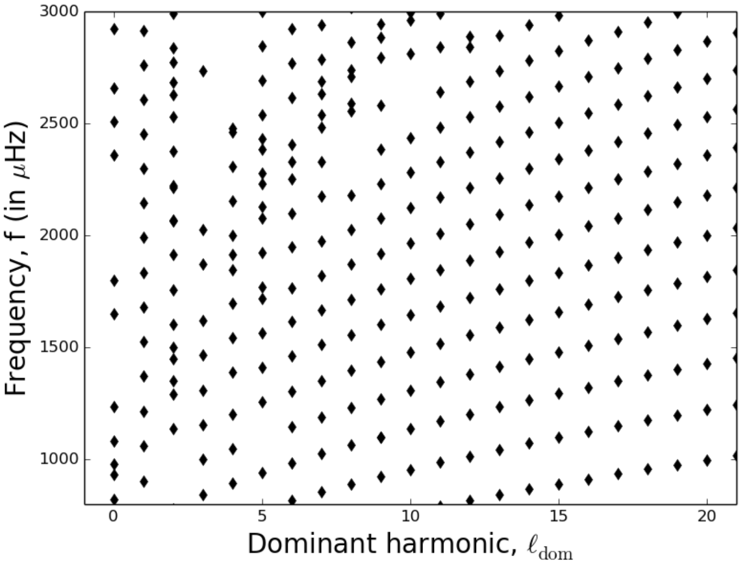
A spectrum


Equatorial velocity v_{eq} (in km/s)
0.0 2.5 5.0 7.5 10.0 12.5
3000 2500 2000 1500 1000 500
Error bar: 1.16 μHz
Frequency f (in μHz)
0.00 0.05 0.10 0.15 0.20 0.25
Rotation rate $\Omega/(GM/R_{eq}^3)^{1/2}$

Maximum perturbative error



Higher ℓ and $|m|$ values



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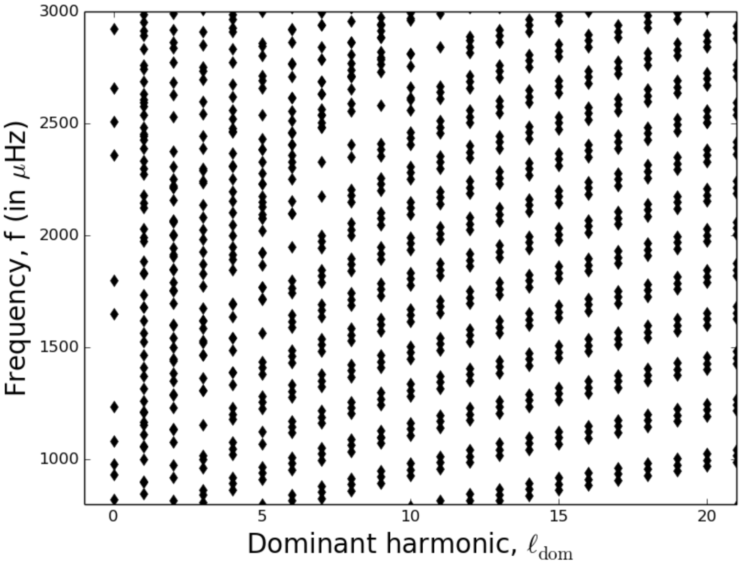
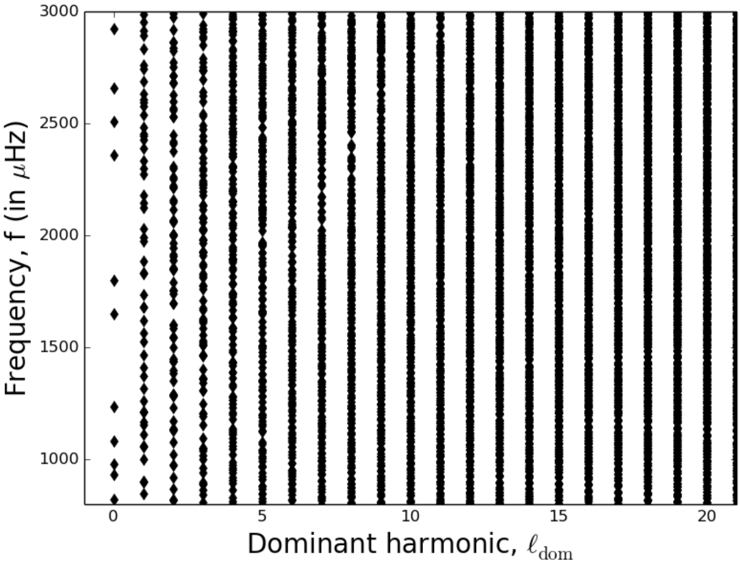
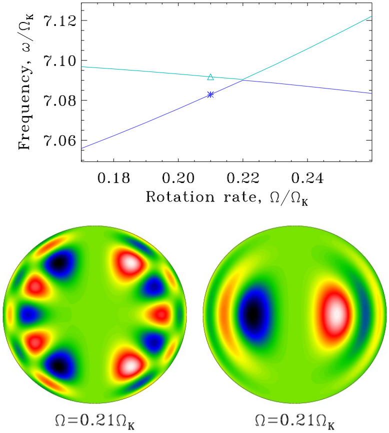


Higher ℓ and $|m|$ values

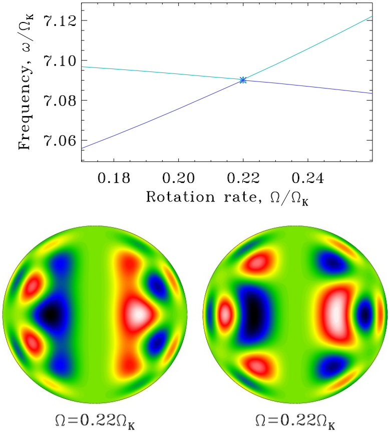


Avoided crossings



- mixing of two coupled modes with close frequencies
- makes mode classification more difficult
- causes frequency deviations

Avoided crossings



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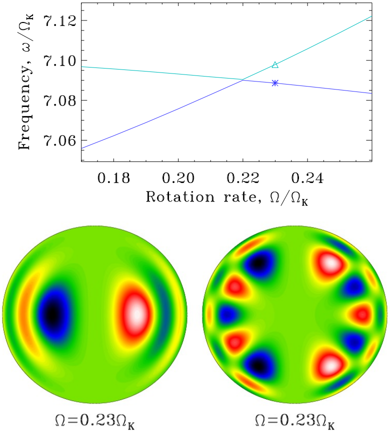

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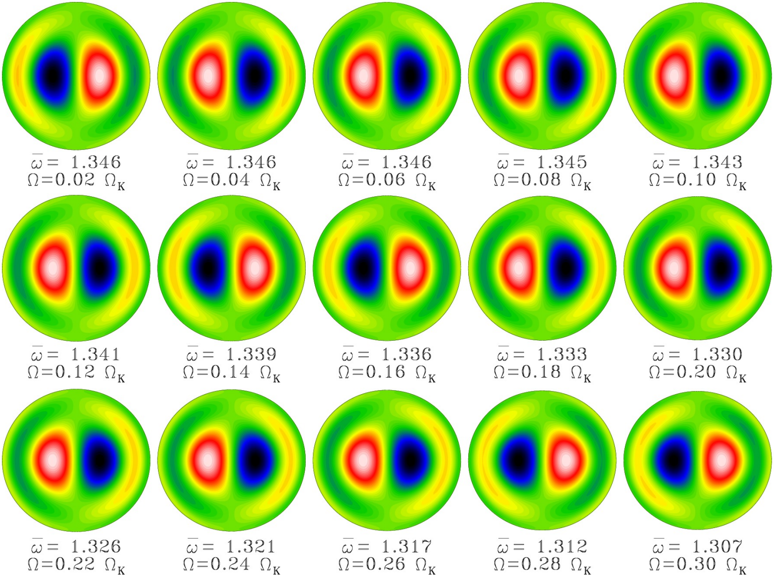
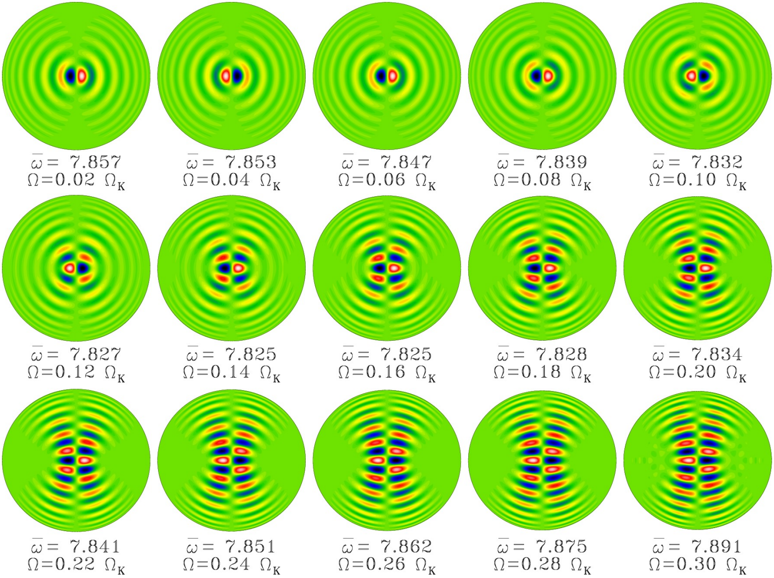
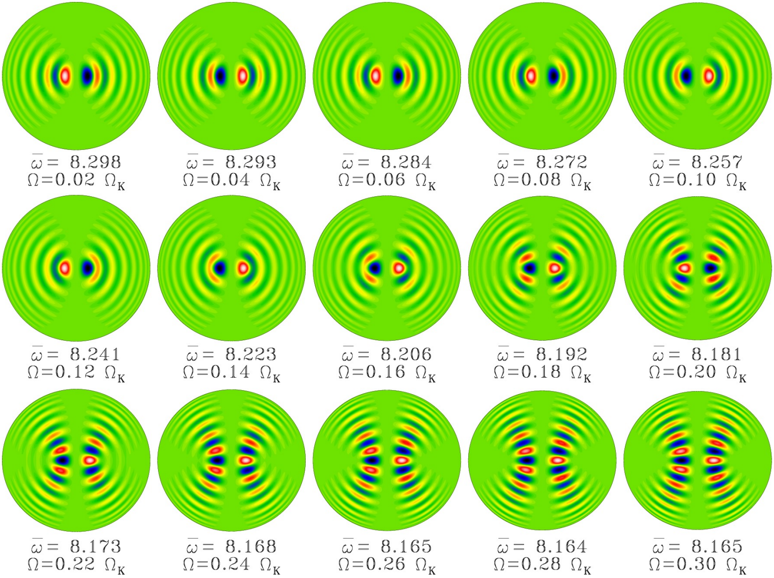
Mode transformation



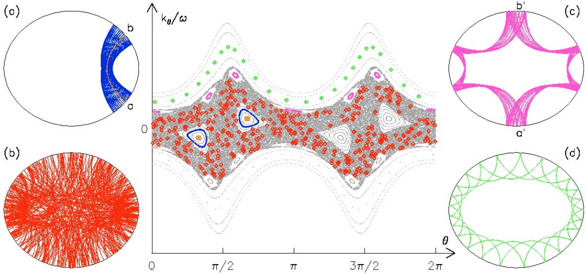
Mode transformation



Mode transformation

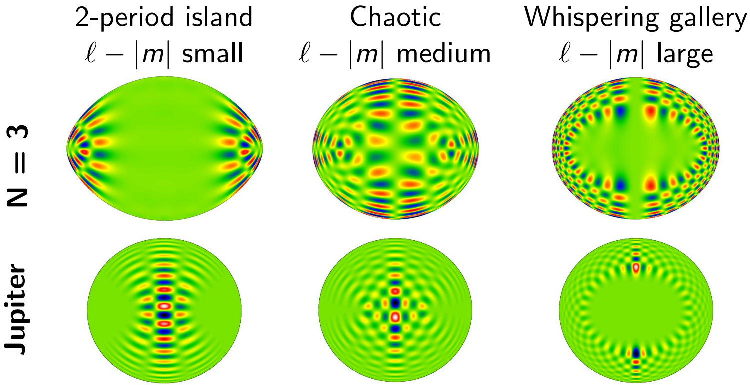


New mode classification

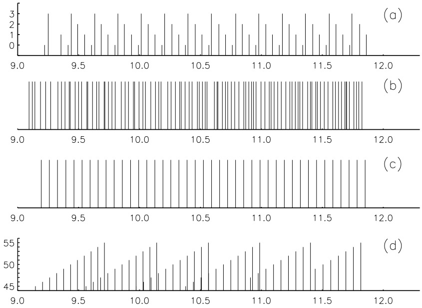


Lignières & Georgeot, 2009

- the study of ray dynamics reveals different classes of modes (Lignières & Georgeot, 2008, 2009)



- mode classification also applies to Jupiter
- some island and chaotic modes probe the centre



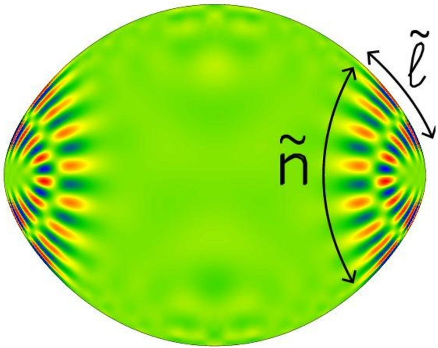
Lignières & Georgeot, 2009

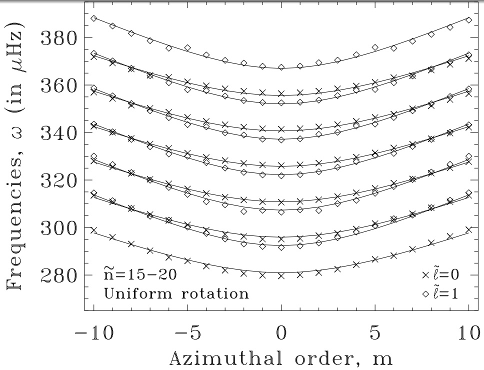
- each class of modes has it's own frequency organisation
  (a) 2-period island modes
  (b) chaotic modes
  (c) 6-period island modes
  (d) whispering gallery modes

2-period island modes

- most visible class of regular modes
- characterised by a different set of quantum numbers, $(\tilde{n}, \tilde{\ell}, \tilde{m})$:

$$
\begin{aligned}
\tilde{n} &= 2n + \varepsilon \\
\tilde{\ell} &= \frac{\ell - |m| - \varepsilon}{2} \\
\tilde{m} &= m \\
\varepsilon &= \ell + m \text{ modulo } 2
\end{aligned}
$$



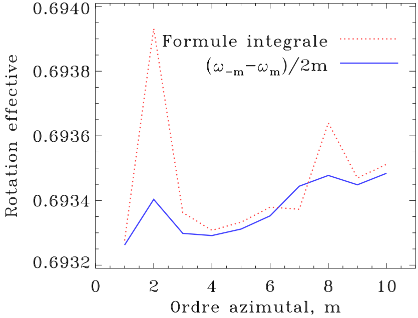


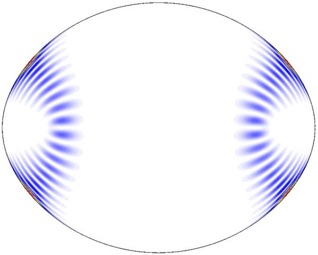
Reese et al., 2009

$$\omega_{\tilde{n},\tilde{\ell},\tilde{m}} \simeq \tilde{n}\Delta_{\tilde{n}} + D_{\tilde{m}}(\tilde{\ell})\sqrt{\tilde{m}^2+\mu(\tilde{\ell})} - \tilde{m}\Omega + \alpha(\tilde{\ell})$$

- $\Delta_{\tilde{n}}$ and $\Delta_{\tilde{\ell}} = \omega_{\tilde{\ell}+1} - \omega_{\tilde{\ell}}$ can be calculated from travel time integrals (Lignières & Georgeot, 2008, 2009, Pasek et al. 2011, 2012)

Probing the rotation profile

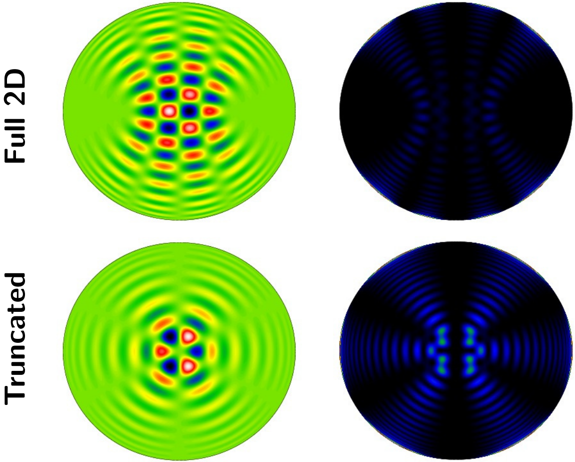




$$\frac{\nu_{-m}-\nu_m}{2m} \simeq \frac{\Omega^{\mathrm{eff}}_{m}+\Omega^{\mathrm{eff}}_{-m}}{2} + \frac{\mathcal{C}_m+\mathcal{C}_{-m}}{2}$$

$$\Omega_{\mathrm{eff}} = \frac{\int_V \Omega\rho_o\|\vec{\xi}\|^2 dV}{\int_V \rho_o\|\vec{\xi}\|^2 dV} \qquad \mathcal{C} = \frac{i}{m}\,\frac{\int_V \rho_o\vec{\Omega}\cdot\left(\vec{\xi}^*\times\vec{\xi}\right)dV}{\int_V \rho_o\|\vec{\xi}\|^2 dV}$$

Probing the rotation profile



- need for full 2D calculations

Conclusion

Rapid rotation

- many additional phenomena
  - avoided crossings, new mode classification, geometry ...
- need to fully include the effects of rotation in Jupiter's pulsations
  - only a 2D approach reproduces correctly the frequencies and the geometry

Exciting prospects

- unlike in stars, modes can be identified in Jupiter
  - possibility of carrying seismology in a rapidly rotating object
  - address interesting science questions (rotation profile, size of the core)